Example: Multiple Standard Addtions

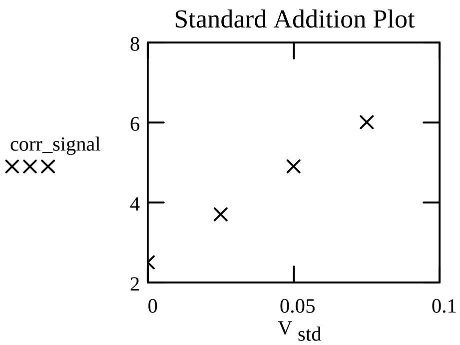
$V_a := 50.00$ volume of sample solution, in mL $C_{std} := 1000$ concentration of lead standard, in ppm

$V_{std} := (0 \;\; 0.025 \;\; 0.050 \;\; 0.075)^T$ volume added standard (converted to mL) $signal := (2.5 \;\; 3.7 \;\; 4.9 \;\; 6.0)^T$ uncorrected signal

We must correct the signal for the effects of dilution.

$i := 0..3$ $corr\_signal_i := signal_i \cdot \dfrac{V_a + V_{std_i}}{V_a}$ $corr\_signal^T = [\,2.5000 \;\; 3.7018 \;\; 4.9049 \;\; 6.0090\,]$

Let's examine the standard addition plot



Looks linear. Here are the LS estimates.

$b_0 := intercept(V_{std}, corr\_signal)$ $b_0 = 2.5194$

$b_1 := slope(V_{std}, corr\_signal)$ $b_1 = 46.9202$

$S_{xx} := 2 \cdot Var(V_{std})$ $xbar := mean(V_{std})$

First let's calculate the point estimate, then its standard error (and the confidence interval)

$Vprime := \dfrac{b_0}{b_1}$ $Vprime = 0.0537$ $C_a := C_{std} \cdot \dfrac{Vprime}{V_a}$ $C_a = 1.0739$ conc analyte, in ppm

$fit := b_1 \cdot V_{std} + b_0$ $res := signal - fit$ $s_{res} := \sqrt{\dfrac{1}{2} \cdot \sum \overrightarrow{res^2}}$ $s_{res} = 0.0394$

$se_{prime} := \dfrac{s_{res}}{b_1} \cdot \sqrt{1 + \dfrac{1}{4} + \dfrac{(Vprime + xbar)^2}{S_{xx}}}$ $se_{prime} = 1.9233 \cdot 10^{-3}$ std error in Vprime

$se_a := se_{prime} \cdot \dfrac{C_{std}}{V_a}$ $se_a = 0.0385$ std error in analyte conc, in ppm

$t := qt(.975, 2)$ $t = 4.3027$ $t \cdot se_a = 0.1655$ width of 95% CI

The lead concentration in the acetic acid leach was 1.07 +/- 0.16 ppm