The Analytics of Technology News Shocks*

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Abstract

This paper constructs several models in which, unlike the standard neoclassical growth model, positive news about future technology generates an increase in current consumption, hours and investment. These models are said to exhibit procyclical news shocks. We find that all models that exhibit procyclical news shocks in our paper have two commonalities. There are mechanisms to ensure that: (I) consumption does not crowd out investment, or vice versa; (II) the benefit of forgoing leisure in response to news shocks outweighs the cost. Among the models we consider, we believe, one model holds the greatest potential for explaining procyclical news shocks. Its critical assumption is that news of the future technology also illuminates the nature of this technology. This illumination in turn permits economic actors to invest in capital that is forward-compatible, i.e. adapted to the new technology. On the technical side, our paper reintroduces the Laplace transform as a tool for studying dynamic economies analytically. Using Laplace transforms we are able to study and prove results about the full dynamics of the model in response to news shocks.

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1 Introduction

The optimal response of aggregate consumption, investment and hours in the neoclassical growth model to an unanticipated permanent (or near permanent) technology increase is well-understood. For most specifications used by researchers, all three variables increase.[1] A technology improvement increases capital's efficiency; thus, the desired capital stock increases. The increase in the actual capital stock towards its desired level is achieved by greater investment. Importantly, greater investment need not come at the cost of a drop in consumption. Rather, since the technology improvement shifts out the production frontier immediately, creating additional consumption and investment is feasible. Moreover, an hours increase is optimal because a higher marginal product of labor induces a substitution effect away from leisure that outweighs the wealth effect, which pushes in the opposite direction.

Next, consider the standard growth model's response to news of a future technology increase. The responses of these variables and the incentives that drive these responses are different. In the standard model, all three variables will not increase. Typically, labor falls upon the arrival of the news. The above-described wealth effect on leisure is operative; however, there is no offsetting substitution effect because the technology increase has not materialized immediately.

With a labor decline, the only way consumption can increase in response to the news is if investment falls. An investment decline is optimal because there is incentive to delay building additional capital stock until technology actually increases. Thus, in the standard model, positive news about future technology can cause a decline in labor and investment, and an increase in consumption (see Figure 1).[2,3]

[Insert Figure 1 here]

[1] Campbell [11] establishes this by simulation using several functional forms of preferences and model parameterizations. He does provide cases where, when preferences are non-separable in consumption and labor, consumption declines in response to technology shocks.

[2] An alternative, but equaling puzzling, response to good news about future technology is that labor hours increase, while consumption declines. This occurs for a small region of growth model's parameter space.

[3] There is some support for procyclical news shocks in U.S. data. Systematic empirical work supporting the news shock explanation includes Schmitt-Grohe and Uribe [29] and Beaudry and Portier [6]. The former estimate a business cycle model with news (anticipated) and current (unanticipated) shocks and find that news shocks explain a greater fraction of output volatility than current shocks. The latter estimate that the component of innovations to stock prices, not correlated with current productivity, is correlated with expected future productivity. Barsky and Sims [2], using a different identification scheme, deliver an opposite result (i.e. news shocks are not procyclical). Other relevant empirical research supporting this explanation includes Beaudry, Dupaigne,and Portier [3], Beaudry and Lucke [4], and Khan and Tsoukalas [22] as well as Leeper, Walker and Yang [24].

This paper studies variants on the standard model that are capable of generating procyclical responses. Each model has mechanism(s) to ensure that: (I) consumption does not crowd out investment, or vice versa; (II) the return to forgoing leisure is sufficiently high.

In our first model, we modify the neoclassical production function to have a convex production frontier between consumption and investment, i.e. production complementarity. In the standard model, the marginal rate of transformation between consumption and investment is fixed at one. Here, this marginal rate of transformation depends upon the consumption-investment ratio. We provide both sufficient and necessary conditions for the model to exhibit procyclical technology news shocks. These conditions depend upon the values of the model's underlying parameters.

With a convex production frontier, greater consumption (investment) increases the marginal product of labor towards the production of investment (consumption). This effect tends to increase investment and labor upon the arrival of the news if there is a consumption boom.[4] This achieves Condition I: consumption does not crowd out investment.

Consumption-investment complementarity causes the two variables to comove; however, it is possible that the two variables might fall rather than increase in response to positive news. In this case, the planner takes leisure over consumption (and capital accumulation) in the short run. This arises if there is too much curvature in the utility function because, in this case, the intertemporal smoothing motive for leisure becomes too strong. As such, there must be sufficiently low curvature in order that hours, consumption and investment comove procyclically in response to positive news. Restricting the curvature to be low ensures our Condition II, that the relative benefit of forgoing leisure is sufficiently strong.

Our second model contains a preference-based mechanism for generating procyclical technology news shocks.[5] Here, we assume a preference externality such that the marginal disutility of own hours worked is falling in the economy-wide average hours worked.[6] The preference externality acts, from each household's perspective, as a preference shock that expands each household's willingness to supply labor. This endogenous labor supply mechanism directly decreases the household utility cost of forgoing leisure (helping satisfy Condition II) and the increase in labor expands output sufficiently for both consumption and investment to increase (satisfying Condition I). The addition of investment adjustment costs also help with

[4]In an extreme but illustrative case, if consumption and investment are produced in a Leontief manner, then consumption and investment comove perfectly.

[5]That is, the production side of this second model is neoclassical.

[6]Despite being non-standard, the preferences are consistent with balanced growth and both consumption and leisure are normal goods from each household's perspective.

satisfying condition II in this model.

After proving a theorem for each model to exhibit procyclical news shocks, we conduct a quantitative analysis. Here, we find a drawback with both models. Each generates quantitatively small consumption booms in response to positive news. Consumption is nearly acyclical.

As a result of the near acyclicality of consumption, we develop two distinct extensions of the production complementarities model. First, we replace balanced-growth preferences with Greenwood, Hercowitz and Huffman [16] (hereafter GHH) preferences. Second, we add investment adjustment costs to the model.[7] With either of these additions we are able to generate quantitatively larger consumption booms and also support procyclical news shocks with greater curvature in the utility function.

In our view, the greatest promise for explaining the phenomenon is a situation where Condition II, i.e. a strong return to forgoing leisure, is achieved because there is a benefit to starting investing early beyond that inherent in the basic neoclassical model. In our view, it is more plausible that this benefit is production-based rather than preference-based. To this end, we construct the final model of the paper, which introduces the concept of "forward-compatible investment."

The starting point for the forward-compatibility model is a neoclassical economy with investment-specific technology (IST) shocks and production complementarity. In addition, we assume investment made between the news arrival and the actual IST increase is partially forward-compatible. By partially forward-compatible we mean that part of the investment made between the news arrival and the actual IST increase will experience an increase in efficiency when the actual IST increase occurs. This causes an investment boom upon the arrival of the news as the social planner builds capital in anticipation of the IST increase, even though there is no immediate technological improvement. This "preparatory-phase" investment is optimal because it allows the planner to smooth consumption while accumulating capital towards the eventual higher steady state.

It is beneficial, in our view, for a modification of the neoclassical model to be as minimal as possible. This makes it less likely that the resulting model does violence to the existing theory of unanticipated shocks. To this end, we note that adding forward compatibility of investment is not a significant departure from the neoclassical model along two key dimensions. First, our solution adds no new state variables to the neoclassical model. This

[7] It is important to note, as will be shown through the series of models in the paper, that GHH preferences and/or investment adjustment costs are neither necessary nor sufficient for generating procyclical news shocks.

is useful because the lack of any new state variables allows for a thorough examination of the mechanism in a two dimensional space; each additional state variable would add two more dimensions to the model (a state and a co-state). Second, our model collapses to the standard neoclassical model with production complementarities when the exogenous driving process is a contemporaneous shock. That is, the forward compatibility does not operate when the business cycle is driven by contemporaneous shocks. This allows our model to be directly comparable to the basic neoclassical model.

To allow for a complete theoretical analysis, we use a continuous time model. A continuous time framework allows us to use the method of Laplace transforms. The Laplace transform is useful for studying linear differential equations with constant coefficients and exogenous (non-homogeneous) terms with discontinuities.[8] Once we log-linearize the growth model, our differential equations take exactly this form. The discontinuity is present because of the forecastable jump in future technology.

There are several existing papers on news-driven cycles in dynamic general equilibrium models. Beaudry and Portier [7] study the difficulty that the neoclassical model has in exhibiting procyclical news shocks. They provide a necessary condition on production sets for news shocks to create consumption and investment comovement. Importantly, they observe that many production technologies used in macro do not satisfy this necessary condition. Also, they calibrate a model with one feature capable of generating news-driven cycles: production complementarities of the kind studied in our paper. Their theoretical work does not explore the analytics underlying the dynamics of news-driven cycles.

Beaudry and Portier [5] generate news-driven cycles by modeling final consumption as a function of non-durables and the capital stock. Jaimovich and Rebelo [19] generate large responses to news shocks, by adding variable capital utilization and two dynamic state variables to the neoclassical model: lagged investment through adjustment costs and time non-separable preferences. Christiano et. al. [13] use investment adjustment costs and habit persistence to generate news-driven business cycles. Wang [34] analyzes and compares three existing models generating procyclical news shocks via a labor market diagram. This graphical analysis is very useful for understanding the static relationships in these models, but not as useful for understanding the models' dynamics. In the same paper, Wang develops a model where an endogenous markup resolves this comovement puzzle.

Nah [27] uses production complementarities and financial frictions to support procyclical

[8] Several introductory textbooks on differential equations describe the Laplace transform, including Boyce and DiPrima [10] and Tenenbaum and Pollard [31]. Early applications of the transform to economics include Judd [20] [21].

news shocks. Chen and Song [12] and Walentin [33] also use financial frictions in their models to help support procyclical news shocks. Gunn and Johri [17] and Qureshi [28] each develop a learning-by-doing model. In response to news about future technological improvement, forward-looking agents increase hours worked and investment immediately in order to build up their stock of knowledge. This amplifies the benefit of the future technology increase. Gunn and Johri [17] show that learning-by-doing combined with variable capital utilization can generate procyclical stock prices. Qureshi [28] shows that learning-by-doing along with an intratemporal adjustment cost can generate sectoral comovement in response to news about neutral and sector-specific technologies. Comin, Gertler and Santacreu [14] develop a model with shocks to the number of new ideas capable of increasing the efficiency of capital and labor. However, resources must be allocated to transform ideas into actual technologies. Tsai [32] uses variable capital utilization and preferences designed to minimize wealth effects on labor supply, along with fixed costs to adopt new vintages of capital. The latter feature in his model has a feel very similar to the forward-compatibility assumption in Section 5 of our paper. A more detailed review of some of these mechanisms and literature can be found in Beaudry and Portier [8] and Lorenzoni [25].

Our paper differs from the above numerical/simulation-based results, along with the theoretical results in Beaudry and Portier [7], in that, to the best of our knowledge, ours is the first paper to study the full dynamics of news shocks analytically. This allows us to shed light on how news shocks in general work.

In the next section, we describe the production complementarity model, characterize its optimal allocation and provide conditions under which the model supports procyclical technology news shocks. In Section 3, we do the analogous examination of a preference-based mechanism capable of supporting these type of news shocks. Section 4 analyzes quantitative and calibration issues with the baseline production complementary model, and develops modifications to address these issues. Section 5 studies a model of forward-compatible investment, and Section 6 concludes.

2 Procyclical News Shocks via Production Complementarity

Consider the following variant of the neoclassical growth model.

The Model

Consumption, $C(t)$, and investment, $I(t)$ are produced according to:

$$F[C(t), I(t)] = K(t)^{\alpha} (A(t) N(t))^{1-\alpha} \tag{1}$$

where $K(t)$ and $N(t)$ represent capital and hours respectively. Assume, as in Huffman & Wynne [18], that

$$F(C, I) \equiv [\theta C^{\upsilon} + (1-\theta) I^{\upsilon}]^{1/\upsilon} \tag{2}$$

where $\alpha, \theta \in (0,1), t \in [0, \infty]$ and $\upsilon \geq 1$.

Our sole departure from the neoclassical model pertains to the definition of $F(C, I)$, which represents the production possibility frontier for consumption and investment given the amounts of inputs. We allow for the possibility of complementarities between the production of consumption and investment goods. If $\upsilon = 1$, the equation collapses to the standard model. As υ increases, the complementarity between the production of the two goods increases. If $\upsilon = \infty$, the production frontier takes a Leontief form.

We can interpret υ as measuring the factor substitutability between the consumption and investment sectors of a more general model. In the basic neoclassical model ($\upsilon = 1$), factors are equally productive in both the consumption and investment sectors. As a result, the relative price of consumption to investment remains constant irrespective of how much resources are being devoted to producing consumption versus investment. In our model, factors are not equally productive in both sectors. As υ increases, a factor productive in one sector is less and less productive in the other sector. For example, a worker that produces goods in the consumption sector, when moved to the investment sector, will become less productive.[9]

The law of motion of capital is:

$$\dot{K}(t) = I(t) - \delta K(t) \tag{3}$$

where δ is the capital depreciation rate.

[9] An alternative mechanism to generate a bowed-out production frontier in equation (2) is intersectoral adjustment costs. Suppose that in the planner problem we replace $F(C, I)$ with

$$F^{ADJ}(C, I) \equiv (C + I) \left[1 + \frac{\psi_Y}{2} \left(\frac{1}{\theta} \frac{C}{I} - 1 \right)^2 \right]$$

Given the above form, the intersectoral adjustment cost and production complementarity models are isomorphic up to the log-linearization.

A social planner ranks utility over different consumption and hours time paths using:

$$U = (1-\sigma)^{-1} \int_0^\infty e^{-\rho t} \left[C(t) \exp(-N(t))\right]^{1-\sigma} dt \tag{4}$$

where $\sigma \geq 0$ is the curvature parameter in the utility function and $\rho > 0$ is the discount rate.[10]

Next, a *positive technology news shock* is an increase in technology arriving at time T that becomes anticipated at time zero. Thus, at time zero, the perfect foresight time path for technology becomes:

$$A(t) = \begin{cases} \bar{A} & \text{for } t \in [0, T) \\ \bar{A} + \epsilon & t \geq T \end{cases} \tag{5}$$

where $\bar{A}$ denotes the initial steady-state technology level. A contemporaneous (or unanticipated) technology shock corresponds to the case when $T = 0$.

It is useful to define the following

Definition 1. *A model* ***exhibits procyclical technology news shocks*** *if an anticipated increase in future technology (i.e. a positive technology news shock) leads to an increase in current consumption, investment and hours for all* $t < T$.

Because we only study technology shocks in this paper, we will often omit the word 'technology' when referring to technology news shocks.

The Planning Problem and Its Solution

The social planner chooses C, I, K and N to maximize U subject to (1), (2) and (3), taking as given the initial condition $K(0)$ and time path of technology given by (5).

The current value Hamiltonian associated with the problem is:

$$H = (1-\sigma)^{-1} C^{1-\sigma} \exp\left[-(1-\sigma) N\right] + \Lambda (I - \delta K) + \Phi \left(K^\alpha (AN)^{1-\alpha} - F(C, I)\right)$$

The first-order necessary conditions at an interior solution satisfy the following:

$$-\frac{U_N}{U_C} = (1-\alpha) \frac{F}{N} (F_C)^{-1} \tag{6}$$

[10]These preferences exhibit balanced growth. Holding fixed hours, σ is the inverse of the intertemporal elasticity of substitution for consumption.

$$\frac{U_C}{\Lambda} = \frac{F_C}{F_I} \tag{7}$$

$$\frac{\dot{\Lambda}}{\Lambda} - \rho = \delta - \alpha \frac{F}{K} (F_I)^{-1} \tag{8}$$

along with an initial condition on capital and a transversality condition.

Equation (6) is the intratemporal Euler equation between consumption and labor hours, equation (7) is the intratemporal Euler equation between consumption and investment, and equation (8) is the optimal capital accumulation equation. All of these equations are similar to their neoclassical counterparts. The sole difference is that $F_C = F_I = 1$ in the basic neoclassical model. With production complementarities F_C and F_I change with level of consumption and investment.

Log-linearizing these equations,[11] we have the following three optimality conditions:

$$n = \upsilon s_I (i - c) \tag{9}$$

$$(\upsilon - 1)(i - c) = \lambda - (-\sigma c - zn) \tag{10}$$

$$\dot{\lambda} = -(\rho + \delta)[\upsilon (1 - s_I)(c - i) + i - k] \tag{11}$$

where $z = (1 - \sigma)(1 - \alpha) / (1 - s_I)$ and $s_I = (\alpha\delta) / (\rho + \delta)$.

Equation (9) ensures an efficient labor allocation. As consumption rises, the marginal utility of consumption falls and the planner increases leisure. An increase in investment shifts out labor supply.

Equation (10) ensures an efficient consumption-investment split. The left-hand side is the price of investment in units of consumption. Because of complementarity, investment becomes more expensive when production of consumption is relatively low. The right-hand side is the marginal utility of investment minus the marginal utility of consumption.

Equation (11) is the intertemporal consumption Euler equation. It differs from the neoclassical model in that λ is not simply the derivative of the marginal utility of consumption. There is an additional relative price effect because of the convex production frontier.

The two resource constraints and the definition of output are given by:

$$(1 - s_I) c + s_I i = \alpha k + (1 - \alpha)(a + n) \tag{12}$$

[11] The system is log-linearized around the initial steady-state, which is consistent with the constant technology $\bar{A}$. A lower case letter denotes the log deviation of that variable from its upper case counterpart.

$$\dot{k} = \delta\,(i - k) \tag{13}$$

$$y = \alpha k + (1-\alpha)\,(a + n) \tag{14}$$

Equation (12) is the static resource constraint. Equation (13) is the law of motion for capital. Equation (14) gives the definition of output.

News-Driven Business Cycles

We next study under what conditions the model exhibits procyclical news shocks. We subdivide our proof into first establishing the procyclicality and comovement between the variables at time zero ($t = 0$), and then the procyclicality and comovement between variables for time $t \in (0, T)$. The latter results for $t \in (0, T)$ distinguish our theoretical work from others.

Lemma 1. *Suppose the economy experiences a positive technology news shock. Consumption, investment and hours will comove at time zero if and only if* $\upsilon > \upsilon^* = (1-\alpha)^{-1}$

Proof. All proofs are contained in Appendix A. □

Lemma 2. *Suppose the economy experiences a positive technology news shock. Consumption, investment and hours will comove procyclically, with respect to the expectations of future technology, at time zero if and only if* $\upsilon > \upsilon^*$ *and* $\lambda(0) > 0$.

[Insert Figure 2 here]

The intuition for Lemmas 1 and 2 can be understood using Figure 2. Figure 2 plots the solution to the static consumption-investment decision holding fixed the marginal utility of investment. It plots this for the cases with and without production complementarity.

Substituting out the optimal hours from the production equation (12), we have:

$$\alpha k + (1-\alpha)\,a = \left(1 - \phi_I^{PC}\right) c + \phi_I^{PC} i \tag{15}$$

where $\phi_I^{PC} = (1 - \upsilon\,(1-\alpha))\,s_I$.

This is plotted as L_1 in Figure 2(a) and Figure 2(b). In the absence of production complementarity, this is a downward-sloping line, as seen in Figure 2(a).[12] Intuitively, when

[12]The figure assumes that the economy is at its steady-state associated with $\bar{A}$ at time zero.

consumption rises, hours cannot optimally rise because leisure is a normal good; therefore, investment must fall. With sufficiently strong complementarity, i.e. $\upsilon > (1-\alpha)^{-1}$, L_1 is upward sloping as seen in panel (b).

This occurs because, with strong complementarity, an increase in investment raises the marginal product of labor in producing the consumption good. This higher marginal product of labor implies that both hours and consumption can increase. An investment decline, on the other hand, will go hand-in-hand with a reduction in consumption.

Next, consider L_2, the consumption-investment Euler equation with optimal hours substituted out:

$$\gamma_I^{PC} i - \left(\sigma + \gamma_I^{PC}\right) c = \lambda \tag{16}$$

where $\gamma_I^{PC} = (\upsilon - 1) - [\upsilon (1-\alpha)(1-\sigma) s_I] / (1 - s_I)$.

In general, the slope of L_2 can be either positive or negative. The slope depends most crucially on ν. To generate procyclical news shocks, ν must be large. To understand why L_2 can be upward-sloping, consider the consumption-leisure Euler equation. For the assumed utility function, consumption equals the real wage (ignoring complementarity in production). Because the real wage is simply labor's share in production, hours are a linear function of the output-consumption ratio. Thus, if the planner decided to increase investment relative to consumption, hours worked increases. This is seen in equation (9). Note that adding production complementarity (i.e. setting $\nu > 1$) increases the hours effect because it increases the marginal product of hours in producing the consumption good.

Next, suppose we consider an increase in the marginal utility of investment, λ, at time zero.[13] First, an increase in $\lambda(0)$ does not shift L_1.Second, an increase in $\lambda(0)$ induces a shift leftward of L_2 either with or without complementarity. As the marginal utility of investment increases, the social planner shifts away from consumption for a given level of investment. Even though L_2 moves in the same direction in either case, the implication for the optimal investment-consumption pair is different between the two cases. Because L_1 is downward sloping without complementarity, investment rises but consumption falls; however, L_1 is upward sloping with complementarity and both investment and consumption rise. Intuitively, the increase in investment raises the marginal product of labor towards consumption when there is production complementarity. The fall in the relative price of consumption leads the planner to increase hours worked.

[13]For now, we take the increase in $\lambda(0)$ as given. Later, starting with Lemma 4, we provide a condition for which time zero news of a technology increase at time T results in an increase in $\lambda(0)$.

Lemma 3. *Suppose the economy experiences a positive technology news shock. Also, assume that $\upsilon > \upsilon^*$. Consumption, investment and hours will comove procyclically for all time $t < T$ if $\forall t < T$, $\dot{\lambda} \geq 0$ and $\dot{k} \geq 0$*

[Insert Figure 3 here]

Equation (15) implies that if k is increasing over time, then production (at the optimal level of labor) also increases over time holding a at its steady state level of zero. As such, $\dot{k} \geq 0$ causes L_1 to progressively shift rightward, shifting out the production frontier. When $\dot{\lambda} \geq 0$, the marginal utility of investment is increasing over time. This causes the planner to shift production away from consumption into investment. This results in a leftward shift in L_2. As illustrated in Figure 3, these two effects cause consumption and investment to continue increasing for all time $t < T$.

We have thus far studied what happens to c, i and n when $\lambda(0)$ increases in response to good news. We now provide conditions on parameters under which this increase in $\lambda(0)$ obtains.

The log-linearized dynamic system is:[14]

$$\begin{bmatrix} \dot{\lambda}(t) \\ \dot{k}(t) \end{bmatrix} = \begin{bmatrix} \Gamma^{PC}_{\lambda,\lambda} & \Gamma^{PC}_{\lambda,k} \\ \Gamma^{PC}_{k,\lambda} & \Gamma^{PC}_{k,k} \end{bmatrix} \begin{bmatrix} \lambda(t) \\ k(t) \end{bmatrix} + \begin{bmatrix} b^{PC}_{\lambda,a} \\ b^{PC}_{k,a} \end{bmatrix} a(t) \tag{17}$$

In the presence of a news shock, there is a discontinuous forcing term in the dynamic system. In equation (17), $a(t)$ is a step function which takes on the value zero for all time $t < T$ and a value of $ln(1.01)$ for all time $t \geq T$.

The presence of a step function means our dynamic system poses a challenge. We can no longer apply standard techniques to solve this differential equation system. Laplace transforms lend themselves nicely here. The Laplace transform of a function is given as

$$F(s) = \mathcal{L}[f(t)] = \int_0^\infty e^{-st} f(t)\, dt$$

Using this transform, we can map our problem from the time domain (t) into the frequency domain (s). A more general way to think about Laplace transforms is that they turn a differential equation into an algebraic equation. In our case this means converting a differential equation with a discontinuity to a continuous algebraic equation that can be

[14] The values of $\Gamma^{PC}_{\cdot,\cdot}$ and $b^{PC}_{\cdot,a}$ can be found in Appendix B.

easily manipulated. Equation (17) after applying the transform simply becomes:

$$\begin{bmatrix} \lambda(s) \\ k(s) \end{bmatrix} = \frac{1}{\left(s-\mu_1^{PC}\right)\left(s-\mu_2^{PC}\right)} \begin{bmatrix} s-\Gamma_{k,k}^{PC} & \Gamma_{\lambda,k}^{PC} \\ \Gamma_{k,\lambda}^{PC} & s-\Gamma_{\lambda,\lambda}^{PC} \end{bmatrix} \left\{ \begin{bmatrix} \lambda(0) \\ k(0) \end{bmatrix} + \begin{bmatrix} b_{\lambda,a}^{PC} \\ b_{k,a}^{PC} \end{bmatrix} \frac{1}{s} e^{-sT} \right\} \tag{18}$$

This is an independent linear system of algebraic equations. The lower row of which can be written as:

$$k(s) = \frac{\Gamma_{k,\lambda}^{PC}\lambda(0) + \left(s-\Gamma_{\lambda,\lambda}^{PC}\right)k(0)}{\left(s-\mu_1^{PC}\right)\left(s-\mu_2^{PC}\right)} + \left[\frac{\Gamma_{k,\lambda}^{PC} b_{\lambda,a}^{PC} + \left(s-\Gamma_{\lambda,\lambda}^{PC}\right) b_{k,a}^{PC}}{s\left(s-\mu_1^{PC}\right)\left(s-\mu_2^{PC}\right)}\right] e^{-sT} \tag{19}$$

For our differential equation system this is the solution for k in the frequency domain. The Laplace transform is bijective. Thus, having solved for k we can apply the inverse Laplace transform to return to the time domain. The inverse Laplace transform is given by:

$$f(t) = \mathcal{L}^{-1}\left[F(s)\right] = \frac{1}{2\pi i}\int_{\gamma+i\infty}^{\gamma-i\infty} e^{st} F(s)\, ds$$

Applying the inverse transform to equation (19) and the corresponding $\lambda(s)$ equation we get the time-paths of $k(t)$ and $\lambda(t)$ for our system in the time domain:

$$k(t) = \begin{cases} \frac{\Gamma_{k,\lambda}^{PC}\lambda(0)}{\mu_1-\mu_2} e^{\mu_1 t} + \frac{\Gamma_{k,\lambda}^{PC}\lambda(0)}{\mu_2-\mu_1} e^{\mu_2 t} & \text{for } t \in [0,T) \\ \frac{\Gamma_{k,\lambda}^{PC}\lambda(0)}{\mu_1-\mu_2} e^{\mu_1 t} + \frac{\Gamma_{k,\lambda}^{PC} b_{\lambda,a}^{PC} - \Gamma_{\lambda,\lambda}^{PC} b_{k,a}^{PC}}{\mu_1\mu_2} + \frac{\Gamma_{k,\lambda}^{PC} b_{\lambda,a}^{PC} + \left(\mu_1-\Gamma_{\lambda,\lambda}^{PC}\right) b_{k,a}^{PC}}{\mu_1(\mu_1-\mu_2)} e^{\mu_1(t-T)} & t \geq T \end{cases} \tag{20}$$

$$\lambda(t) = \begin{cases} \frac{\left(\mu_1-\Gamma_{k,k}^{PC}\right)\lambda(0)}{\mu_1-\mu_2} e^{\mu_1 t} + \frac{\left(\mu_2-\Gamma_{k,k}^{PC}\right)\lambda(0)}{\mu_2-\mu_1} e^{\mu_2 t} & \text{for } t \in [0,T) \\ \frac{\left(\mu_1-\Gamma_{k,k}^{PC}\right)\lambda(0)}{\mu_1-\mu_2} e^{\mu_1 t} + \frac{\Gamma_{\lambda,k}^{PC} b_{k,a}^{PC} - \Gamma_{k,k}^{PC} b_{\lambda,a}^{PC}}{\mu_1\mu_2} + \frac{\Gamma_{\lambda,k}^{PC} b_{k,a}^{PC} + \left(\mu_1-\Gamma_{k,k}^{PC}\right) b_{\lambda,a}^{PC}}{\mu_1(\mu_1-\mu_2)} e^{\mu_1(t-T)} & t \geq T \end{cases} \tag{21}$$

where μ_1 and μ_2 are the eigenvalues of the Γ^{PC} matrix.[15] Without loss of generality, let $\mu_1 < 0$ and $\mu_2 > 0$. [16]

The Laplace transform provides a way to solve complicated differential equations with discontinuities such as the one that news shocks introduce to our models. The use of Laplace transforms allows us to analytically study the full dynamics of our system.

The solutions for the time paths of k and λ show that the dynamics of the system

[15] The Appendix B contains the full derivations of (20) and (21)

[16] In Appendix B, we prove that μ_1 and μ_2 are real with one being positive and the other negative.

before time T are being determined not only by the stable eigenvalue, but also the unstable eigenvalue. This is important. Without a role for the unstable eigenvalue, the system would be on a new stable manifold, corresponding to a higher permanent level of $a(t)$, for all $t < T$. Along the stable manifold capital and the shadow value of investment do not comove. This will result in a negative comovement between the variables for $t \in [0, T)$. After time T, the system is on a new stable path.

The above solution has one undetermined variable, $\lambda(0)$. We seek a path for (λ, k) that is not explosive. In order to achieve this, we choose $\lambda(0)$ such that the explosive root μ_2 does not determine the evolution of the system for $t > T$. This ensures that we are on the stable path. Otherwise, the path for k will be explosive. This restriction on $\lambda(0)$ is:

$$\frac{\Gamma_{k,\lambda}^{PC}\lambda(0) + \left(\mu_2 - \Gamma_{\lambda,\lambda}^{PC}\right) k(0)}{(\mu_2 - \mu_1)} = -\frac{\Gamma_{k,\lambda}^{PC} b_{\lambda,a}^{PC} + \left(\mu_2 - \Gamma_{\lambda,\lambda}^{PC}\right) b_{k,a}^{PC}}{\mu_2 (\mu_2 - \mu_1)} e^{-\mu_2 T}$$

Studying the above equation in conjunction with the time paths for k and λ, it can be seen that the discontinuity of $a(t)$ does not cause a discontinuity in the time path of λ or k. Instead, the discontinuity manifests itself as a non-differentiability (kink) at time T.

Using the time paths of k and λ above, along with the restrictions for a stable solution, we have the following lemmas.

Lemma 4. *Suppose the economy experiences a positive technology news shock and* $\upsilon > \upsilon^*$. $\dot{\lambda} \geq 0$ *and* $\dot{k} \geq 0$ $\forall t < T$ *if and only if* $\lambda(0) > 0$.

Lemma 5. *Suppose the economy experiences a positive technology news shock and* $\upsilon > \upsilon^*$. $\lambda(0) > 0$ *if and only if* $\sigma < \sigma^*$, *where* σ^* *solves*

$$\mu_2^{PC} = (\rho + (1-\alpha)\delta)\, \upsilon / (\gamma_I + \sigma)$$

and $\mu_2^{PC} = \mu_2^{PC}(\sigma)$ *is the positive eigenvalue of* Γ^{PC}.

The condition that $\sigma < \sigma^*$ implies that in order to generate procyclical technology news shocks the model requires a relatively low curvature parameter σ.

Lemmas 1 through 5 lead to the following theorem.

Theorem 1. *The production complementarity model exhibits procyclical technology news shock if and only if* $\upsilon > \upsilon^*$ *and* $\sigma < \sigma^*$.

[Insert Figure 4 here]

To better understand this theorem, Figure 4 plots the phase diagram for four cases. For now, we focus on the first three: (a) no production complementarity, (b) complementarity with a high curvature parameter, (c) complementarity with a low curvature parameter. Only case (c) results in procyclical news shocks.

Initially before the news, as seen in in Figure 4(a), the capital-multiplier pair (k_{ss}, λ_{ss}) lies on the initial manifold $\mathcal{M}$ and is at the initial steady state. Upon the time zero news arrival, the multiplier falls below λ_{ss} because there is an immediate consumption boom. Because capital is a predetermined variable, $k_0 = k_{ss}$. After time zero and before T (which is the instant of the technology arrival), the capital stock falls indicating that investment is below the steady-state. The consumption boom comes at the expense of investment. This is one indicator of the comovement problem in the standard model: investment declines upon the arrival of good news.

Note that between time zero and time T, the capital-multiplier pair flow in the opposite direction of the stable manifold. This is due to influence of the explosive root (μ_2) before the technology change occurs. At time T, the model is on the new stable manifold $\mathcal{M}'$ and the system then converges monotonically to the new steady-state. The $\mathcal{M}$ and $\mathcal{M}'$ manifolds are parallel to each other because the technology shock does not change the coefficients multiplying the endogenous variables.

Figure 4(b) contains the phase diagram with production complementarity, but a high curvature parameter. As in the case without complementarity, investment initially falls in response to the news. Investment eventually increases once the technology change actually occurs. At this point, the system is on the manifold and capital converges monotonically to the new higher steady state. The desire for smooth consumption, due to the high curvature parameter, is evident in the path of the multiplier. It jumps downward on impact and then moves monotonically to the new steady state.

Figure 4(c) contains the phase diagram for the case of greatest interest. It represents the case when both conditions of Theorem 1 are satisfied: strong production complementarity and a low curvature parameter. Note that the marginal utility of investment jumps up rather than down in response to the good news.

Both σ and δ play important roles in affecting λ on impact, through equation (10).[17] As a starting point, note first that according to the consumption-hours Euler equation (9), n increases if and only if $i - c$ increases.

[17] The L_2 line is exactly equation (10) once the labor term is substituted out using (9).

Next, rewriting equation (10), we have:

$$\underbrace{\lambda}_{MU_I} = \underbrace{(v-1)\,(i-c)}_{price_I} + \underbrace{\left(-\sigma c - \frac{(1-\sigma)\,(1-\alpha)}{(1-s_I)} n\right)}_{MU_C} \tag{22}$$

First, a good news shock that causes hours to rise is accompanied by an increase in the price of investment (in units of consumption) when $v > 1$. This works to raise the marginal utility of investment. Next, consumption also rises if the news shock is procyclical. The increases in c causes the marginal utility of consumption to fall, which offsets the price effect on λ. This effect is dampened when σ is close to zero. This is a straightforward channel operating in the standard neoclassical model.[18] Intuitively, when σ is close to zero the timing of investment is governed by production efficiency concerns and not a desire to smooth consumption.

Finally, n appears in the MU_C term because consumption and hours are non-separable in the utility function. σ plays a different role in the term pre-multiplying n. Here, it effects the degree of complementarity between n and c in preferences. If $\sigma = 1$, preferences are separable and the n term drops out. If $\sigma < 1$, then leisure and consumption are complements, which puts downward pressure on λ. It must be the case that the effect of σ on c dominates its effect on n.

Thus, a low σ (through a dampened consumption effect) and the production complementarity lead to an overall increase in the shadow value of investment. This makes the return to forgoing leisure in order to produce the investment good high.

Although investment jumps up at time zero, the new steady state must involve $k'_{ss} > k_{ss}$ and $\lambda'_{ss} < \lambda_{ss}$. This occurs in case (c) because the new manifold eventually crosses into the fourth quadrant of the phase space.

It is important to note that the conditions of Theorem 1 are independent of the value of T. As a result, the model preserves the ability to generate procyclical comovements in response to traditional time zero unanticipated shocks and news shocks that change the level of technology for any time T in the future. Many models that can generate procyclical comovements in response to news shocks are sensitive to the value T.

In addition to providing both the necessary and sufficient conditions for solving the news shock puzzle, Theorem 1 provides insight into understanding how news shocks work. We present our understanding via the following observation.

[18] If consumption and hours were separable, then σ is simply the inverse of the intertemporal elasticity of substitution of consumption.

Main Observation *A variant of the neoclassical model will exhibit procyclical technology news shocks if it has one or more features that ensures:*

I. consumption does not crowd out investment, or vice versa, and

II. the benefit to forgoing leisure is sufficiently strong.

[Insert Figure 5 here]

Figure 5 plots how the critical value of the utility curvature parameter, $\sigma*$, changes with the capital share in production and the depreciation rate.[19]

As the capital share increases, the critical value of the utility curvature parameter increases. This is because, as the capital share rises, the benefit to building capital increases, inducing the planner to forgo leisure in order to produce investment. This greater willingness to forgo leisure implies the model can support procyclical news shocks with less reliance on low curvature in utility.

Next, consider the depreciation rate. As δ falls, the capital produced via investment in response to the news will have suffered less depreciation by the time that technology increase is realized. Thus, the planner is more willing to forgo leisure in order to produce investment. Once again, this greater willingness to forgo leisure implies the model can support procyclical news shocks with less reliance on low curvature in utility.

More formally, for the depreciation rate near zero, the second condition for a model to exhibit procyclical news shocks simplifies dramatically.

Lemma 6. *For δ sufficiently close to zero, the model exhibits procyclical news shocks if $\upsilon > \upsilon^*$ and $\sigma < 1$.*

Next, we plot the impulse responses for a specific model parameterization. Our model calibration meets the two conditions: $\upsilon > \upsilon^*$ and $\sigma < \sigma^*$. First, $\upsilon = 1.8$. Vall'es [35] finds that $\upsilon = 1.8$ best matches the estimated responses of investment to various shocks. Sims [30] uses a similar $F(C, I)$ function and chooses $\upsilon = 3$.

The value of σ in our baseline calibration is 0.5, which implies less curvature than the oft-used 1.0 (i.e. log utility). However, $\sigma = 0.5$ is within the range of some empirical estimates

[19]Remember $\sigma < \sigma*$ supports procyclical news shocks. Holding labor fixed, σ is the inverse of the intertemporal elasticity of substitution. Thus a larger σ corresponds to greater curvature in the utility function

(e.g. Beaudry & Wincoop [9], Vissing-Jorgensen & Attanasio [36] and Mulligan [26]). In Section 4, we consider modifications of the production complementarity model that allow for greater curvature in the utility function. The remaining parameters are less crucial and entirely in line with existing research. All parameters are reported in Table 1.

[Insert Table 1 here]

[Insert Figure 6 here]

The impulse responses are given in Figure 6. At time zero, capital is at the initial steady state and agents receive news of an expected one-percent permanent increase in technology that will arrive at $T = 4$. Examining panels (b), (c) and (d), we see that consumption, hours and investment all increase on impact. Moreover, as our phase diagram and theorem dictate, the shadow value of investment λ increases upon the arrival of the news (see panel (f)).

The response of consumption, seen in panel (b), is positive but nearly zero. This may be viewed as a deficiency of the model, although we note that non-durable consumption, the closest analogue in actual data to consumption in our model, contributes very little to empirical business cycles. In Section 4, we examine how adding various features to the baseline production complementarity model can affect the impulse responses quantitatively.

3 Procyclical News Shocks via a Positive Labor Externality

This section modifies the preference side of the growth model as a way to sustain procyclical news shocks. Specifically, we replace the momentary utility function in equation (4) with:

$$W\left(C, N, \bar{N}\right) = (1-\sigma)^{-1}\left[C\exp\left(-N\bar{N}^{-\gamma_N}\right)\right]^{1-\sigma}$$

where $0 < \gamma_N < 1$ and $\bar{N}$ is the average economy-wide labor input.[20] Thus, there is an external effect of employment on utility. In particular, the marginal disutility of labor falls as average labor rises. From an individual's perspective, he would prefer to work additional

[20]To characterize the resource allocation in the presence of the externality, we have the social planner take the time path of $\bar{N}$ as given when choosing the time paths of (C, I, K, N). Thus, we are studying a constrained optimal plan. It is straightforward to show that this allocation is the same as what would obtain in a competitive equilibrium with the externality.

hours when others are working. In turn, the labor externality will act, from the household perspective, similarly to a preference shock that shifts out labor supply. This mechanism will ensure a low relative cost of forgoing leisure.

Also, we add investment adjustment costs by replacing (3) with

$$\dot{K} = I - \delta K - \frac{\psi_I}{2}\left(1 - \frac{I}{\delta K}\right)^2 I, \tag{23}$$

In absence of adjustment costs (or an alternative appropriate mechanism), no stable solution exists at low values of σ. With the employment externality, production complementarity is not necessary to support procyclical news shocks; as such, we set $\nu = 1$.

The log-linearized equations that characterize a solution to the constrained planner's problem are now:

$$n = \frac{s_I}{1-\gamma_N}(i - c) \tag{24}$$

$$\underbrace{\lambda}_{MU_I} = \underbrace{\psi_I (i-k)}_{price_I} + \underbrace{(-\sigma c - zn)}_{MU_C} \tag{25}$$

$$\dot{\lambda} = -(\rho + \delta)\left[(1 - s_I)(c - i) + i - k\right] + \rho\psi_I (i - k) \tag{26}$$

as well as the resource constraints given by equations (12) and (13).

The term γ_N only appears in (24), the consumption-labor Euler equation.[21]and the addition of investment adjustment costs only alters equations (25) and (26).

We can substitute (24) into (12) to get a new consumption-investment production frontier (new L_1 line):

$$\left(1 - \phi_I^{LE}\right)c + \phi_I^{LE} i = \alpha k + (1-\alpha)a \tag{27}$$

Here, $\phi_I^{LE} = \left(1 - \frac{1-\alpha}{1-\gamma_{\bar{N}}}\right)s_I$. In this model, L_1 is upward sloping if $\phi_I^{LE} < 0$. This requirement, which simplifies to $\gamma_N > \alpha$, is necessary and sufficient for comovement (although not necessarily procyclical) in response to a news shock.

Lemma 7. *Suppose the economy experiences a positive technology news shock. Consumption, investment and hours comove at time zero if and only if* $\gamma_N > \gamma_N^* = \alpha$.

[21] In fact, if one were to replace $1/(1-\gamma_N)$ with the value ν on the left-hand side, the equation would be identical to (9) from the production complementarity model.

Lemma 7 is the labor externality counterpart to Lemma 1 for the production complementarity model.

The steps in characterizing the allocation under labor externalities are very similar to those (previously done) under production complementarities. Two of the five equations, as noted above, are identical across the two cases. The remaining three equations contain nearly the same endogenous variables as in the previous model. Differences between the two sets of preferences are limited to the coefficients multiplying the endogenous variables. As such, we can apply the previous technique to this model.

The optimal solution satisfies the following conditions: [22]

$$x = \tau^{LE}_{x,k} k + \tau^{LE}_{x,\lambda} \lambda + \tau^{LE}_{x,a} a \text{ for } x = c, i, n$$

$$\begin{bmatrix} \dot{k} \\ \dot{\lambda} \end{bmatrix} = \Gamma^{LE} \begin{bmatrix} k \\ \lambda \end{bmatrix} + b^{LE} a \tag{28}$$

As mentioned previously for a model with labor externalities there exists no stable dynamic solution for low values of σ. The addition of investment adjustment costs alleviates this problem. The next lemma holding fixed all other parameter values gives the minimum value of ψ_I, the investment adjustment cost parameter, under which a stable dynamic solution exists.

Lemma 8. *Suppose the economy experiences a positive technology news shock. Also, assume that $\gamma_N > \gamma_N^*$. Then a stable solution to equation (28) exists if*

$$\psi_I > \psi_I^+ = -\frac{\gamma_I^{LE} + \phi_I^{LE} \sigma}{1 - \phi_I^{LE}}$$

Lemma 9. *Suppose the economy experiences a positive technology news shock. Also, assume that $\gamma_N > \gamma_N^*$ and $\psi_I > \psi_I^+$. $\lambda(0) > 0$ if and only if $\psi_I > \psi_I^*$. where, ψ_I^* solves equality*

$$\mu_2^{LE} = \frac{\Gamma^{LE}_{\lambda,\lambda} b^{LE}_{k,a} - \Gamma^{LE}_{k,\lambda} b^{LE}_{\lambda,a}}{b^{LE}_{k,a}}$$

and $\mu_2^{LE} = \mu_2^{LE}(\psi_I)$ is the positive eigenvalue of Γ^{LE}.

[22] The explicit formulas for $\tau^{LE}_{\cdot,\cdot}$, b^{LE} and Γ^{LE} are given in Appendix B.

Theorem 2. *The labor externality model exhibits procyclical technology news shock if:* $\gamma_N > \gamma_N^*$ *and* $\psi_I > Max\left\{\psi_I^+, \psi_I^*\right\}$.

Here is the intuition and relation to our main observation. As labor increases, the marginal product of labor falls, which reduces the incentive to work. Without an external effect, the marginal disutility of labor is increasing in labor. Thus, these two effects work in the same direction. On the other hand, with the external effect, in a symmetric equilibrium, the effective marginal disutility of labor is falling in labor. If the labor externality, measured by γ_N, has a stronger positive labor supply effect than the negative diminishing returns to labor demand effect, measured by $1-\alpha$, then the employment increase will be sufficiently large to support both a consumption and investment increase.[23] Thus, the labor externality mechanism achieves Condition I.

Next, equation (25) is critical for ensuring that the marginal utility of investment rises on impact. The intuition here is identical to that of the production complementarity model with one difference. In the current model, the price of investment (see the right-hand side of equation (25)) increases because of investment adjustment costs term, $\psi_I\left(i-k\right)$. In the production complementarity model, the price of investment (see the right-hand side of equation (22)) increased because of the production complementarity term, $v\left(i-c\right)$. The adjustment cost mechanism achieves Condition II.

Although the labor externality model has a different mechanism than the production complementarity model, the relevant diagrams for the two models are identical. Figure 4(c) is the correct phase diagram for the new model; Figures 2 and 3 illustrate the static relationships for the new model.

Next, we plot the impulse responses for a specific model parameterization. Our model calibration meets the requirement $\gamma_{\bar{N}} > \alpha$ and the two conditions on ψ_I. We set $\gamma_N = 0.332, \psi_I = 20$ and $\sigma = 1$. All parameters values are reported in Table 1.

The values of σ and ψ_I are in line with those of existing research. On the other hand, γ_N is chosen somewhat arbitrarily because existing research provides no guidance for choosing its value. This parameter choice is motivated by our desire to demonstrate one mechanism capable of supporting procyclical news shocks.

[Insert Figure 7 here]

[23] For parameter pairs (γ_N, α) that satisfy the comovement condition, note that there does not exist an interior solution to the first-best resource allocation. With $\gamma_N > 0$, the social marginal disutility of labor is declining, rather than rising, as labor increases. With $\gamma_N > \alpha$, this decline occurs more rapidly than the increase in marginal cost associated with diminishing returns.

The impulse responses are given in Figure 7. Consumption, hours, investment as well the shadow value of investment all increase upon the arrival of the news. Quantitatively, we view the results as disappointing. Each of the three main variables is nearly acyclical in response to the positive news.[24]

4 Quantitative Issues and Calibration Issues

Section 2 established that adding production complementarity to the neoclassical growth model was sufficient to support procyclical news shocks. The mechanism, by itself, has two potential drawbacks: the resulting consumption boom is miniscule and it requires low curvature in the utility function. The former drawback is also present in the preference-externality model. This section restricts attention to the production complementarity model and shows how either changing preferences or adding investment adjustment costs can mitigate these difficulties.

Greenwood-Hercowitz-Huffman Preferences

Suppose we replace the King, Plosser and Rebello preferences [23] (hereafter KPR), used earlier in this paper, with those of Greenwood, Hercowitz and Huffman [16], GHH preferences. The instantaneous utility function becomes

$$V(C,N) = \left(C - \xi \frac{N^{1+\psi}}{1+\psi}\right)^{1-\omega} / (1-\omega)$$

where $\omega, \xi, \psi > 0$. The set-up is otherwise identical to the production-complementarity model.

The critical feature of these preferences is that the marginal disutility of labor is falling in consumption. Recall that, in absence of a low σ under the KPR preferences, it was optimal for the planner to delay the hours boom until the new technology arrives. Under GHH preferences, the marginal disutility of work is falling in consumption; as such, there is an preference-driven incentive to work more during a consumption boom.

The log-linearized conditions for an optimum consists of five equations, three of which are identical to the baseline production-complementarity model.[25] The first equation (that does

[24]It should be noted that in our simulation the marginal product of labor, a measure of real wage, falls in response to a positive news shock, however, similar to the other variables the quantitative response is nearly acyclical.

[25]The unchanged equations are (11), (12), and (13).

change) is the intratemporal consumption-hours Euler condition. Under GHH preferences, it is:

$$n = vs_I\left(i - c\right) + c \tag{29}$$

For comparison, we restate the corresponding equation for the baseline preferences:

$$n = vs_I\left(i - c\right)$$

The production-complementarity effect, reflected by the $vs_I\left(i - c\right)$ term, is present in both equations. GHH preferences, additionally, imply that the planner works more hours when consumption is high. This augments the model's ability to achieve Condition I.

The second equation (that does change) is the consumption-investment Euler condition. Under GHH preferences, it is [26]

$$\lambda = \left(\nu - 1\right)\left(i - c\right) + \underbrace{\frac{1}{1 - \frac{s_n}{1+\psi}}\left[-\omega c + \omega s_n n\right]}_{MU_C} \tag{30}$$

For the baseline preferences, we have

$$\lambda = \left(v - 1\right)\left(i - c\right) + \underbrace{\left[-\sigma c - \left(1 - \sigma\right)\left(\frac{1-\alpha}{1 - s_I}\right) n\right]}_{MU_C}$$

Recall that in order for comovement to be *procyclical* with respect to positive news, λ must increase upon arrival of the news. This is an implication of our Condition II. Under the baseline preferences, we showed that this was qualitatively possible if v was sufficiently large and σ was sufficiently close to zero; however, quantitatively the consumption boom was nearly very small. Examining the KPR preferences, one sees that λ is decreasing in consumption. Roughly speaking, the planner attempts to allocate output to consumption and investment to equalize their marginal benefit. If consumption rises too much, then the marginal utility of consumption, MU_c, will fall too much. In turn, the marginal utility of investment would also have to fall.

GHH preferences help support an increase in λ in response to positive news. This is because, while MU_c is falling in consumption, it is rising in hours worked for any ω. Since labor and consumption comove, this leads to an offsetting effect on MU_c.

[26] Here $s_n = \frac{(1-\alpha)}{1-s_I}$

The steps in characterizing the optimal allocation under GHH preferences are very similar to those described previously. Three of the five equations, as noted above, are identical across the two cases. The remaining two equations, (29) and (30), for the GHH-preferences case, contain the same endogenous variables as in the KPR case. Differences between the two preference assumptions are limited to the coefficients multiplying the endogenous variables. As such, we can apply the previous technique.

The optimal solution satisfies the following conditions: [27]

$$x = \tau_{x,k}^{GHH} k + \tau_{x,\lambda}^{GHH} \lambda + \tau_{x,a}^{GHH} a \text{ for } x = c, i, n$$

$$\begin{bmatrix} \dot{k} \\ \dot{\lambda} \end{bmatrix} = \Gamma^{GHH} \begin{bmatrix} k \\ \lambda \end{bmatrix} + b^{GHH} a$$

[Insert Figure 8 here]

Next, we examine the quantitative implications of applying GHH preferences. The three new model parameters are set at $\omega = 0.5$, $\psi = 0.01$, and the scale parameter $\xi = 6.96$ to match a steady state value of labor hour, $N = 0.3$. We calibrate the remaining parameters at the values used in Section 2. Figure 8 plots the impulse responses. When compared to the earlier models the magnitude of the initial responses are larger.

Investment Adjustment Costs

Condition II to generate procyclical news shocks requires a model feature that will ensure the benefit to forgoing leisure sufficiently outweighs the cost. Parameterizing our model with a high intertemporal elasticity of substitution led to us satisfying this condition in our basic production-based model. Further analysis of the marginal effect of other parameters in that model showed that increasing the marginal returns to investment, either by increasing the capital share in production or lowering the depreciation rate, also led to an increase in the benefit to forgoing leisure, albeit not large enough [28]. The natural extension thus would be to include a feature that generates a very high return to investment, such as investment adjustment costs.

We make one modification to the baseline production-complementarity model. We introduce investment adjustment costs by replacing (3) with (23).

[27] The explicit formulas for $\tau_{\cdot,\cdot}^{G}$ and Γ^{G} are given in Appendix B. This appendix also contains the conditions on the underlying parameters $(\alpha, \omega, \rho, \delta, \nu, \xi)$ required for the model to exhibit procyclical news shocks.

[28] We still need higher than normal values of the intertemporal elasticity of substitution

This addition alter equations (10) and (11) in our log-linearized system by adding a new term:

$$\lambda = \underbrace{(\upsilon - 1)(i - c) + \psi_I (i - k)}_{price_I} + \left(-\sigma c - (1 - \sigma) \left(\frac{1 - \alpha}{1 - s_I} \right) n \right) \tag{31}$$

$$\dot{\lambda} = -(\rho + \delta) [\upsilon (1 - s_I)(c - i) + i - k] + \rho \psi_I (i - k) \tag{32}$$

The remaining three equations are unchanged.

Equation (31) gives us the optimal consumption-investment decision. It is identical to the baseline production-complementarity model except there is an additional component to the price of investment, $\psi_I (i - k)$. This component is due to the investment adjustment costs, is increasing in i and works to increase λ. The phase diagrams in Section 2 showed why an increase in λ upon arrival of the news is required in order that a model support *procyclical* comovement. Adjustment costs help ensure that increase in λ, and, therefore, Condition II.

Equation (32) is the intertemporal consumption Euler equation. The sole difference in this equation from the baseline production-complementarity model is that the rate of change in the marginal utility of investment reflects the investment adjustment cost.

Next, consider the impact of adjustment costs on Condition I. Even though the addition of investment adjustment costs alters equation (10), and thus the static system, it can be shown that the Lemmas 1 through 4 still hold. The L_1 line, equation (15), remains the same, and while the magnitude of the slope of L_2 changes quantitatively, there is no qualitative change. Figures 2(b) and 3 still reflect (qualitatively) the static and required dynamic relationship in the adjustment cost model.

With investment adjustment costs the dynamic analysis is slightly altered. Lemma 5, and thus Theorem 1, now place a different parameter restriction required to generate a positive $\lambda(0)$. Most importantly the restriction on the critical value of σ is relaxed. High investment adjustment costs lead to an increase in the returns to investment, which in turn leads to more capital, which further in turn leads to an increase in the benefit of supplying labor and forgoing leisure. Finally, the dynamic analysis in the (k, λ) space is qualitatively still given by Figure 4.

In figure 8 we examine the quantitative implications of adding investment adjustment costs. We set $\sigma = 1$ and $\psi_I = 10$, the remaining parameter values remain the same as before (see table 1). The addition of investment adjustment costs lets us generate procyclical news shocks with higher curvature in utility.

5 Forward-Compatible Investment

One observation, thus far, has been that in order to generate procyclical news shocks we require a feature or features that increases $\lambda(0)$, which is an alternative expression of Condition II. In this section, we describe how this can be achieved by introducing forward-compatible investment. Physical investment is forward-compatible if additions to the capital made between the arrival of the news and the actual technology increase are particularly well-suited to the future technology. For example, if an IT firm is laying down fiber optic cables and it knows a new, better standard will be in effect in a year then it may be able to ensure that the fiber optic cables currently being installed can take advantage of the new standard. This will mean that at least part of the investment done in the preparatory phase ($t \in [0, T)$) will be able to have an advantage of the new technology improvement once time T arrives. [29]

The Model

Suppose that at time zero, news arrives of a future investment-specific technology shock.[30] The technology increase will arrive at $T > 0$ and will be permanent:

$$Q\left(t\right) = \begin{cases} \bar{Q} & \text{for } t \in [0, T) \\ \tilde{Q} = 1.01 \times \bar{Q} & t \geq T \end{cases} \tag{33}$$

We shall refer to time between zero and T as the *preparatory phase*.

This shock appears in the capital law of motion:

$$\dot{K}\left(t\right) = Q\left(t\right) I\left(t\right) - \delta K\left(t\right) + \left(K\left(t\right) - e^{-\delta T} \bar{K}\right) P\left(\tilde{Q}, t, T, \epsilon\right) \tag{34}$$

where $\bar{K}$ is the initial capital stock, which we assume is at the steady-state consistent with $\bar{Q}$.

Here q can be interpreted as the level of technology embodied in the capital created at a point in time. The function P represents the idea that capital might embody additional

[29] Baron & Schmidt [1] provide evidence for this mechanism. They document how the adoption of new standards both gives a signal about future technological change - "news shocks" - and subsequently results in slow diffusion of technology.

[30] Investment-specific technology shocks by themselves cannot generate news-driven procyclical business cycles, established in Beaudry and Portier [7].

technology that does not become useful until a future date - forward compatibility of capital with future technology.[31]

The right-hand side of (34) decomposes the time derivative of capital into three terms. The first term is the contribution of investment multiplied by the current efficiency of investment. The second is the capital lost due to depreciation. The last term embodies the model's key assumption. It is the multiple of two functions. The first function is $K(t) - e^{-\delta T}\bar{K}$, representing the investment accumulated during the preparatory phase that has not depreciated. For convenience, define $\mathcal{K}(t) = K(t) - e^{-\delta T}\bar{K}$. The second function, $P(Q, t, T, \varepsilon)$, is a positive technology shock. It will be constructed so that it permanently increases the productivity of investment made during the preparatory phase. Thus, investment made after the arrival of news will be forward-compatible with the yet-to-arrive technology.

This second function will be non-zero only in a small neighborhood of T. Thus, this positive shock will not come online until time T. The contribution of the third term in (34) will only apply to $\mathcal{K}(T)$. We refer to $\mathcal{K}(T)$ as *partially-adapted capital.* We plot the log deviations of the functions Q and P in Figure 9.

[Insert Figure 9 here]

P depends on t because the forward-compatible effect is only operative for a certain interval of time. The term ε is a small positive number. It will define the neighborhood (of the time interval) in which the "forward-compatibility"shock occurs. Mathematically, $P(Q, t, T, \varepsilon) \neq 0$ only if $t \in [T, T + \varepsilon]$. A technical detail necessitates ε.[32] Later, we will drive ε to zero at the appropriate rate. We choose a particular form for P to aid calibration:

$$P\left(\tilde{Q}, t, T, \varepsilon\right) = \begin{cases} 0 & \text{for } t \in [0, T) \cup (T + \epsilon, \infty) \\ \frac{\tau \tilde{Q}}{(1 - e^{-\epsilon})\bar{Q}} & t \in [T, T + \epsilon] \end{cases}$$

For notational simplicity, we will sometimes use $P_\varepsilon(t)$ and suppress the function's dependence on $\tilde{Q}$ and T.

A positive investment-specific technology shock is isomorphic to a positive neutral technology shock combined with a capital depreciation shock. Forward compatability, in the face of an investment-specific news shock, mitigates the "capital depreciation" shock component.

[31] The relationship between investment-specific technology and capital embodiment is discussed in Greenwood, Hercowitz and Huffman [16].

[32] If P were positive and finite only at one instant, then P will have zero effect on the capital. Instead, as we let ϵ approach zero, the P will become infinite at the instant T and cause K to jump upward. This technical detail is not needed in a discrete time model.

That is, capital put in place following the news will not depreciate to the same degree as the already in-place capital. Thus, forward compatability, by itself, boosts the relative benefit to forgoing leisure to produce investment upon the news arrival.

Production of consumption occurs according to equations (1) and (2); that is, there is production complementarity between consumption and investment.[33] The utility function is (4), which is taken from the baseline model of Section 2.

The Planning Problem and Its Solution

The social planner chooses C, I, K and N to maximize U subject to (1), (2) and (34), taking as given the initial condition $K(0)$ and time path of technology. The current value Hamiltonian for the problem is:

$$H = (1-\sigma)^{-1} C^{1-\sigma} \exp\left[-(1-\sigma) N\right] + \Lambda \left(QI - \delta K + P_{\varepsilon}\mathcal{K}\right) + \Phi \left(K^{\alpha} N^{1-\alpha} - F(C, I)\right)$$

The log-linearized system that solves this Hamiltonian is given by five equations.[34] Two equations, (9) and (12), are identical to those from the baseline production complementarity model. These are the consumption-hours Euler equation and the production function. The three new equations are:

$$\lambda = (\upsilon - 1)(i - c) + (-\sigma c - z\nu) - q \tag{35}$$

$$\dot{\lambda} = -(\rho + \delta)\left[v(1 - s_I)c + q + i - k\right] - p_{\varepsilon} \tag{36}$$

$$\dot{k} = \delta(q + i - k) + (1 - e^{-\delta T})p_{\varepsilon} \tag{37}$$

Equation (35) ensures an efficient consumption-investment split. It is identical to the corresponding equation from the baseline production complementarity except for the final term on the right-hand side, $-q$. This is a relative price effect because technology is investment-specific. Before time T, however, $q(t) = 0$ because the technology improvement has not yet arrived.

Equation (36) is the intertemporal consumption Euler equation. It differs from the neoclassical model in two ways. First, λ is not simply the derivative of the marginal utility of consumption. There is an additional relative price effect because of the convex production frontier. Second, λ jumps down at T as a result of p_{ε}, that is in the limit as $\varepsilon \to 0$.

[33] We assume that $A_t = 1$, since technology improvements are only investment specific in this model.

[34] The non-linear first-order conditions are presented in Appendix B.

Equation (37) is the law of motion for capital. On the right-hand side, the first term is standard and the second reflects an increase in capital at time T. This occurs when the forward-compatible capital built during the preparatory phase becomes utilized with the new technology. Recall that p_ε is positive, and otherwise zero, only in a neighborhood of T. Below, we take the limit as $\varepsilon \to 0$. Then, p_ε becomes "infinite at an instant," causing an upward jump in the capital stock. The optimal solution also satisfies a standard transversality condition and an initial condition on capital.

Recall that, in order to support procyclical news shocks, a model must have mechanism(s) that ensure: (I) consumption does not crowd out investment, and (II) a sufficiently large return to forgoing leisure.

Production complementarity is sufficient to ensure Condition I due to the derivations in the benchmark production complementarity model. Lemma 1 holds for this model.

Following the same techniques as for the benchmark production complementarity model [35], we have expressions for the jump variables (consumption, investment and hours) as functions of the state variables.

$$x = \tau^{FC}_{x,k} k + \tau^{FC}_{x,\lambda} \lambda + \tau^{FC}_{x,q} q \text{ for } x = c, i, n \tag{38}$$

Substituting (38) into the $\dot{k}$ and $\dot{\lambda}$ equations, (36) and (37), we have:[36]

$$\begin{bmatrix} \dot{\lambda}(t) \\ \dot{k}(t) \end{bmatrix} = \Gamma^{FC} \begin{bmatrix} \lambda(t) \\ k(t) \end{bmatrix} + b^{FC}_q q(t) + b^{FC}_p p(\tilde{q}, t, T, \varepsilon) \tag{39}$$

Solving the differential equation (39), the time-paths of (k, λ) are:[37]

$$k(t) = \begin{cases} \frac{\Gamma_{k,\lambda}\lambda(0)}{\mu_1-\mu_2} e^{\mu_1 t} + \frac{\Gamma_{k,\lambda}\lambda(0)}{\mu_2-\mu_1} e^{\mu_2 t} & \text{for } t \in [0,T) \\ \frac{\Gamma_{k,\lambda}\lambda(0)}{\mu_1-\mu_2} e^{\mu_1 t} + \frac{\Gamma_{k,\lambda} b_{\lambda,q} - \Gamma_{\lambda,\lambda} b_{k,q}}{\mu_1\mu_2} + \frac{\Gamma_{k,\lambda}\left(b_{\lambda,q}+\tau\mu_1 b_{\lambda,p_\epsilon}\right)+\left(\mu_1-\Gamma_{\lambda,\lambda}\right)\left(b_{k,q}+\tau\mu_1 b_{k,p_\epsilon}\right)}{\mu_1(\mu_1-\mu_2)} e^{\mu_1(t-T)} & t \geq T \end{cases} \tag{40}$$

$$\lambda(t) = \begin{cases} \frac{\left(\mu_1-\Gamma_{k,k}\right)\lambda(0)}{\mu_1-\mu_2} e^{\mu_1 t} + \frac{\left(\mu_2-\Gamma_{k,k}\right)\lambda(0)}{\mu_2-\mu_1} e^{\mu_2 t} & \text{for } t \in [0,T) \\ \frac{\left(\mu_1-\Gamma_{k,k}\right)\lambda(0)}{\mu_1-\mu_2} e^{\mu_1 t} + \frac{\Gamma_{\lambda,k} b_{k,q} - \Gamma_{k,k} b_{\lambda,q}}{\mu_1\mu_2} + \frac{\Gamma_{\lambda,k}\left(b_{k,q}+\tau\mu_1 b_{k,p_\epsilon}\right)+\left(\mu_1-\Gamma_{k,k}\right)\left(b_{\lambda,q}+\tau\mu_1 b_{\lambda,p_\epsilon}\right)}{\mu_1(\mu_1-\mu_2)} e^{\mu_1(t-T)} & t \geq T \end{cases} \tag{41}$$

where μ_1 and μ_2 are the eigenvalues of the Γ^{FC} matrix.[38]

[35] Details can be found in Appendix B.

[36] The explicit formulas for $\tau^{FC}_{\cdot,\cdot}, \Gamma^{FC}, b^{FC}_q$ and b^{FC}_p are given in Appendix B.

[37] Appendix B contains the derivation of (40) and (41)

[38] In the above expressions and the expression below, we suppress the superscript FC from several variables

The above solution has one undetermined variable $\lambda(0)$. We seek a path for (λ, k) that is not explosive. We choose $\lambda(0)$ such that the explosive root μ_2 does not determine the evolution of the system for $t > T$; otherwise, the path would be explosive. This restriction on $\lambda(0)$ is:

$$\frac{\Gamma_{k,\lambda}\lambda(0) + (\mu_2 - \Gamma_{\lambda,\lambda})\, k(0)}{(\mu_2 - \mu_1)} = -\frac{\Gamma_{k,\lambda}\left(b_{\lambda,q} + \tau\mu_2 b_{\lambda,p_\epsilon}\right) + (\mu_2 - \Gamma_{\lambda,\lambda})\left(b_{k,q} + \tau\mu_2 b_{k,p_\epsilon}\right)}{\mu_2\,(\mu_2 - \mu_1)} e^{-\mu_2 T}$$

Next, we provide a theorem concerning procyclical news shocks in the forward-compatibility model.[39]

Theorem 3. *The forward-compatible investment model exhibits procyclical technology news shocks if and only if* $\upsilon\,(1-\alpha) > 1$ *and*

$$\tau > \frac{\Gamma^{FC}_{k,\lambda} b^{FC}_{\lambda,q} + \left(\mu^{FC}_2 - \Gamma^{FC}_{\lambda,\lambda}\right) b^{FC}_{k,q}}{\Gamma^{FC}_{k,\lambda}\mu^{FC}_2 b^{FC}_{\lambda,p_\epsilon} + \left(\mu^{FC}_2 - \Gamma^{FC}_{\lambda,\lambda}\right)\mu^{FC}_2 b^{FC}_{k,p_\epsilon}} \tag{42}$$

where μ^{FC}_2 *is the positive eigenvalue of* Γ^{FC}.

Let us understand this result: Theorem 3 requires that there is both sufficient complementarity in production (Condition I) and the new capital accumulated between time 0 and T must be sufficiently forward compatible (Condition II).

Panel (d) of Figure 4 contains the phase diagram for the forward-compatibility model when the conditions of Theorem 3 are satisfied. Before the news shock, the economy is at its initial steady state (k_{ss}, λ_{ss}). The initial capital stock is lower and the shadow value of investment is larger than their long-run, post-shock counterparts (k'_{ss}, λ'_{ss}). $\mathcal{M}$ is the pre-shock stable manifold and $\mathcal{M}'$ is the corresponding manifold after time T.

At the instant of the news arrival, the shadow value of investment increases. This occurs because new investment will be more productive relative to previous investment, albeit not until time T. Capital does not jump instantaneously; however $\dot{k}\,(0) > 0$. This shows that investment increases in response to the shock—the first of three requirements for procyclical news shocks. The reader should return to Figure 2(b) to see graphically that the second requirement–that consumption rises–is satisfied. That figure shows that the increase in λ

to avoid notational clutter.

[39]It should be noted that Lemmas 2-4 also hold for the forward compatible investment model.

results in higher consumption.[40] The third requirement—that hours increase– holds because, otherwise, both consumption and investment could not increase.

At instant T, the actual increase in investment specific technology occurs. All future investment produces an additional one percent of capital. Moreover, because of forward compatibility, investment made during the preparatory phase becomes τ percent more productive. This latter effect causes the capital to jump up at T. This latter effect also causes λ to jump down from T^{-} to T^{+}. When the capital stock jumps up, the shadow value of investment, i.e. adding to that capital stock, declines in a discrete fashion. During the preparatory phase, the explosive root is operative, causing λ to rise.

Next, at and after time T^{+}, k and λ lie on the new stable manifold $\mathcal{M}'$. If $(k(t), \lambda(t))$ did not lie on $\mathcal{M}'$ for $t > T^{+}$, then the pair would diverge. Intuitively, going from T^{+} onward, k and λ must be on the new manifold because technology has reached its new permanently higher level.[41]

The dynamics of news shocks of every model in this paper are contained in Figure 4. All models discussed in the paper, excluding the forward-compatible model, achieve $\lambda(0) > 0$ by causing the stable manifold to shift upwards. With the forward compatibility, this is not necessary. We can maintain the stable manifold movements as in the standard neoclassical model and yet generate procyclical news shocks by instead adding in a friction that results in a discontinuity in the time path of the state and co-state variables. This allows a fair amount of flexibility in adding other non-related frictions and still generating procyclical news shocks.

Including a feature that adds a discontinuity to the state variable allows for richer dynamics and places fewer restrictions on the movements of the stable manifold vis-a-vis the inclusion of other economic frictions.

Quantitative Analysis of the Forward-Compatibility Model

[40] The intuition for this effect is discussed early in the paper.

[41] The partial forward-compatible assumption in this model is similar to the time-to-build assumption where $\dot{K}(t) = Q(t) I(t-\xi) - \delta K(t)$ (here ξ gives a measure of how long it takes to build capital). It can be shown that in a model with production complementarity and time-to-build if $\xi > T$ then under certain plausible calibrations the model is able to generate procyclical technology news shocks. These models are however distinct. The two key differences are: (i) under time-to-build investment made today doesn't come online until ξ periods into the future, whereas; partially forward-compatible investment is available immediately after it is created. (ii) under the partially forward-compatible assumption only part of the investment created before the technology increase will experience an increase in efficiency when the actual technology increases, whereas; under time-to-build with $\xi > T$ all the investment created before the technology increase will experience an efficiency increase when the investment becomes available.

The only new parameter is τ, the degree of forward compatibility. This is clearly a difficult parameter to calibrate. We set $\tau = 0.66$, a little above the critical value of $\tau \approx 0.5$ to demonstrate the models ability to generate procyclical news shocks. We can now also set the utility curvature parameter to a higher value of $\sigma = 1$. The remaining parameters are set at values used earlier in the paper.

[Insert Figure 10 here]

Figure 10 plots the responses of key variables to a positive news shock. As seen in panels (b)-(d), consumption, investment and hours all increase on impact upon the arrival of the news. The three then increase smoothly until the beginning of quarter four. Then technology actually increases; each of the three jumps upward because the output cost of producing newly installed capital, $1/Q$, falls.

Although we set $T = 4$ (i.e. one year) in our baseline calibration, the model can exhibit procyclical news shows for much higher values of T. Holding all other parameters fixed at their benchmark values calculations based on Theorem 3 imply that procyclical news shocks obtain for any $T < 21.3$.

Next, the relative price of investment to consumption, seen in panel (e), rises on impact due to production complementarities along with a larger increase in investment relative to consumption in response to the news. Panel (f) shows that the shadow value of investment also increases on impact.

With respect to existing research, we note that Flóden [15] has a paper related to ours. He constructs a two-period model that is capable of resolving the comovement puzzle using variable capacity utilization and vintage capital. In his paper, news arrives in the initial period and technology arrives in the second period. The news increases the efficiency of investment made in the initial period toward producing capital in the second period, which is similar to our forward-compatibility.

Value of the Firm

We are additionally interested in the value of the capital stock, which can be interpreted as the value of the stock market in a typical decentralization of our social planner's problem. Our interest is partly empirical: using a VAR strategy, Beaudry and Portier [6] find that the real value of the S&P500 is procyclical with respect to technology news shocks.

[Insert Figure 11 here]

Figure 11 plots the value of the firm in each of our five model economies. For our first four models the value of the firm rises in response to news.

Here is the intuition. Recall that the models presented in panel (a) and (c) all contain production complementarities. The presence of production complementarities increase the price of investment relative to consumption when the planner moves to increase investment in response to a news shock. An increase in the price of investment raises the value of installed capital. Investment adjustment costs, present in the models of panels (b) and (c), increase the price of investment during the investment boom, thereby reinforcing the production complementarity channel. Thus, the increased price of investment in all four models results in an increase in the value of the capital stock.[42]

The dashed line in panel (d) plots the value of the capital stock with forward compatible investment for our benchmark calibration. The value of the firm falls even though, as with the models in panels (a) and (c), this model also contains production complementarities. The reason that the firm's value falls is, unlike the first three panels, the exogenous news in (d) is regarding investment-specific technology, rather than neutral technology.

In the standard neoclassical model, a contemporaneous positive investment-specific technology shock causes the value of the firm to fall. Recall the value of the firm is simply the value of its capital. Higher productivity in the capital producing sector depresses the price of capital. Thus, the value of the firm's existing capital must fall on impact. This channel is operational for news about investment-specific technology as well.

These results suggest that models with neutral-technology, as opposed to investment-specific technology, news shocks do have one advantage in terms of generating procyclical stock prices.

6 Conclusion

In this paper, we have made the case that an analytic approach to understanding news shocks in stochastic growth models is insightful. Up to a first-order approximation, we have provided analytic solutions to five variants on the standard neoclassical model. In each case, we have provided conditions under which the model exhibits procyclical news shocks. For each, we have provided a plausible calibration and then examined the quantitative features

[42]We use the benchmark calibration in Table 1 to compute these impulse responses. In addition it can easily be shown that for the production complementarity model, the labor externality model, and the investment adjustment cost model of panels (a)-(c) the time 0 value of the firm will increase for all parameters under which those models exhibit procyclical news shocks.

of the model's impulse response functions.

By examining several models, we have identified commonalities across their respective mechanisms that generate resolutions of the news-shock comovement puzzle. In particular, each model has a mechanism or mechanisms that ensure that consumption and investment do not crowd each other out and that the relative benefit of forgoing leisure outweighs its cost.

As a result of our analysis, we have further developed a view on what type of mechanisms are best suited for achieving the above conditions. We contend that it is more reasonable to attribute procyclical news shocks to production-based rather than preference-based components. For example, GHH preferences help significantly boost the size of a consumption boom in the production complementary model; however, using these preferences creates other problems, not related to news shock. These include an absence of balanced growth with respect to neutral technological change. On the other hand, the addition of production complementarity, which has an interpretation as a two-sector neoclassical model, seems, in our view, to be a plausible production-based mechanism.

Bibliography

[1] Baron, Justus, and Julia Schmidt, "Technological Standardization, Endogenous Productivity and Transitory Dynamics," *mimeo*, Northwestern University, 2014.

[2] Barsky, Robert B., and Eric Sims, "News Shocks and Business Cycles," *Journal of Monetary Economics*, 58:273-289, 2011.

[3] Beaudry, Paul, Martial Dupaigne, and Franck Portier, "Modeling News-Driven International Business Cycles," *Review of Economic Dynamics*, 14:72-91, 2011.

[4] Beaudry, Paul and Bernd Lucke, "Letting Different Views about Business Cycles Compete," *NBER Macroeconomics Annual 2009*, 24:413-455, 2010.

[5] Beaudry, Paul and Franck Portier, "An Exploration into Pigou's Theory of Cycles," *Journal of Monetary Economics*, 51:1183-1216, 2004.

[6] Beaudry, Paul and Franck Portier, "Stock Prices, News, and Economic Fluctuations," *American Economic Review*, 96:1293-1307, 2006.

[7] Beaudry, Paul and Franck Portier, "When Can Changes in Expectations Cause Business Cycle Fluctuations in Neo-Classical Settings?," *Journal of Economic Theory*, 135:458-477, 2007.

[8] Beaudry, Paul and Franck Portier, "News Driven Business Cycles: Insights and Challenges," *NBER Working Paper 19411*, 2013.

[9] Beaudry, Paul, and Eric van Wincoop, "The Intertemporal Elasticity of Substitution: An Exploration using a US Panel of State Data," *Economica*, 63:495-512, 1996.

[10] Boyce, William E., and Richard C. DiPrima, *Elementary Differential Equations and Boundary Value Problems*, John Wiley & Sons, New York, 1969.

[11] Campbell, John Y., "Inspecting the Mechanism: An Analytical Approach to the Stochastic Growth Model," *Journal of Monetary Economics*, 33:463-506, 1994.

[12] Chen, Kaiji, and Zheng Song, "Financial Frictions on Capital Allocation: A Transmission Mechanism of TFP Fluctuations," *Journal of Monetary Economics*, forthcoming.

[13] Christiano, Lawrence, Cosmin Ilut, Roberto Motto, and Massimo Rostagno, "Monetary Policy and Stock Market Boom-Bust Cycles," *Working Paper Series 955*, European Central Bank, 2008.

[14] Comin, Diego A., Mark Gertler, and Ana Maria Santacreu, "Technology Innovation and Diffusion as Sources of Output and Asset Price Fluctuations," *Harvard Business School Working Paper*, 09-134, 2009.

[15] Flóden, Martin, "Vintage Capital and Expectations Drive Business Cycles," CEPR working paper, 2007.

[16] Greenwood, Jeremy, Zvi Hercowitz, Gregory W. Huffman, "Investment, Capacity Utilization, and the Real Business Cycle," *American Economic Review*, 78:402-417, 1988.

[17] Gunn, Christopher, and Alok Johri, "News and Knowledge Capital," *Review of Economic Dynamics*, 14:92-101, 2011.

[18] Huffman, Gregory and Mark Wynne, "The Role of Intratemporal Adjustment Costs in a Multisector Economy," *Journal of Monetary Economics*, 43:317-350, 1999.

[19] Jaimovich, Nir, and Sergio Rebelo, "Can News About the Future Drive the Business Cycle?," *American Economic Review*, 99:1097-1118, 2009

[20] Judd, Kenneth, "An Alternative to Steady-State Comparisons in Perfect Foresight Models," *Economic Letters*, 10:55-59, 1982.

[21] Judd, Kenneth, "Short-Run Analysis of Fiscal Policy in a Simple Perfect Foresight Model," *Journal of Political Economy*, 93:298-319, 1985.

[22] Khan, Hashmat and John Tsoukalas, "The Quantitative Importance of News Shocks in

an Estimated DSGE Model," *Journal of Money, Credit, and Banking*, 44:1535-1561, 2012.

[23] King, Robert G., Charles I. Plosser and Sergio T. Rebelo, "Production, Growth and Business Cycles I. The Basic Neoclassical Model," *Journal of Monetary Economics*, 21:195-232, 1988.

[24] Leeper, Eric M., Todd Walker, and Shu-Chun (Susan) Yang, "Foresight and Information Flows," *Econometrica*, 81:1115-1145, 2013.

[25] Lorenzoni, Guido, "News and Aggregate Demand Shocks," *Annual Review of Economics*, 3:537-557, 2011.

[26] Mulligan, Casey, "Capital, Interest, and Aggregate Intertemporal Substitution," *NBER Working Paper 9373*, 2002.

[27] Nah, Seungho, "Financial Frictions, Intersectoral Adjustment Costs, and News-Driven Business Cycles," *mimeo*, The Ohio State University, 2009.

[28] Qureshi, Hammad, "News Shocks and Learning-by-Doing," *mimeo*, The Ohio State University, 2009.

[29] Schmitt-Grohe, Stephanie and Martin Uribe, "What's News in Business Cycles," *Econometrica*, 80:2733-2764, 2012.

[30] Sims, Christopher A., "Models and their Uses,"*American Journal of Agricultural Economics*, 71:489-494, 1989.

[31] Tenenbaum, Morris and Harry Pollard, *Ordinary Differential Equations*, Dover, 1985.

[32] Tsai, Yi-Chan, "News Shocks and Costly Technology Adoption," *mimeo*, The University of Tokyo, 2009.

[33] Walentin, Karl "Expectation Driven Business Cycles with Limited Enforcement," *Sveriges Riksbank Working Paper No. 229*, 2012.

[34] Wang, Pengfei, "Understanding Expectation-Driven Fluctuations—A Labor Market Approach," *Journal of Money, Credit and Banking*, 44:487-506, 2012.

[35] Vall'es, Javier, "Aggregate Investment in a Business Cycle Model with Adjustment Costs," *Journal of Economic Dynamics and Control*, 21:1181-1198, 1997.

[36] Vissing-Jorgensen, Annette, and Orazio P. Attanasio, "Stock Market Participation, Intertemporal Substitution, and Risk Aversion," *American Economic Review*, 93:383-391, 2003.

Table 1: Parameterization for computing impulse responses from various models

Parameter	Value	Description
		Production complementarity
$\beta = \frac{1}{\rho+1}$	0.985	Subjective discount factor (quarterly)
α	0.33	Capital share in production
δ	0.025	Depreciation rate of capital
υ	1.8	Degree of complementarity between c and i
σ	0.5	Utility curvature
θ	0.253	Calibrated to match a steady state price of investment of 1
T	4	Quarters between news arrival and actual technology increase
		Labor Externality
σ	1	Utility curvature
γ_N	0.332	Degree of labor externality
ψ_I	20	Investment adjustment cost
		GHH preferences
ω	0.5	Utility curvature
$\frac{1}{\psi}$	100	Frisch Elasticity
ξ	6.96	Calibrated to match steady state fraction of time spent working = 0.3
		Investment Adjustment Costs
σ	1	Utility curvature
ψ_I	10	Investment adjustment cost
		Forward Compatible Investment
σ	1	Utility curvature
τ	0.66	Degree of forward compatibility

Figure 1: Impulse Response to a Positive News Shock - Basic Neoclassical Model

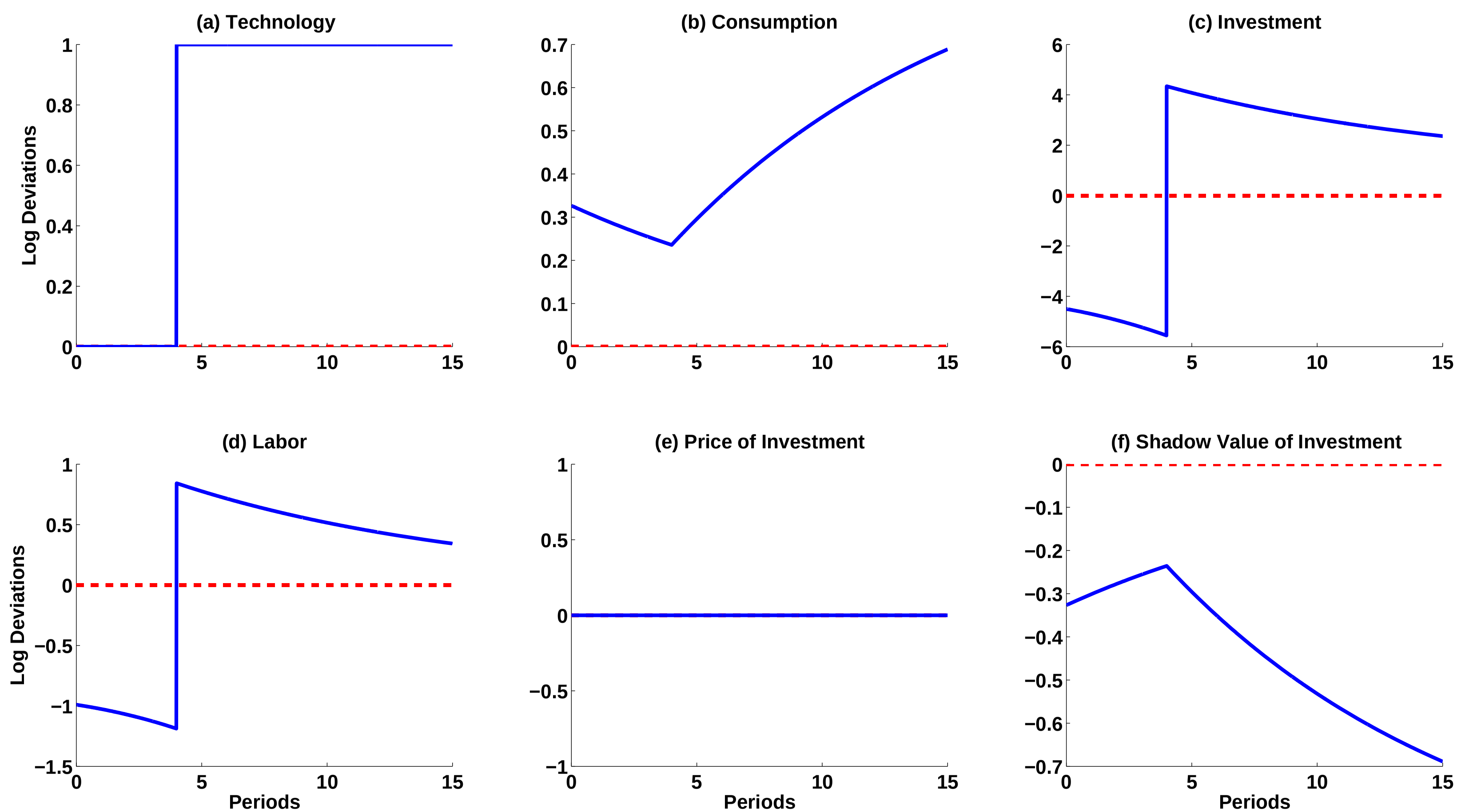


Figure 2: L_1 and L_2 in i-c Space, and an increase in the marginal utility of investment (λ)

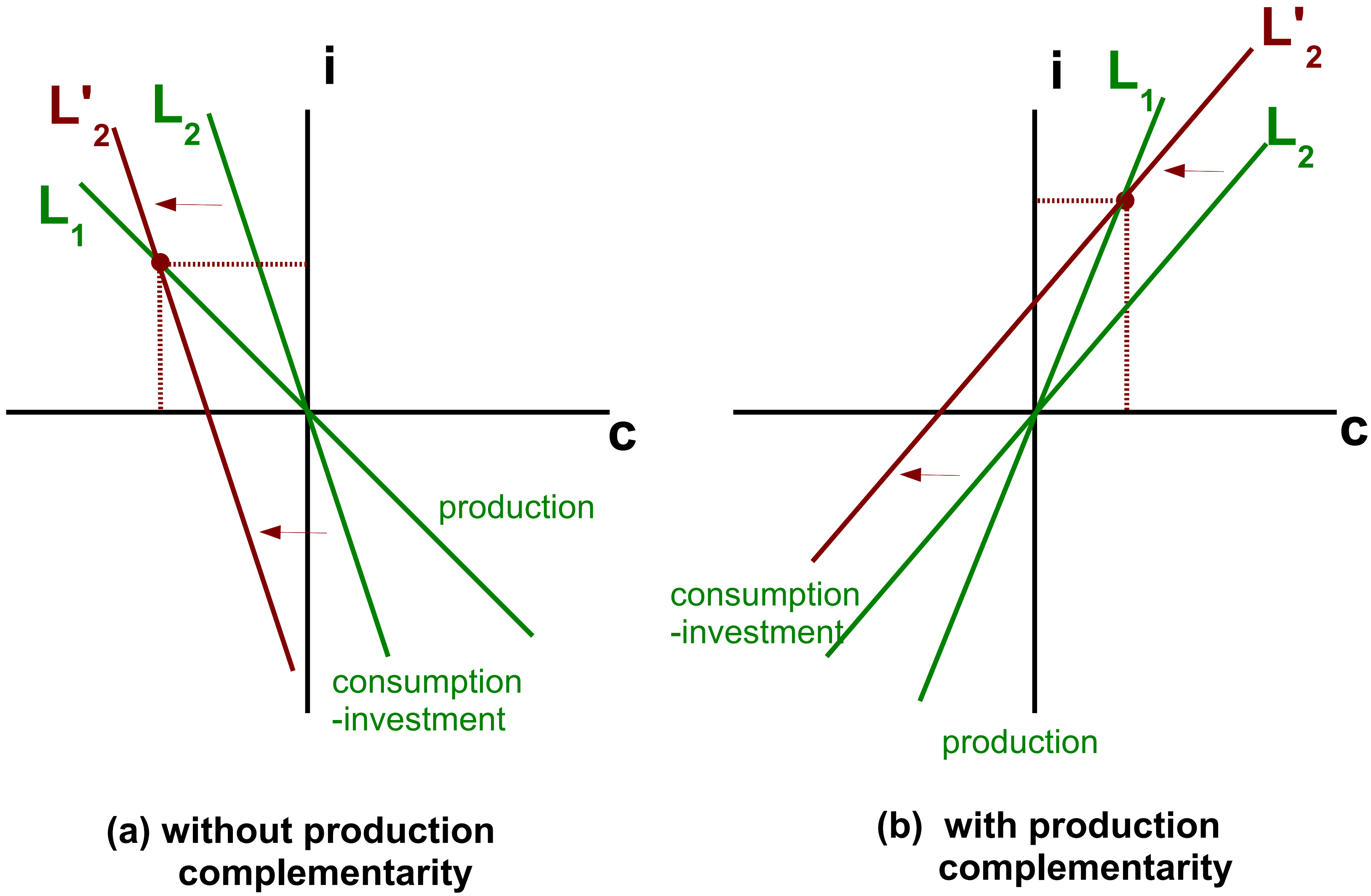


Figure 3: L_1 and L_2 in i-c Space, and an increase in both the marginal utility of investment (λ) and capital (k)

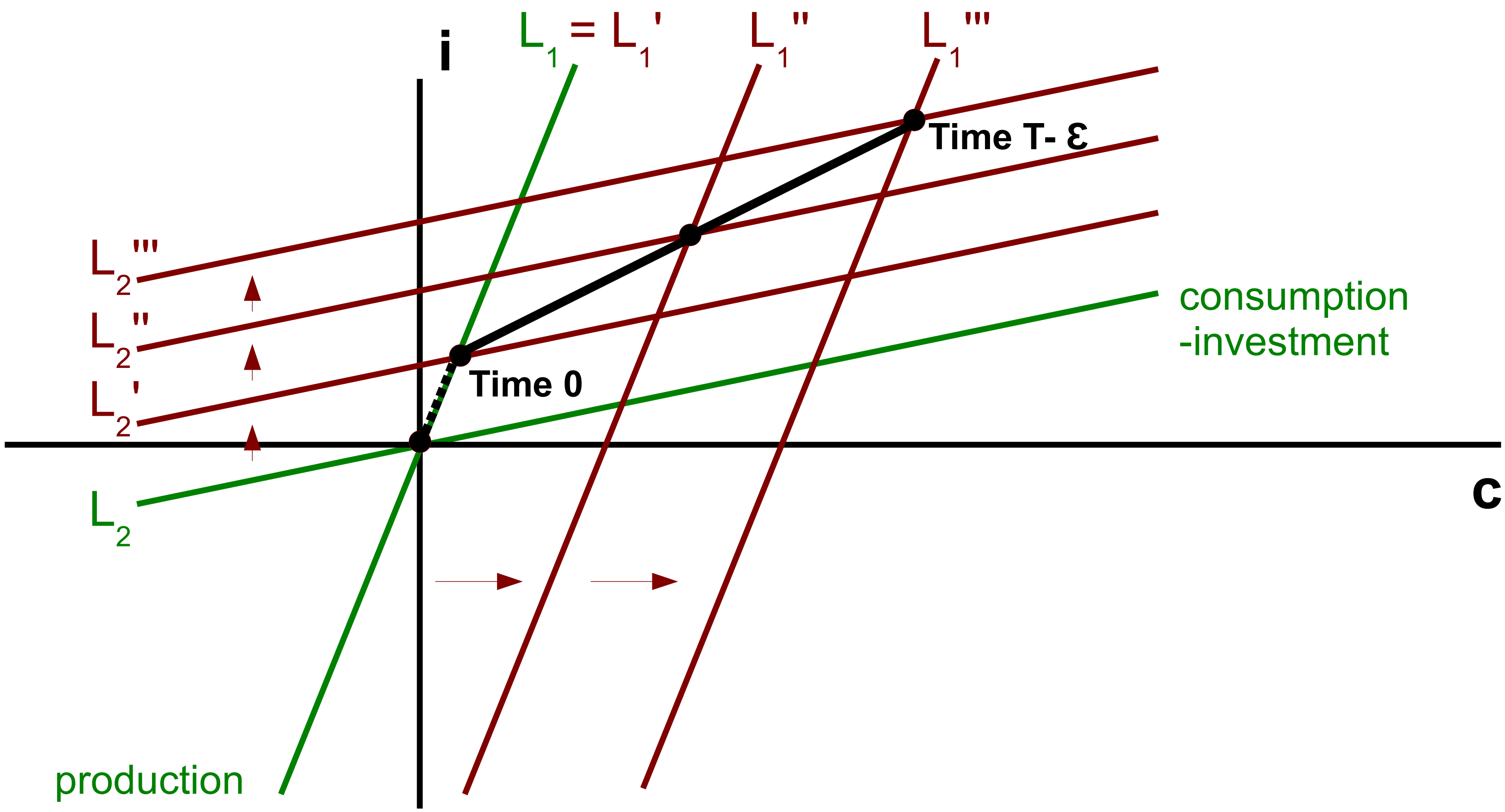
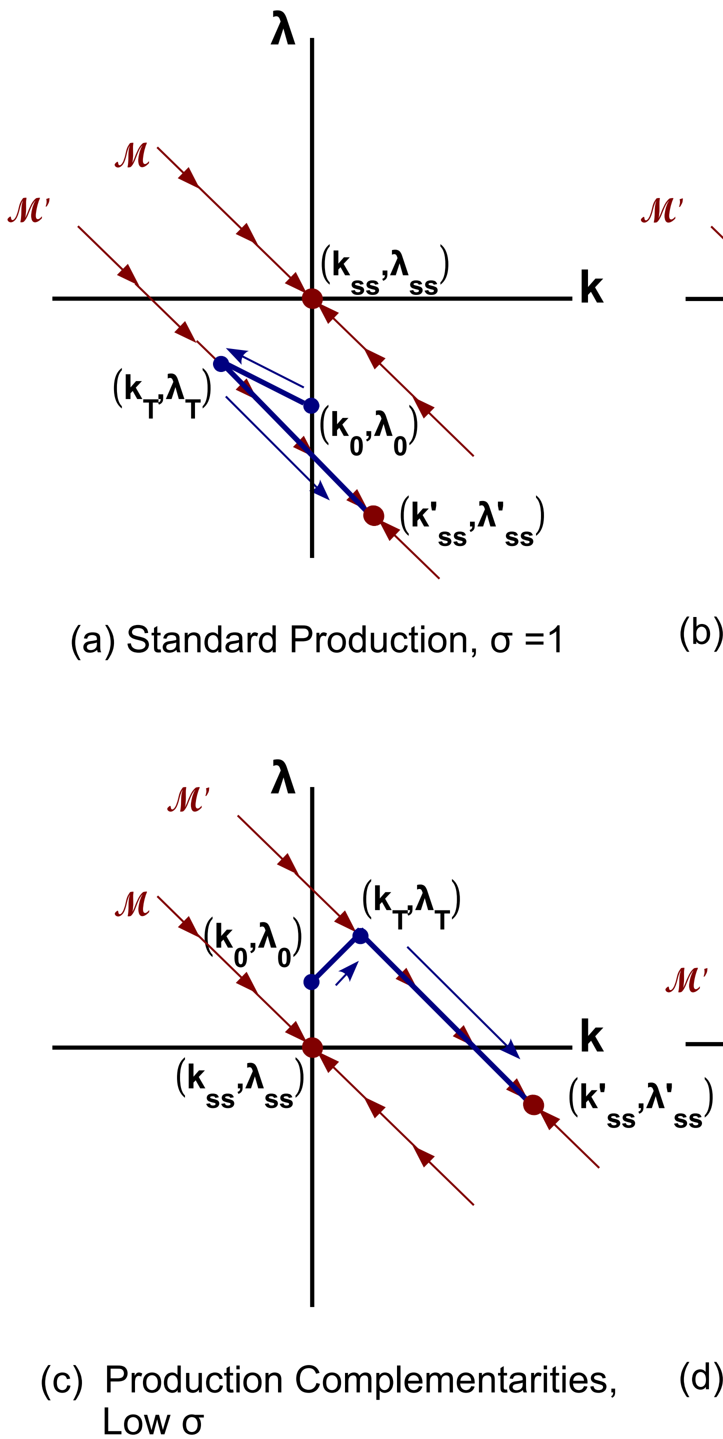


Figure 4: Phase diagrams in (k, λ) space for a positive news shock



(a) Standard Production, σ =1

(b) Production Complementarities, High σ

(c) Production Complementarities, Low σ

(d) Production Complementarities, Partial Forward Compatibility

Figure 5: Critical Utility Curvature (σ^*) vs. Other Model Parameters

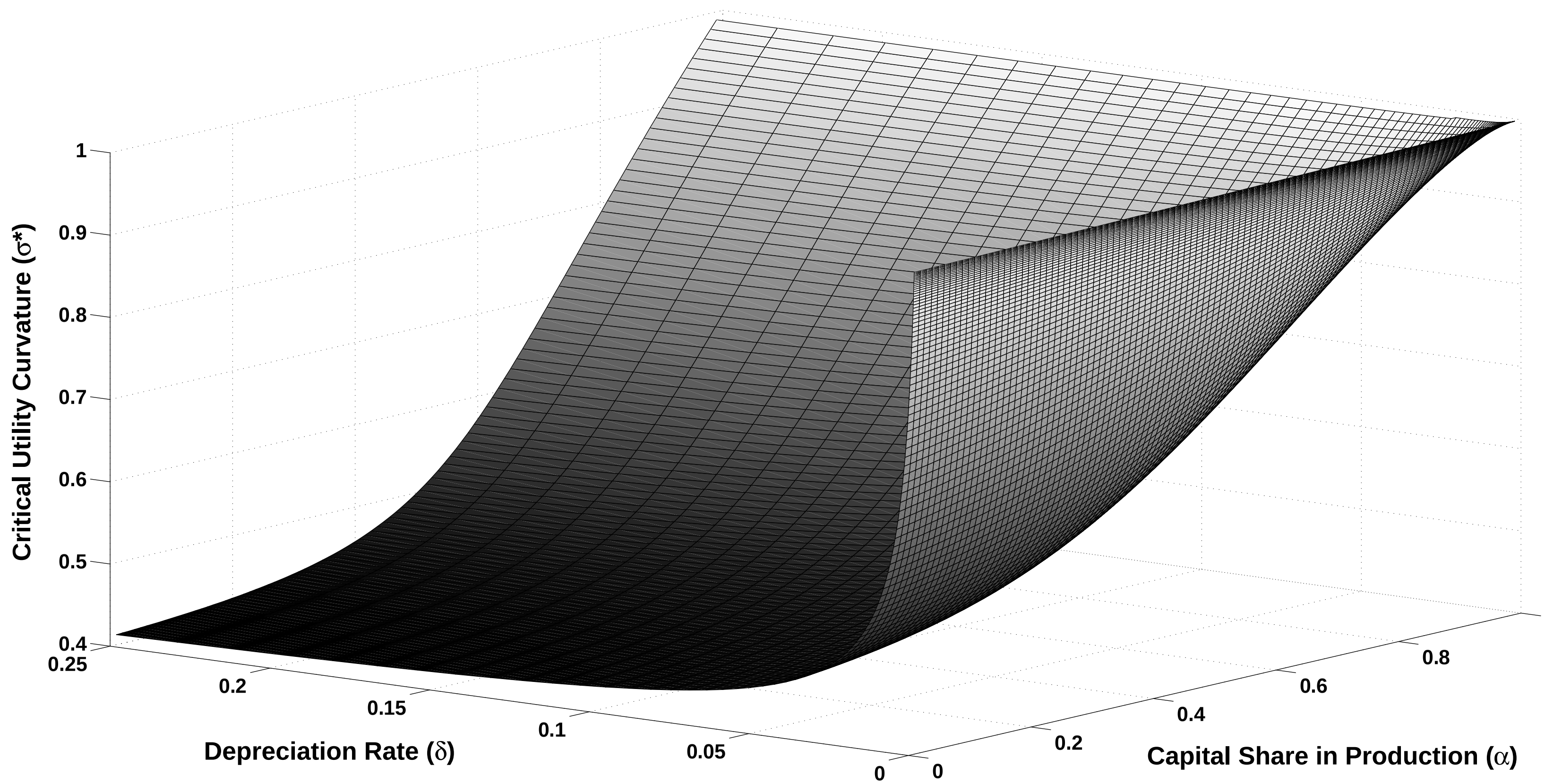


Figure 6: Impulse Response to a Positive News Shock - Benchmark Production Complementarity Model

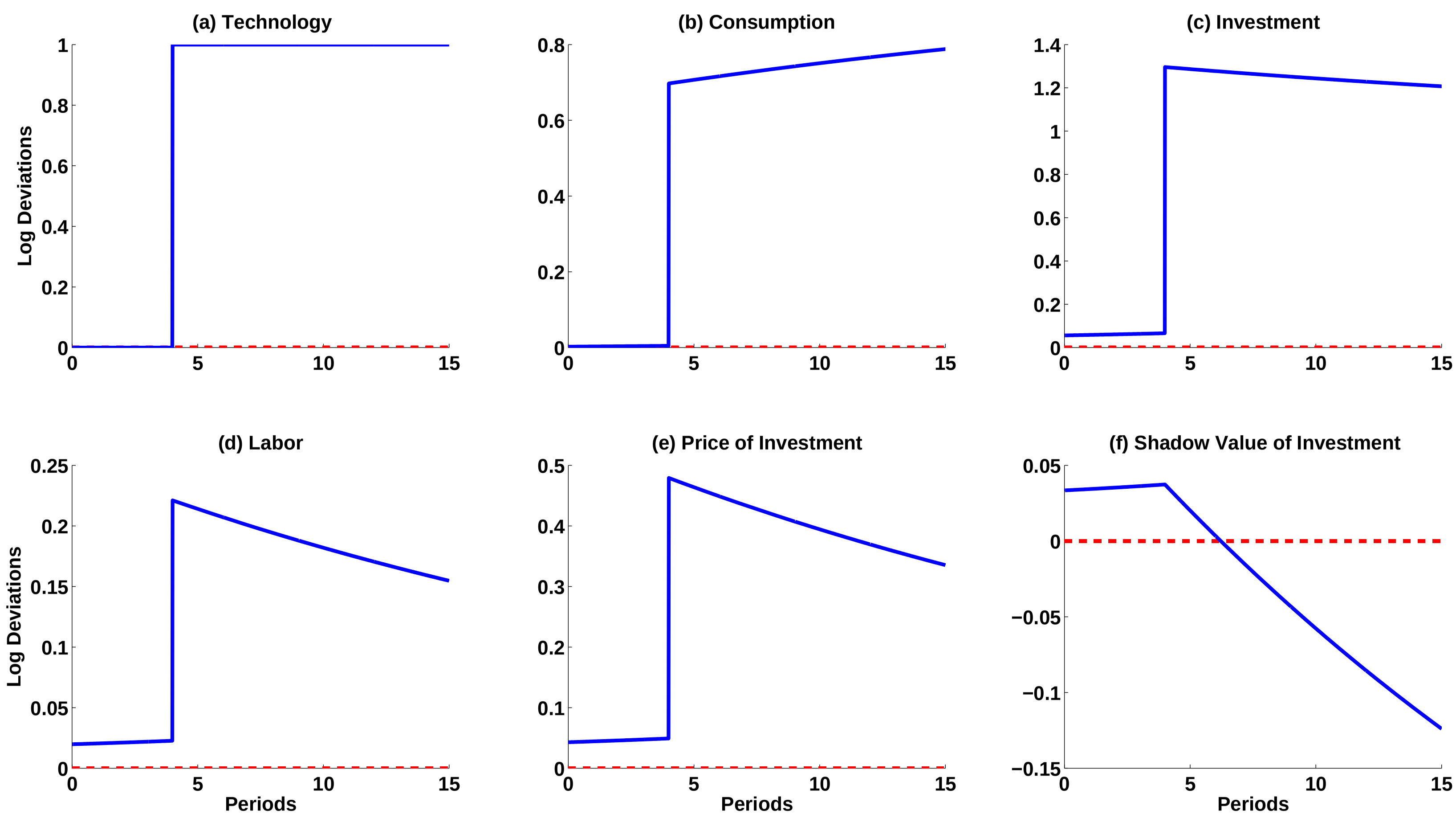


Figure 7: Impulse Response to a Positive News Shock - Labor Externality Model)

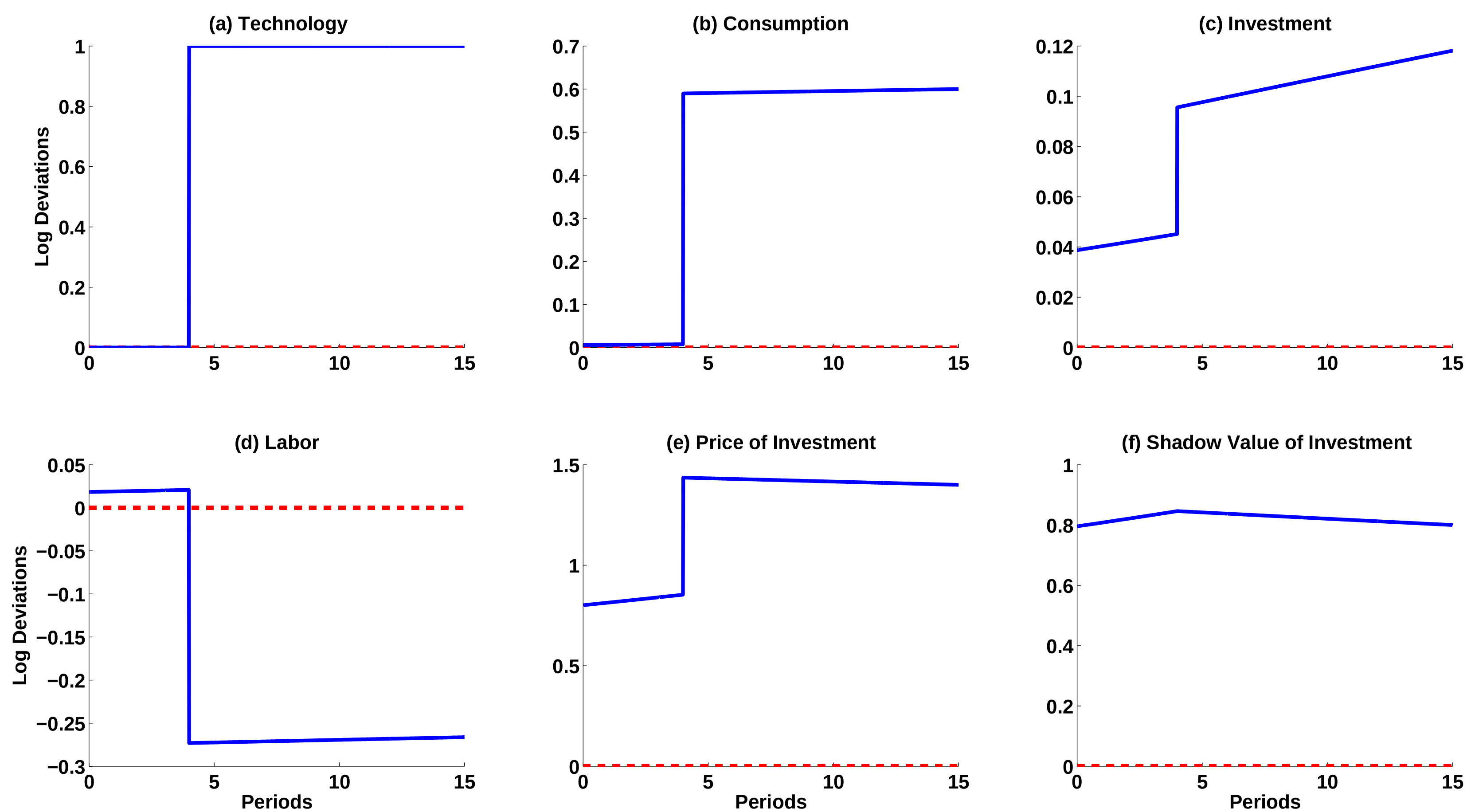


Figure 8: Impulse Response to a Positive News Shock - Variants on the Production Complementarity Model: GHH Preferences and Investment Adjustment Costs

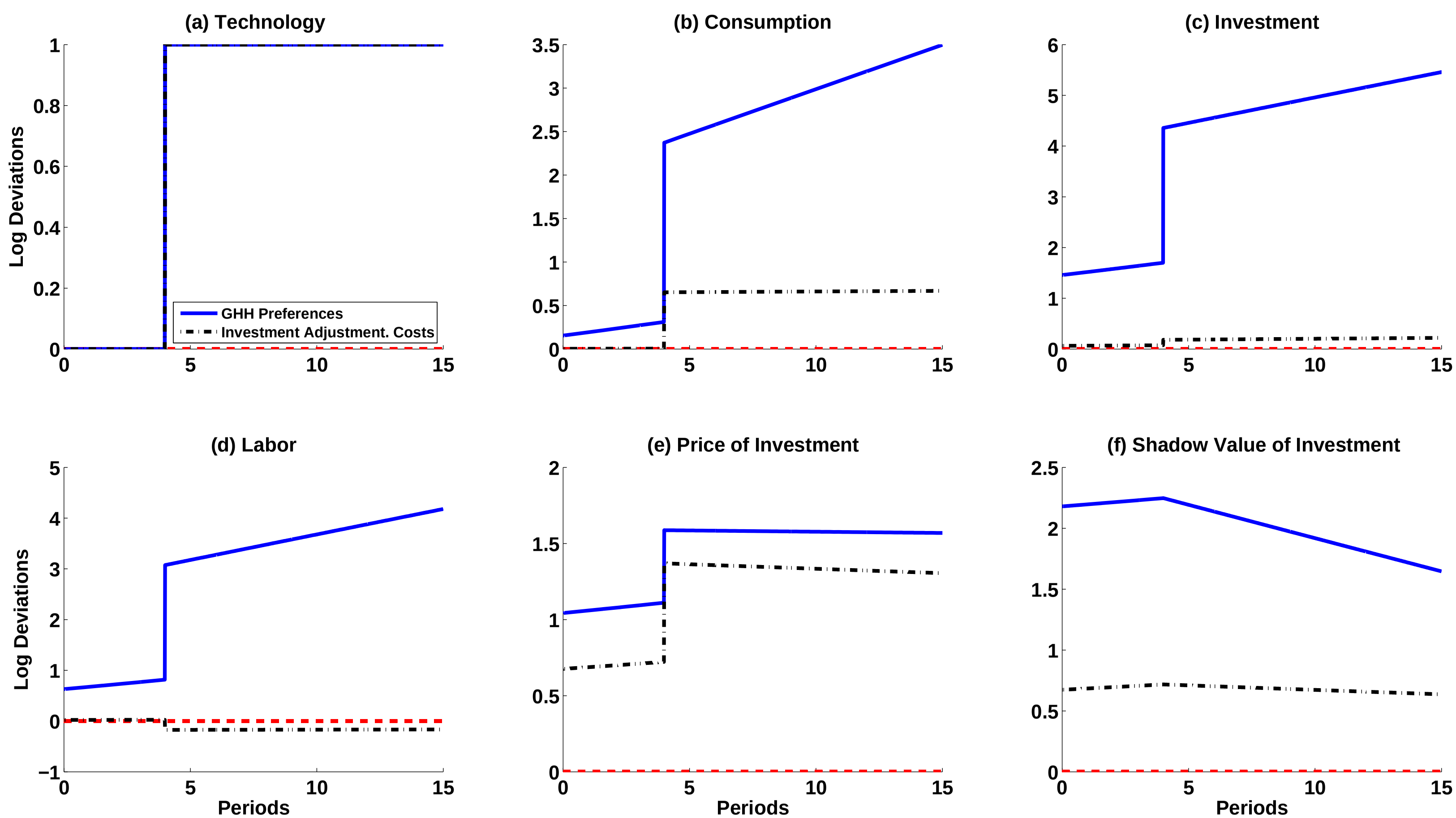
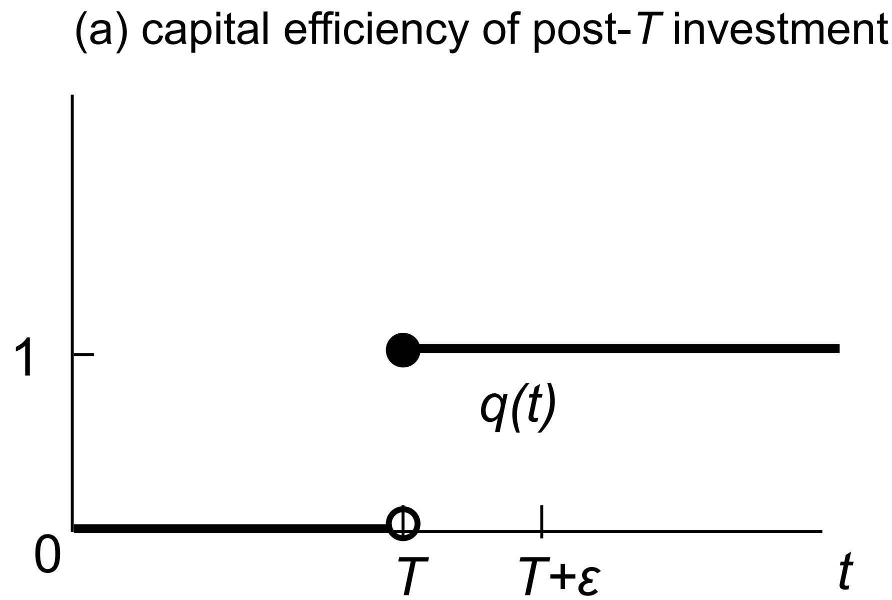


Figure 9: Efficiency of Different Types of Capital



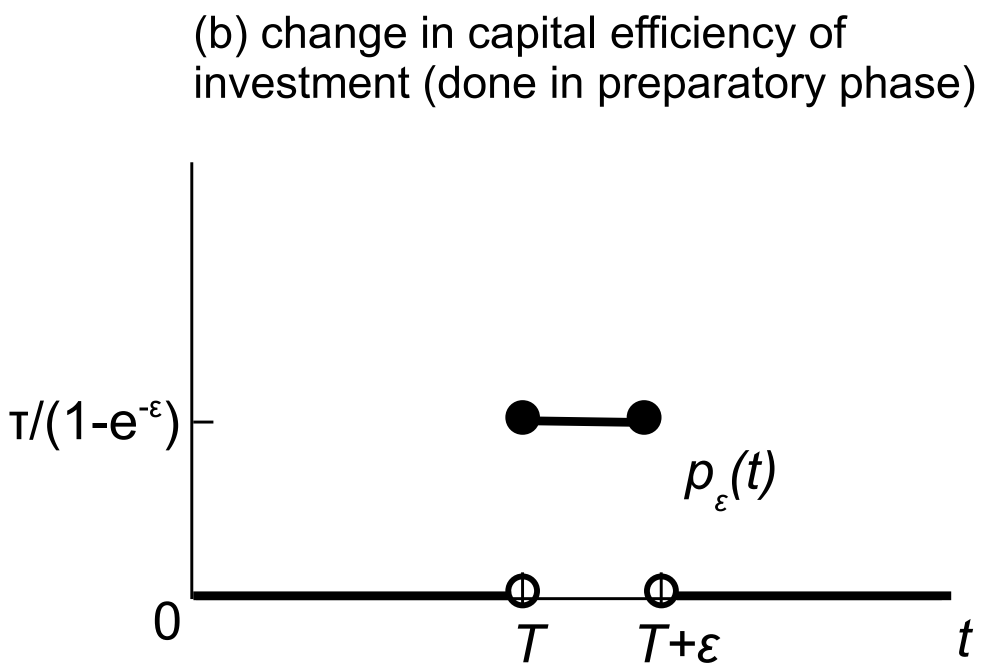


Figure 10: Impulse Response to a Positive News Shock - Forward Compatible Investment

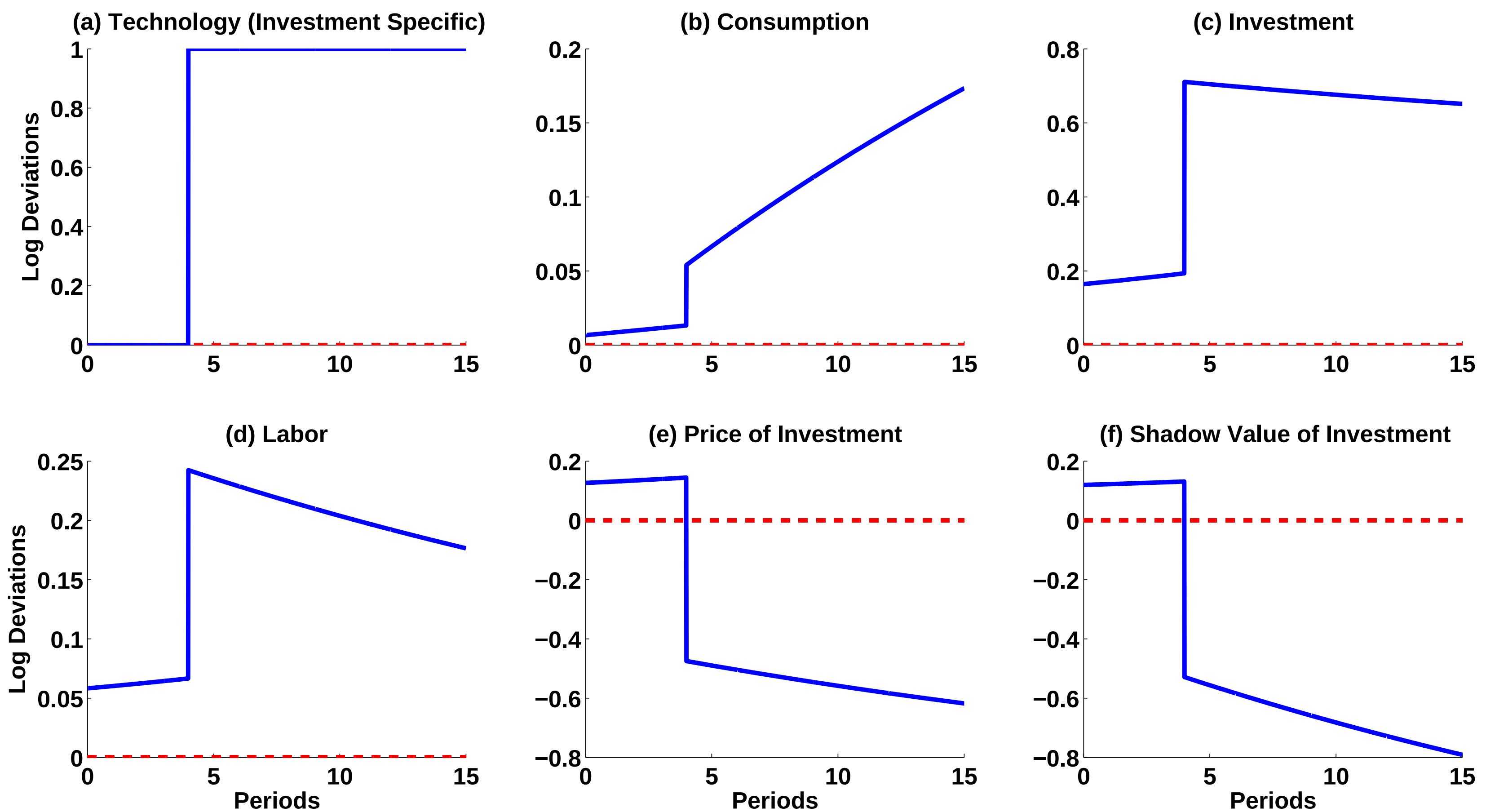
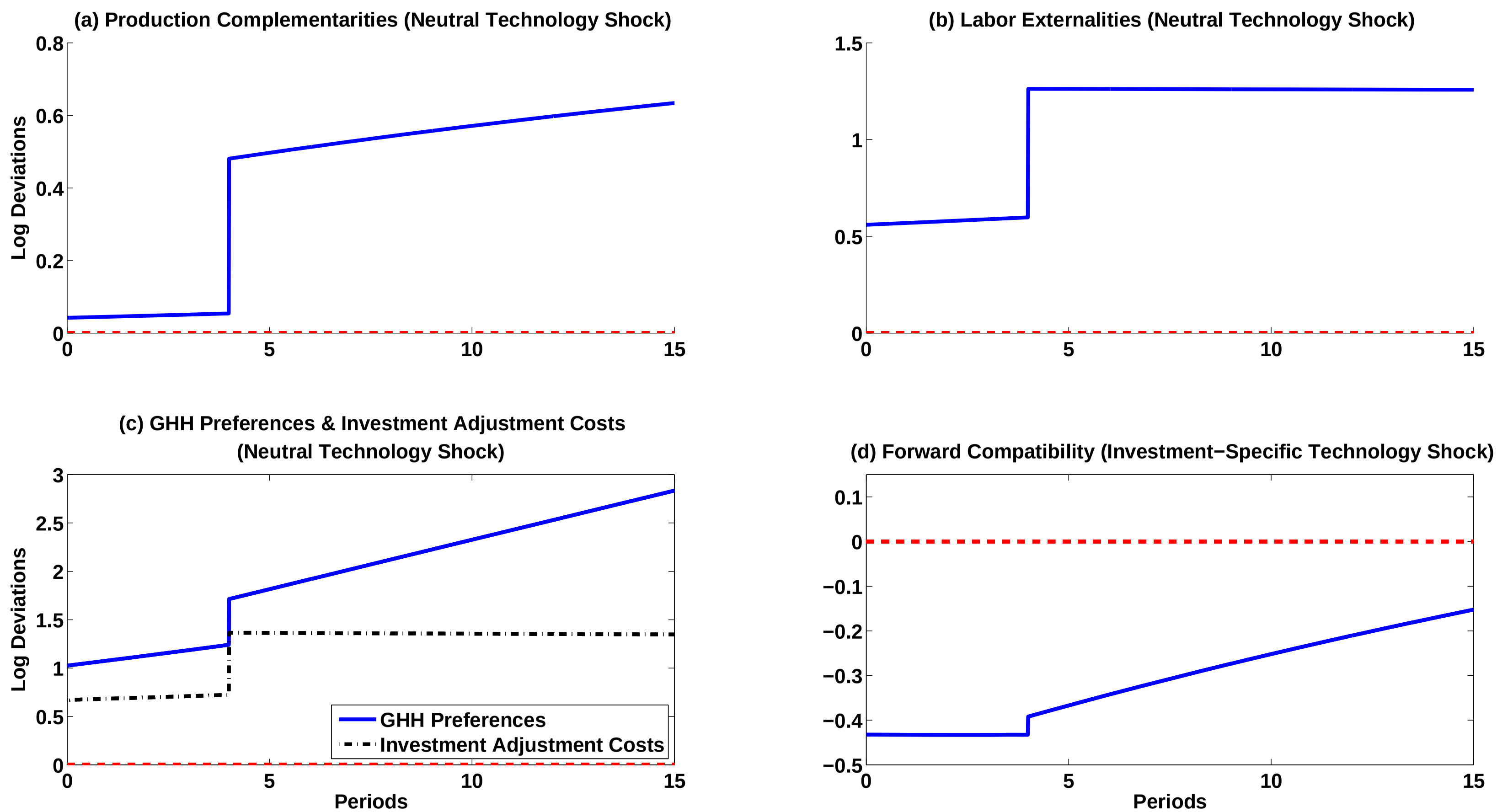


Figure 11: Impulse Response to a Positive News Shock - Value of the Firm



A Lemmas and Theorems

Proof of Lemma 1. Substituting (9) into (12):

$$\begin{aligned}\alpha k + (1-\alpha)\, a &= (1-(1-\upsilon(1-\alpha))\, s_I)\, c + (1-\upsilon(1-\alpha))\, s_I i \\ &= \left(1-\phi_I^{PC}\right) c + \phi_I^{PC} i \end{aligned} \tag{A.1}$$

Here $\phi_I^{PC} = (1-\upsilon(1-\alpha))\, s_I$.

$k(0) = 0$ and $a(0) = 0$, therefore (A.1) can be written as:

$$0 = \left(1-\phi_I^{PC}\right) c(0) + \phi_I^{PC} i(0) \tag{A.2}$$

First, if $\upsilon(1-\alpha) > 1$, then $\phi_I^{PC} < 0$ & $\left(1-\phi_I^{PC}\right) > 0$. Therefore, if $\upsilon(1-\alpha) > 1$ and $c(0)$ increases then for (A.2) to hold $i(0)$ must also increase.

Further, if $\upsilon(1-\alpha) > 1$, then $\left(1-\phi_I^{PC}\right) = \left(-\phi_I^{PC}+1\right) > -\phi_I$. Therefore, if $c(0)$ increases, then for (A.2) to hold $i(0)$ must increase by a larger magnitude than $c(0)$, this implies $(i(0)-c(0))$ increases when $c(0)$ increases, which in turn due to (9) implies that $n(0)$ must increase.

Therefore, if $\upsilon(1-\alpha) > 1$, then consumption, investment, and hours will comove at time zero.

Second, if $\upsilon(1-\alpha) < 1$, then $\phi_I^{PC} \in (0,1)$ and $\left(1-\phi_I^{PC}\right) > 0$. Therefore, if $c(0)$ increases then for (A.2) to hold $i(0)$ must decrease. Therefore, if $\upsilon(1-\alpha) < 1$, then consumption, investment and hours will not comove at time zero. □

Proof of Lemma 2. Substituting (9) into (10):

$$\gamma_I^{PC} i - \left(\sigma + \gamma_I^{PC}\right) c = \lambda \tag{A.3}$$

Here $\gamma_I^{PC} = (\upsilon - 1) - (\upsilon(1-\alpha)(1-\sigma)\, s_I) / (1-s_I)$.

Further, substituting (A.1) into (A.3) and solving for c at time 0 leads to:

$$c(0) = \frac{-\phi_I^{PC}}{\gamma_I^{PC} + \phi_I^{PC}\sigma} \lambda(0) \tag{A.4}$$

First, from the proof of lemma 1 we know that $-\phi_I^{PC} > 0$ if $\upsilon(1-\alpha) > 1$. Also, $\gamma_I^{PC} + \phi_I^{PC}\sigma > 0$ if $\upsilon(1-\alpha) > 1$.[43] From equation (A.4) if $\lambda(0) > 0$ then $c(0)$ will increase. If $c(0) > 0$ then from the proof of lemma 1 we know that both $i(0)$ and $n(0)$ will also increase.

As a result, if $\upsilon(1-\alpha) > 1$ and $\lambda(0) > 0$, then consumption, investment, and labor hours will comove procyclically at time zero in response to a news shock about technology in time $T > 0$.

Second, by Lemma 1 we also know that if $\upsilon(1-\alpha) < 1$ and $\lambda(0) > 0$, then consumption and investment will not comove at time zero. □

Proof of Lemma 3. Solving (A.1) and (A.3) simultaneously for the values of c and i:

$$c = \tau_{c,k}^{PC} k + \tau_{c,\lambda}^{PC} \lambda + \tau_{c,a}^{PC} a \tag{A.5}$$

$$i = \tau_{i,k}^{PC} k + \tau_{i,\lambda}^{PC} \lambda + \tau_{i,a}^{PC} a \tag{A.6}$$

and substituting into (9):

$$n = \tau_{n,k}^{PC} k + \tau_{n,\lambda}^{PC} \lambda + \tau_{n,a}^{PC} a \tag{A.7}$$

Here $\tau_{c,k}^{PC}, \tau_{c,\lambda}^{PC} \tau_{i,k}^{PC}, \tau_{i,\lambda}^{PC}, \tau_{n,k}^{PC}$, and $\tau_{n,\lambda}^{PC}$ are all positive.[44]

It follows directly that if $\dot{\lambda} \geq 0$ and $\dot{k} \geq 0$ $\forall t < T$ then $\dot{c} \geq 0$, $\dot{i} \geq 0$, and $\dot{n} \geq 0$ for all $t < T$. Again, remember for $\forall t < T$, $a(t) = 0$. □

Proof of Lemma 4. Recall $k(0) = 0$. As a result, the time derivatives of the $k(t)$ and $\lambda(t)$ paths for all $t < T$:

$$\dot{k}(t) = \frac{\Gamma_{k,\lambda}^{PC}\left(\mu_2^{PC} e^{\mu_2^{PC} t} - \mu_1^{PC} e^{\mu_1^{PC} t}\right)}{\mu_2^{PC} - \mu_1^{PC}} \lambda(0)$$

$$\dot{\lambda}(t) = \left[\frac{\left(\mu_2^{PC} - \Gamma_{k,k}^{PC}\right)}{\mu_2^{PC} - \mu_1^{PC}} \mu_2^{PC} e^{\mu_2^{PC} t} - \frac{\left(\mu_1^{PC} - \Gamma_{k,k}^{PC}\right)}{\mu_2^{PC} - \mu_1^{PC}} \mu_1^{PC} e^{\mu_1^{PC} t}\right] \lambda(0)$$

First, for $0 \leq t < T$: $\left(\Gamma_{k,\lambda}^{PC}\left(\mu_2^{PC} e^{\mu_2^{PC} t} - \mu_1^{PC} e^{\mu_1^{PC} t}\right)\right) / \left(\mu_2^{PC} - \mu_1^{PC}\right)$ is positive as $\Gamma_{k,\lambda}^{PC} > 0$, [45] and we know that $\mu_2^{PC} > 0$ and $\mu_1^{PC} < 0$. Therefore the $sign(\dot{k}(t)) = sign(\lambda_0)$.

[43] For the proof see Lemma B.3 in Appendix B.
[44] For the proof see Lemma B.4 in Appendix B.
[45] For the proof see Lemma B.5 in Appendix B.

Second, for the $\dot{\lambda}$ equation: $\left(\mu_2^{PC} - \Gamma_{k,k}^{PC}\right)\mu_2^{PC} e^{\mu_2^{PC} t} / \left(\mu_2^{PC} - \mu_1^{PC}\right)$ is positive because $\mu_2^{PC} - \Gamma_{k,k}^{PC} = \Gamma_{\lambda,\lambda}^{PC} - \mu_1^{PC}$,[46] and we know $\mu_1^{PC} < 0$ and $\Gamma_{\lambda,\lambda}^{PC} > 0$.[47]

$\left(\mu_1^{PC} - \Gamma_{k,k}^{PC}\right)\mu_1^{PC} e^{\mu_1^{PC} t} / \left(\mu_2^{PC} - \mu_1^{PC}\right)$ may be either positive or negative. If $\mu_1^{PC} - \Gamma_{k,k}^{PC} > 0$, then the second term on the right-hand side is positive. In this case, $\dot{\lambda}(t) > 0$. However, if $\mu_1^{PC} - \Gamma_{k,k}^{PC} < 0$, then $\left(\mu_1^{PC} - \Gamma_{k,k}^{PC}\right)\mu_1^{PC} e^{\mu_1^{PC} t} / \left(\mu_2^{PC} - \mu_1^{PC}\right)$ is negative. In this case, we must show that $\left(\mu_2^{PC} - \Gamma_{k,k}^{PC}\right)\mu_2^{PC} e^{\mu_2^{PC} t} / \left(\mu_2^{PC} - \mu_1^{PC}\right)$ is larger than $\left(\mu_1^{PC} - \Gamma_{k,k}^{PC}\right)\mu_1^{PC} e^{\mu_1^{PC} t} / \left(\mu_2^{PC} - \mu_1^{PC}\right)$ in order that $\dot{\lambda}(t) > 0$. Because $\mu_2^{PC} > 0 > \mu_1^{PC}$, in this second case, the smallest value for $\dot{\lambda}(t)$ occurs at $t = 0$.

$$
\begin{aligned}
\dot{\lambda}(0) &= \frac{\lambda(0)}{\mu_2^{PC} - \mu_1^{PC}}\left[\mu_2^{PC}\left(\mu_2^{PC} - \Gamma_{k,k}^{PC}\right) - \mu_1^{PC}\left(\mu_1^{PC} - \Gamma_{k,k}^{PC}\right)\right] \\
&= \lambda_0\left[\mu_2^{PC} + \mu_1^{PC} - \Gamma_{k,k}^{PC}\right] \\
&= \lambda_0\left[\Gamma_{k,k}^{PC} + \Gamma_{\lambda,\lambda}^{PC} - \Gamma_{k,k}^{PC}\right] \\
&= \lambda_0 \Gamma_{\lambda,\lambda}^{PC}
\end{aligned}
$$

As $\Gamma_{\lambda,\lambda}^{PC} > 0$, this establishes that $sign(\dot{\lambda}(t)) = sign(\lambda_0)$. □

Proof of Lemma 5. Recall $\mu_2^{PC} > 0$ and

$$
k(t) = \begin{cases}
\frac{\Gamma_{k,\lambda}^{PC}\lambda(0) + \left(\mu_1^{PC} - \Gamma_{\lambda,\lambda}^{PC}\right)k(0)}{\mu_1^{PC} - \mu_2^{PC}} e^{\mu_1^{PC} t} + \frac{\Gamma_{k,\lambda}^{PC}\lambda(0) + \left(\mu_2^{PC} - \Gamma_{\lambda,\lambda}^{PC}\right)k(0)}{\mu_2^{PC} - \mu_1^{PC}} e^{\mu_2^{PC} t} & \text{for } t \in [0, T) \\
\frac{\Gamma_{k,\lambda}^{PC}\lambda(0) + \left(\mu_1^{PC} - \Gamma_{\lambda,\lambda}^{PC}\right)k(0)}{\mu_1^{PC} - \mu_2^{PC}} e^{\mu_1^{PC} t} + \frac{\Gamma_{k,\lambda}^{PC} b_{\lambda,a}^{PC} - \Gamma_{\lambda,\lambda}^{PC} b_{k,a}^{PC}}{\mu_1^{PC}\mu_2^{PC}} + \frac{\Gamma_{k,\lambda}^{PC} b_{\lambda,a}^{PC} + \left(\mu_1^{PC} - \Gamma_{\lambda,\lambda}^{PC}\right)b_{k,a}^{PC}}{\mu_1^{PC}\left(\mu_1^{PC} - \mu_2^{PC}\right)} e^{\mu_1^{PC}(t-T)} & t \geq T
\end{cases}
$$

Then as $k(0) = 0$ a non-explosive path for $[\lambda\ k]'$ requires that we choose $\lambda(0)$ such that the terms involving the explosive root μ_2^{PC} in the exponential are 'zeroed out' for all $t > T$. Otherwise the path for $k(t)$ will be explosive. This imposes the following restriction on $\lambda(0)$:

$$
\left(\frac{\Gamma_{k,\lambda}^{PC}}{\mu_2^{PC} - \mu_1^{PC}}\right)\lambda_0 = -\frac{\Gamma_{k,\lambda}^{PC} b_{\lambda,a}^{PC} + \left(\mu_2^{PC} - \Gamma_{\lambda,\lambda}^{PC}\right) b_{k,a}^{PC}}{\mu_2^{PC}\left(\mu_2^{PC} - \mu_1^{PC}\right)} e^{-\mu_2^{PC} T}
$$

[46] This follows because $tr(\Gamma^{PC}) = \mu_1^{PC} + \mu_2^{PC}$.

[47] For the proof see Lemma B.5 in Appendix B.

This can be re-written as:

$$\lambda_0 = -\left[\frac{\Gamma^{PC}_{k,\lambda} b^{PC}_{\lambda,a} + \left(\mu^{PC}_2 - \Gamma^{PC}_{\lambda,\lambda}\right) b^{PC}_{k,a}}{\Gamma^{PC}_{k,\lambda}\mu^{PC}_2}\right] e^{-\mu^{PC}_2 T} \tag{A.8}$$

Because $\Gamma^{PC}_{k,\lambda} > 0$, $\lambda(0) > 0$ if and only if $\Gamma^{PC}_{k,\lambda} b^{PC}_{\lambda,a} + \left(\mu_2 - \Gamma^{PC}_{\lambda,\lambda}\right) b^{PC}_{k,a} < 0$. Also, $\Gamma^{PC}_{k,\lambda} b^{PC}_{\lambda,a} + \left(\mu_2 - \Gamma^{PC}_{\lambda,\lambda}\right) b^{PC}_{k,a} < 0$ algebraically simplifies to $\mu^{PC}_2 < (\rho + (1-\alpha)\,\delta)\,\upsilon / (\gamma_I + \sigma)$. □

Proof of Theorem 1. $\Leftarrow$. If $\upsilon(1-\alpha) > 1$ and $\mu^{PC}_2 < (\rho + (1-\alpha)\,\delta)\,\upsilon / \left(\gamma^{PC}_I + \sigma\right)$, then a technology news shock is procyclical. Lemmas 2 and 5 prove the procyclical comovement at $t = 0$, while Lemmas 3, 4 and 5 establish the procyclical comovement for $0 < t < T$.

$\Rightarrow$. If $\upsilon(1-\alpha) < 1$ or $\mu^{PC}_2 < (\rho + (1-\alpha)\,\delta)\,\upsilon / \left(\gamma^{PC}_I + \sigma\right)$, then a technology news shock is not procyclical. This follow trivially from Lemma 2, as the procyclical comovement will not occur at time $t = 0$ if either of the above conditions are not met. □

Proof of Lemma 6. The condition $\mu^{PC}_2 < (\rho + (1-\alpha)\,\delta)\,\upsilon / \left(\gamma^{PC}_I + \sigma\right)$ can be rewritten implicitly as $\sigma < \sigma^*$. As $\delta \to 0$ we have $\sigma^* \to 1$. The above lemma thus follows directly from Theorem 1. □

Proof of Lemma 7. Substituting (24) into (12):

$$\begin{aligned} \alpha k + (1-\alpha)\,a &= \left(1 - \left(1 - \frac{1-\alpha}{1-\gamma_N}\right) s_I\right) c + \left(1 - \frac{1-\alpha}{1-\gamma_N}\right) s_I i \\ &= \left(1 - \phi^{LE}_I\right) c + \phi^{LE}_I i \end{aligned} \tag{A.9}$$

Here $\phi^{LE}_I = \left(1 - \frac{1-\alpha}{1-\gamma_{\bar{N}}}\right) s_I$.

$k(0) = 0$ and $a(0) = 0$, therefore (A.9) can be written as:

$$0 = \left(1 - \phi^{LE}_I\right) c(0) + \phi^{LE}_I i(0) \tag{A.10}$$

First, if $\gamma_N > \alpha$, then $\phi^{LE}_I < 0$ & $\left(1 - \phi^{LE}_I\right) > 0$. Therefore, if $\gamma_N > \alpha$ and $c(0)$ increases then for (A.10) to hold $i(0)$ must also increase.

Further, if $\gamma_N > \alpha$, then $\left(1 - \phi_I^{LE}\right) = \left(-\phi_I^{LE} + 1\right) > -\phi_I^{LE}$. Therefore, if $c(0)$ increases, then for (A.10) to hold $i(0)$ must increase by a larger magnitude than $c(0)$, this implies $(i(0) - c(0))$ increases when $c(0)$ increases, which in turn due to (24) implies that $n(0)$ must increase.

Therefore, if $\gamma_N > \alpha$, then consumption, investment, and hours will comove at time zero.

Second, if $\gamma_N < \alpha$, then $\phi_I^{LE} > 0$ and $\left(1 - \phi_I^{LE}\right) > 0$. Therefore, if $c(0)$ increases then for (A.10) to hold $i(0)$ must decrease. Therefore, if $\gamma_N < \alpha$, then consumption, investment and hours will not comove at time zero. □

Proof of Lemma 8. For a stable solution to exist one eigenvalue of Γ^{LE} should be positive and the other negative. The product of the eigenvalues is given by the determinant of the Γ^{LE} matrix.

$$\det\left(\Gamma^{LE}\right) = \frac{-\delta\left(\rho + \delta\right)}{\left(\phi_I^{LE}\sigma + \gamma_I^{LE} + \psi_I\left(1 - \phi_I^{LE}\right)\right)}\left[\left(1 - s_I\right)\left(1 - \alpha\right)\right]$$

First, if $\psi_I > \psi_I^+ = -\frac{\gamma_I^{LE} + \phi_I^{LE}\sigma}{1 - \phi_I^{LE}}$ then the product of the eigenvalues is negative and it follows that the eigenvalues have opposite signs. Further, it can be shown that $\operatorname{tr}\left(\Gamma^{LE}\right) = \rho$ which gives the sum of the two eigenvalues.

Second, if $\psi_I < \psi_I^+ = -\frac{\gamma_I^{LE} + \phi_I^{LE}\sigma}{1 - \phi_I^{LE}}$ then the product of the eigenvalues is positive and with $\operatorname{tr}\left(\Gamma^{LE}\right) = \rho$, which gives the sum of the two eigenvalues, it follows that the eigenvalues are both positive. □

Proof of Lemma 9. Recall $\mu_2^{LE} > 0$. Also, $\Gamma_{k,\lambda}^{LE} > 0$.[48] For a stable solution we need:

$$\lambda_0 = -\left[\frac{\Gamma_{k,\lambda}^{LE} b_{\lambda,a}^{LE} + \left(\mu_2^{LE} - \Gamma_{\lambda,\lambda}^{LE}\right) b_{k,a}^{LE}}{\Gamma_{k,\lambda}^{LE}\mu_2^{LE}}\right] e^{-\mu_2^{LE} T} \tag{A.11}$$

As a result $\lambda(0) > 0$ if and only if $\Gamma_{k,\lambda}^{LE} b_{\lambda,a}^{LE} + \left(\mu_2^{LE} - \Gamma_{\lambda,\lambda}^{LE}\right) b_{k,a}^{LE} < 0$. □

Proof of Theorem 2. Given the proofs and results of lemmas 7 through 9, to prove this theorem we must establish that when $\gamma_N > \gamma_N^*$ and $\psi_I > \psi_I^+$ three results hold: (1) $c(0) > 0$ if and only if $\lambda(0) > 0$. (2) Consumption, investment and hours will comove procyclically for all time $t < T$ if $\forall t < T$, $\dot{\lambda} \geq 0$ and $\dot{k} \geq 0$. (3) if $\lambda(0) > 0$ then $\dot{\lambda} \geq 0$ and $\dot{k} \geq 0$.

[48] For the proof see Lemma B.10 in Appendix B.

(1): $c(0) > 0$ if and only if $\lambda(0) > 0$ and $\gamma_N > \gamma_N^*$ follows from the observation that we can substitute 24 into 25, and the result into A.10 to get an equation of the form $c(0) = \zeta^{LE}\lambda(0)$ where $\zeta^{LE} = \frac{-\phi_I^{LE}}{\gamma_I^{LE}+\phi_I^{LE}\sigma+\psi_I\left(1-\phi_I^{LE}\right)}$. $\zeta^{LE} > 0$ follows trivially from $\psi_I > \psi_I^+$ and $\gamma_N > \gamma_N^* \Rightarrow \phi^{LE} < 0$.

(2): We can solve for and define $x = \tau_{x,k}^{LE}k + \tau_{x,\lambda}^{LE}\lambda + \tau_{x,a}^{LE}a$ for $x = c, i, n$. Here $\tau_{c,k}^{LE}$, $\tau_{c,\lambda}^{LE}$, $\tau_{i,k}^{LE}$, $\tau_{i,\lambda}^{LE}$, $\tau_{n,k}^{LE}$, and $\tau_{n,\lambda}^{LE}$ are all positive[49], as result it trivially follows that if $\forall t < T$, $\dot{\lambda} \geq 0$ and $\dot{k} \geq 0$ then consumption, investment and hours will comove procyclically for all time $t < T$.

(3): The dynamic system given by (28) takes the same form as the dynamic system given by (17). As a result showing that $\dot{\lambda} \geq 0$ and $\dot{k} \geq 0$ if $\lambda(0) > 0$ amounts, exactly as in lemma 4, to proving that $\Gamma_{k,\lambda}^{LE} > 0$ and $\Gamma_{\lambda,\lambda}^{LE} > 0$. [50] $\Gamma_{k,\lambda}^{LE} > 0$ and $\Gamma_{\lambda,\lambda}^{LE} > 0$ follow from $\psi_I > \psi_I^+$ and $\gamma_N > \gamma_N^* \Rightarrow \phi^{LE} < 0$.

Results (1) - (3) together establish that if $\gamma_N > \gamma_N^*$, $\psi_I > \psi_I^+$, and $\lambda(0) > 0$ then the labor externality model exhibits procyclical technology news shocks. From lemma 9 we further know that $\lambda(0) > 0$ if and only if $\psi_I > \psi_I^*$. □

Proof of Theorem 3. $\Gamma^{FC} = \Gamma^{PC}$ and $\tau_{x,y}^{FC} = \tau_{x,y}^{PC}$ for $x = i, c, n$ and $y = k, \lambda$. Hence, for a model with forward compatible investment lemmas 1 through 4 still hold as before.

Now, recall $\mu_2^{FC} = \mu_2^{PC} > 0$. Also, $\Gamma_{k,\lambda}^{FC} = \Gamma_{k,\lambda}^{PC} > 0$. For a stable solution we need:

$$\lambda(0) = -\left[\frac{\Gamma_{k,\lambda}^{FC}\left(b_{\lambda,q}^{FC} + \tau\mu_2^{FC}b_{\lambda,p}^{FC}\right) + \left(\mu_2^{FC} - \Gamma_{\lambda,\lambda}^{FC}\right)\left(b_{k,q}^{FC} + \tau\mu_2^{FC}b_{k,p}^{FC}\right)}{\Gamma_{k,\lambda}^{FC}\mu_2^{FC}}\right]e^{-\mu_2^{FC}T} \tag{A.12}$$

As a result $\lambda(0) > 0$ if and only if $\tau > \frac{\Gamma_{k,\lambda}^{FC}b_{\lambda,q}^{FC}+\left(\mu_2^{FC}-\Gamma_{\lambda,\lambda}^{FC}\right)b_{k,q}^{FC}}{\Gamma_{k,\lambda}^{FC}\mu_2^{FC}b_{\lambda,p_\epsilon}^{FC}+\left(\mu_2^{FC}-\Gamma_{\lambda,\lambda}^{FC}\right)\mu_2^{FC}b_{k,p_\epsilon}^{FC}}$.

$\Leftarrow$. If $\upsilon(1-\alpha) > 1$ and $\tau > \frac{\Gamma_{k,\lambda}^{FC}b_{\lambda,q}^{FC}+\left(\mu_2^{FC}-\Gamma_{\lambda,\lambda}^{FC}\right)b_{k,q}^{FC}}{\Gamma_{k,\lambda}^{FC}\mu_2^{FC}b_{\lambda,p_\epsilon}^{FC}+\left(\mu_2^{FC}-\Gamma_{\lambda,\lambda}^{FC}\right)\mu_2^{FC}b_{k,p_\epsilon}^{FC}}$, then a investment technology news shock is procyclical. Lemmas 2 and the result above prove the procyclical comovement at $t = 0$, while Lemmas 3, 4 and the result above establish the procyclical comovement for $0 < t < T$.

$\Rightarrow$. If $\upsilon(1-\alpha) < 1$ or $\tau > \frac{\Gamma_{k,\lambda}^{FC}b_{\lambda,q}^{FC}+\left(\mu_2^{FC}-\Gamma_{\lambda,\lambda}^{FC}\right)b_{k,q}^{FC}}{\Gamma_{k,\lambda}^{FC}\mu_2^{FC}b_{\lambda,p_\epsilon}^{FC}+\left(\mu_2^{FC}-\Gamma_{\lambda,\lambda}^{FC}\right)\mu_2^{FC}b_{k,p_\epsilon}^{FC}}$, then a technology news shock is not procyclical. This follow trivially from Lemma 2, as the procyclical comovement will not

[49] For the proof see Lemma B.9 in Appendix B.

[50] For the proof see Lemma B.10 in Appendix B.

occur at time $t = 0$ if either of the above conditions are not met. $\square$