Supplementary Materials for:

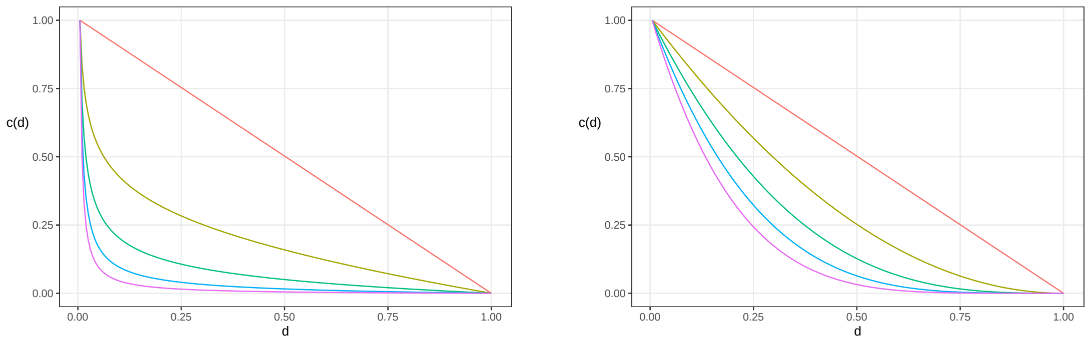
Siting Noxious Facilities: Efficiency and Majority Rule Decisions

Timothy L. Hamilton* and Amit Eynan†

Contents

*University of Richmond; thamilt2@richmond.edu (corresponding author)
†University of Richmond; aeynan@richmond.edu

A: Disamenity Cost Function



(a) $\delta \in [0, -1]; \gamma = 1$

(b) $\delta = 0; \gamma \in [1, 5]$

Figure A.1: Disamenity Cost Function.

B: Derivation of Mean Costs

Normalizing the maximum distance in a locality to 1, mean costs are found by integrating costs (as a function of distance) over the density function of distance,

$$\bar{c} = \int_0^1 x^\gamma (1-x)^\delta x^{\alpha-1} (1-x)^{\beta-1} \frac{1}{B(1;\alpha,\beta)} \, \mathrm{d}x \tag{B.1}$$

$$= \frac{1}{B(1;\alpha,\beta)} \int_0^1 x^{\alpha+\gamma-1} (1-x)^{\beta+\delta-1} \, \mathrm{d}x. \tag{B.2}$$

The final integral is simply the definition of the beta function, $B(1; \alpha+\gamma, \beta+\delta)$, so that

$$\bar{c} = \frac{B(1;\alpha+\gamma,\beta+\delta)}{B(1;\alpha,\beta)}. \tag{B.3}$$

C: Mean-Cost Voter

With a linear disamenity cost function, the voter at the mean distance will suffer mean costs.

$$
\begin{aligned}
c(\bar{d}) &= 1 - \bar{d} && \text{(C.1)} \\
&= 1 - \int_0^1 d f(d; \alpha, \beta) \mathrm{d}d && \text{(C.2)} \\
&= \int_0^1 f(d; \alpha, \beta) \mathrm{d}d - \int_0^1 d f(d; \alpha, \beta) \mathrm{d}d && \text{(C.3)} \\
&= \int_0^1 (1 - d) f(d; \alpha, \beta) \mathrm{d}d && \text{(C.4)} \\
&= \int_0^1 c(d) f(d; \alpha, \beta) \mathrm{d}d && \text{(C.5)} \\
&= \bar{c} && \text{(C.6)}
\end{aligned}
$$

D: Efficient Supermajority With Linear Costs

Mean distance from the Beta distribution is $\bar{d} = \frac{\alpha}{\alpha+\beta}$. To find the efficient supermajority for the specified parameter values, we evaluate the cumulative distribution function at this average distance,

$$F(\bar{d};\alpha,\beta) = \frac{B(\bar{d};\alpha,\beta)}{B(\alpha,\beta)} = \frac{\int_0^{\bar{d}} t^{\alpha-1}(1-t)^{\beta-1}\ \mathrm{d}t}{\int_0^1 t^{\alpha-1}(1-t)^{\beta-1}\ \mathrm{d}t} = x^{\alpha}, \tag{D.1}$$

and calculate an efficient supermajority $M = 1 - F(\bar{d};\alpha,\beta)$.

D.1 $\alpha \in [2,\infty], \beta = 1$

For $\beta = 1$, Equation (D.1) reduces to $\bar{d}^{\alpha}$ and the solution for the efficient supermajority is $M = 1 - \left(\frac{\alpha}{\alpha+1}\right)^{\alpha}$. For $\alpha = 2$, $M = \frac{5}{9}$. To calculate M for cases of populations concentrated away from the facility, we take the limit of M as α goes to infinity and $\beta = 1$:

$$\lim_{\alpha\to\infty} \left(\frac{\alpha}{1+\alpha}\right)^{\alpha} = \lim_{\alpha\to\infty} \frac{1}{\left(1+\frac{1}{\alpha}\right)^{\alpha}} = \frac{1}{e}. \tag{D.2}$$

Therefore,

$$\lim_{\alpha\to\infty} M = 1 - \frac{1}{e} \approx 0.6321. \tag{D.3}$$

D.2 $\alpha \in (3,\infty], \beta = 3$

For $\beta = 3$, Equation (D.1) can be evaluated at mean costs $\bar{d} = \frac{\alpha}{\alpha+3}$ as

$$M = 1 - \frac{1}{2}\left(\frac{\alpha}{\alpha+3}\right)^{\alpha}\left(\frac{18+39\alpha+17\alpha^2}{(\alpha+3)^2}\right). \tag{D.4}$$

For $\alpha = 3$, $M = 0.5$. We then take the limit of this expression as α approaches infinity. Define the variable L such that

$$\ln L = \lim_{\alpha\to\infty} \ln\left(\frac{\alpha}{\alpha+3}\right)^{\alpha} = \lim_{\alpha\to\infty} \alpha \ln\left(\frac{\alpha}{\alpha+3}\right) = \lim_{\alpha\to\infty} \frac{3\ln\left(\frac{1}{1+\frac{3}{\alpha}}\right)}{\frac{3}{\alpha}}. \tag{D.5}$$

Define the variable $t = \frac{3}{\alpha}$, with $\lim_{\alpha\to\infty} t = 0$, and substitute into the above equation so that we are now interested in

$$\lim_{t\to 0} \frac{3\left(-\ln(1+t)\right)}{t} = \lim_{t\to 0} \frac{-3}{1+t} = -3. \tag{D.6}$$

Furthermore,

$$\lim_{\alpha\to\infty} \left(\frac{18+39\alpha+17\alpha^2}{(\alpha+3)^2}\right) = 17. \tag{D.7}$$

Hence,

$$\lim_{\alpha\to\infty} M = 1 - \left(\frac{1}{2}\right)\left(e^{-3}\right)(17) = 0.5768. \tag{D.8}$$

E: Estimating Beta Distribution Parameters

Given a set of observed voter distances $d = \{d_1, ...d_n\}$, the log-likelihood function for the Beta distribution parameters (α, β) is

$$(\alpha - 1)\sum_{i=1}^{n} \ln(d_i) + (\beta - 1)\sum_{i=1}^{n} \ln(1 - d_i) - n \ln B(\alpha, \beta), \tag{E.1}$$

where $B(\alpha, \beta)$ is the beta function,

$$B(\alpha, \beta) = \int_0^1 t^{\alpha-1}(1-t)^{\beta-1} dt \tag{E.2}$$

There is no closed-form solution that maximizes Equation (E.1), so numerical methods are required. However, most software packages are equipped with functions to estimate parameters from observed data (see the `fitdistr` function from the `MASS` package in *R*; `UNIVARIATE` procedure in *SAS*; or the `bayesmh` command in *STATA*). Parameteres α and β can alternatively be estimated using method of moments estimation, though such estimates are less efficient than maximum likelihood estimates. Method of moment estimates for the Beta distribution are

$$\hat{\alpha} = \bar{d}\left(\frac{\bar{d}}{s_d^2} - 1\right) \tag{E.3}$$

$$\hat{\beta} = (1 - \bar{d})\left(\frac{\bar{d}}{s_d^2} - 1\right), \tag{E.4}$$

where $\bar{d}$ is sample mean of d and s_d^2 is the sample variance of d.