

Final Exam - Solutions to Extra Derivations

For each of the following construct a proof demonstrating that the validity claim holds.

1. $\{(\forall x)(\sim Fx \vee \sim Gx), Fa\} \vdash \sim (\forall x)(\sim Fx \vee Gx)$

1	$(\forall x)(\sim Fx \vee \sim Gx)$	Assumption
2	Fa	Assumption
3	$\sim Fa \vee \sim Ga$	1, $\forall E$
4	$\sim \sim Fa$	2, DN
5	$\sim Ga$	3, 4, $\vee E$
6	$\sim \sim Fa \& \sim Ga$	4, 5, $\&I$
7	$\sim (\sim Fa \vee Ga)$	6, DeM
8	$(\exists x) \sim (\sim Fx \vee Gx)$	7, $\exists I$
9	$\sim (\forall x)(\sim Fx \vee Gx)$	8, QN

2. $\{(\forall x)(Fx \supset Gx), Fm \vee Fn\} \vdash (\exists x)Gx$

1	$(\forall x)(Fx \supset Gx)$	Assumption
2	$Fm \vee Fn$	Assumption
3	$\sim (\exists x)Gx$	Assumption
4	$(\forall x) \sim Gx$	3, QN
5	$Fm \supset Gm$	1, $\forall E$
6	$\sim Gm$	4, $\forall E$
7	$\sim Fm$	5, 6, MT
8	Fn	2, 7, $\vee E$
9	$Fn \supset Gn$	1, $\forall E$
10	Gn	8, 9, $\supset E$
11	$\sim Gn$	4, $\forall E$
12	$(\exists x)Gx$	3-11, $\sim E$

3. $\{(\forall x)(Gx \supset (\exists y) \sim Kxy), (\exists x)(Fx \& (\forall y)Kxy)\} \vdash (\exists x)(Fx \& \sim Gx)$

1	$(\forall x)(Gx \supset (\exists y) \sim Kxy)$	Assumption
2	$(\exists x)(Fx \& (\forall y)Kxy)$	Assumption
3	$(Fa \& (\forall y)Kay)$	Assumption
4	$Ga \supset (\exists y) \sim Kay$	1, $\forall E$
5	$(\forall y)Kay$	3, $\&E$
6	$\sim \sim (\forall y)Kay$	5, DN
7	$\sim (\exists y) \sim Kay$	6, QN
8	$\sim Ga$	4, 7, MT
9	Fa	3, $\&E$
10	$Fa \& \sim Ga$	8, 9, $\&I$
11	$(\exists x)(Fx \& \sim Gx)$	10, $\exists I$
12	$(\exists x)(Fx \& \sim Gx)$	2, 3-11, $\exists E$

4. $\{(\forall x)[(Fx \equiv Hx) \supset Gx]\} \vdash (\forall x)(\sim Fx \& \sim Hx) \supset (\forall x)Gx$

1	$(\forall x)[(Fx \equiv Hx) \supset Gx]$	Assumption
2	$(\forall x)(\sim Fx \& \sim Hx)$	Assumption
3	$\sim Fa \& \sim Ha$	2, $\forall E$
4	$[(Fa \equiv Ha) \supset Ga]$	1, $\forall E$
5	$(Fa \& Ha) \vee (\sim Fa \& \sim Ha)$	4, $\vee I$
6	$Fa \equiv Ha$	5, Equiv
7	Ga	4, 6, $\supset E$
8	$(\forall x)Gx$	7, $\forall I$
9	$(\forall x)(\sim Fx \& \sim Hx) \supset (\forall x)Gx$	2-8, $\supset I$

5. $\{(\forall x)[Ax \supset (\forall y)(By \supset Cxy)], Am \& Bn\} \vdash Cmn$

1	$(\forall x)[Ax \supset (\forall y)(By \supset Cxy)]$	Assumption
2	$Am \& Bn$	Assumption
3	$Am \supset (\forall y)(By \supset Cmy)$	1, $\forall E$
4	Am	2, $\&E$
5	$(\forall y)(By \supset Cmy)$	3,4, $\supset E$
6	$Bn \supset Cmn$	5, $\forall E$
7	Bn	2, $\&E$
8	Cmn	6, 7, $\supset E$

6. $\{(\forall y)(My \supset Ay), (\exists y)(Cy \& My)\} \vdash (\exists y)(Cy \& Ay)$

1	$(\forall y)(My \supset Ay)$	Assumption
2	$(\exists y)(Cy \& My)$	Assumption
3	$Ca \& Ma$	Assumption
4	Ma	3, $\&E$
5	$Ma \supset Aa$	1, $\forall E$
6	Aa	4, 5, $\supset E$
7	Ca	3, $\&E$
8	$Ca \& Aa$	6, 7, $\&I$
9	$(\exists y)(Cy \& Ay)$	8, $\exists I$
10	$(\exists y)(Cy \& Ay)$	2, 3-9, $\equiv E$

7. $\{(\forall x)(Ax \vee Bx), (\forall x)(Bx \supset Ax)\} \vdash (\forall x)Ax$

1	$(\forall x)(Ax \vee Bx)$	Assumption
2	$(\forall x)(Bx \supset Ax)$	Assumption
3	$Aa \vee Ba$	1, $\forall E$
4	$Ba \supset Aa$	2, $\forall E$
5	$\sim \sim Aa \vee Ba$	3, DN
6	$\sim Aa \supset Ba$	5, Imp
7	$\sim Aa \supset Aa$	4, 6, HS
8	$\sim \sim Aa \vee Aa$	7, Imp
9	$Aa \vee Aa$	8, DN
10	Aa	9, Idem
11	$(\forall x)Ax$	10, $\forall I$

8. $\vdash (\forall x)(\forall y)[(Fx \supset Fy) \equiv (Fy \vee \sim Fx)]$

1	$Fa \supset Fb$	Assumption
2	$\sim Fa \vee Fb$	1, Imp
3	$Fb \vee \sim Fa$	2, Com
4	$Fb \vee \sim Fa$	Assumption
5	$\sim Fa \vee Fb$	4, Com
6	$Fa \supset Fb$	5, Imp
7	$(Fa \supset Fb) \equiv (Fb \vee \sim Fa)$	1-3, 4-6, $\equiv I$
8	$(\forall y)[(Fa \supset Fy) \equiv (Fy \vee \sim Fa)]$	7, $\forall I$
9	$(\forall x)(\forall y)[(Fx \supset Fy) \equiv (Fy \vee \sim Fx)]$	8, $\forall I$

9. $\{(\forall y)(Jy \supset (Ky \& Ly)), (\exists x) \sim Kx\} \vdash (\exists z) \sim Jz$

1	$(\forall y)(Jy \supset (Ky \& Ly))$	Assumption
2	$(\exists x) \sim Kx$	Assumption
3	$\sim Ka$	Assumption
4	$Ja \supset (Ka \& La)$	1, $\forall E$
5	$\sim Ka \vee \sim La$	3, $\vee I$
6	$\sim (Ka \& La)$	5, DeM
7	$\sim Ja$	4, 6, MT
8	$(\exists z) \sim Jz$	7, $\exists I$
9	$(\exists z) \sim Jz$	2, 3-8, $\equiv E$

10. $\{(\exists x)Fx \supset (\exists x)(Bx \& Cx), (\exists x)(Cx \vee Dx) \supset (\forall x)Ax\} \vdash (\forall x)(Fx \supset Ax)$

1	$(\exists x)Fx \supset (\exists x)(Bx \& Cx)$	Assumption
2	$(\exists x)(Cx \vee Dx) \supset (\forall x)Ax$	Assumption
3	Fa	Assumption
4	$(\exists x)Fx$	3, $\exists I$
5	$(\exists x)(Bx \& Cx)$	1, 4, $\supset E$
6	$Bb \& Cb$	Assumption
7	Cb	6, $\&E$
8	$Cb \vee Db$	7, $\vee I$
9	$(\exists x)(Cx \vee Dx)$	8, $\exists I$
10	$(\exists x)(Cx \vee Dx)$	5, 6-9, $\exists E$
11	$(\forall x)Ax$	2, 10, $\supset E$
12	Aa	11, $\forall E$
13	$Fa \supset Aa$	3-12, $\supset I$
14	$(\forall x)(Fx \supset Ax)$	13, $\forall I$

11. $\{(\forall x)Fx \supset (\exists x)Gx, (\forall x) \sim Gx\} \vdash (\exists x) \sim Fx$

1	$(\forall x)Fx \supset (\exists x)Gx$	Assumption
2	$(\forall x) \sim Gx$	Assumption
3	$\sim (\exists x)Gx$	2, QN
4	$\sim (\forall x)Fx$	1, 3, MT
5	$(\exists x) \sim Fx$	4, QN

12. $\{(\forall x)(Ax \& \sim Bx) \supset (\exists x)Cx, \sim (\exists x)(Cx \vee Bx)\} \vdash \sim (\forall x)Ax$

1	$(\forall x)(Ax \& \sim Bx) \supset (\exists x)Cx$	Assumption
2	$\sim (\exists x)(Cx \vee Bx)$	Assumption
3	$(\forall x) \sim (Cx \vee Bx)$	2, QN
4	$\sim (Ca \vee Ba)$	3, $\forall E$
5	$\sim Ca \& \sim Ba$	4, DeM
6	$\sim Ca$	5, $\&E$
7	$(\forall x) \sim Cx$	6, $\forall I$
8	$\sim (\exists x)Cx$	7, QN
9	$(\forall x)Ax$	Assumption
10	Aa	10, $\forall E$
11	$\sim Ba$	5, $\&E$
12	$Aa \& \sim Ba$	10, 11, $\&E$
13	$(\forall x)(Ax \& \sim Bx)$	12, $\forall I$
14	$(\exists x)Cx$	1, 13, $\supset E$
15	$\sim (\exists x)Cx$	8, R
16	$\sim (\forall x)Ax$	8-15, $\sim I$

13. $\{(\exists x)(\exists y)Rxy, (\forall x)(\forall y)(Rxy \supset (\forall z)Rxz), (\forall x)((\forall z)Rxz \supset (\forall y)Ryx)\} \vdash (\forall x)(\forall y)Rxy$ $\{(\forall x)Ax \equiv (\exists x)(Bx \& Cx), (\forall x)(Cx \supset Bx)\} \vdash (\forall x)Ax \equiv (\exists x)Cx$

1	$(\exists x)(\exists y)Rxy$	Assumption	1	$(\forall x)Ax \equiv (\exists x)(Bx \& Cx)$	Assumption
2	$(\forall x)(\forall y)(Rxy \supset (\forall z)Rxz)$	Assumption	2	$(\forall x)(Cx \supset Bx)$	Assumption
3	$(\forall x)((\forall z)Rxz \supset (\forall y)Ryx)$	Assumption	3	$(\forall x)Ax$	Assumption
4	$(\exists y)Ray$	Assumption	4	$(\exists x)(Bx \& Cx)$	1, 3, $\equiv E$
5	Rab	Assumption	5	$(Ba \& Ca)$	Assumption
6	$(\forall y)(Ray \supset (\forall z)Raz)$	2, $\forall E$	6	Ca	5, $\&E$
7	$Rab \supset (\forall z)Raz$	6, $\forall E$	7	$(\exists x)Cx$	6, $\exists I$
8	$(\forall z)Raz$	5, 7, $\supset E$	8	$(\exists x)Cx$	4, 5-7, $\exists E$
9	$(\forall z)Raz \supset (\forall y)Rya$	3, $\forall E$	9	$(\exists x)Cx$	Assumption
10	$(\forall y)Rya$	8, 9, $\supset E$	10	Cb	Assumption
11	$(\forall y)(Rcy \supset (\forall z)Rcz)$	2, $\forall E$	11	$Cb \supset Bb$	2, $\forall E$
12	$Rca \supset (\forall z)Rcz$	11, $\forall E$	12	Bb	10, 11, $\supset E$
13	Rca	10, $\forall E$	13	$Bb \& Cb$	10, 12, $\&I$
14	$(\forall z)Rcz$	12, 13, $\supset E$	14	$(\exists x)(Bx \& Cx)$	13, $\exists I$
15	$(\forall z)Rcz$	4, 5-14, $\exists E$	15	$(\exists x)(Bx \& Cx)$	9, 10-14, $\exists E$
16	$(\forall z)Rcz$	1, 4-15, $\exists E$	16	$(\forall x)Ax$	1, 15, $\equiv E$
17	Rcd	16, $\forall E$	17	$(\forall x)Ax \equiv (\exists x)Cx$	3-8, 9-16, $\equiv I$
18	$(\forall y)Rcy$	17, $\forall I$			
19	$(\forall x)(\forall y)Rxy$	18, $\forall I$			

15. $\{(\exists x) \sim Ax \vee (\exists x) \sim Bx, (\forall x)Bx\} \vdash \sim (\forall x)Ax$

1	$(\exists x) \sim Ax \vee (\exists x) \sim Bx$	Assumption
2	$(\forall x)Bx$	Assumption
3	$\sim \sim (\forall x)Bx$	2, DN
4	$\sim (\exists x) \sim Bx$	3, QN
5	$(\exists x) \sim Ax$	1, 4, $\vee E$
6	$\sim (\forall x)Ax$	5, QN

16. $\vdash (\forall x)(Ax \& (\exists y) \sim Bxy) \equiv \sim (\exists x)[\sim Ax \vee (\forall y)(Bxy \& Bxy)]$

1	$(\forall x)(Ax \& (\exists y) \sim Bxy)$	Assumption
2	$(\forall x)(\sim \sim Ax \& (\exists y) \sim Bxy)$	1, DN
3	$(\forall x)[\sim \sim Ax \& (\exists y)(\sim Bxy \vee \sim Bxy)]$	2, Idem
4	$(\forall x)[\sim \sim Ax \& (\exists y) \sim (Bxy \& Bxy)]$	3, Dem
5	$(\forall x)[\sim \sim Ax \& \sim (\forall y)(Bxy \& Bxy)]$	4, QN
6	$(\forall x) \sim [\sim Ax \vee (\forall y)(Bxy \& Bxy)]$	5, DeM
7	$\sim (\exists x)[\sim Ax \vee (\forall y)(Bxy \& Bxy)]$	6, QN
8	$\sim (\exists x)[\sim Ax \vee (\forall y)(Bxy \& Bxy)]$	Assumption
9	$(\forall x) \sim [\sim Ax \vee (\forall y)(Bxy \& Bxy)]$	8, QN
10	$(\forall x)[\sim \sim Ax \& \sim (\forall y)(Bxy \& Bxy)]$	9, DeM
11	$(\forall x)[\sim \sim Ax \& (\exists y) \sim (Bxy \& Bxy)]$	10, QN
12	$(\forall x)[\sim \sim Ax \& (\exists y)(\sim Bxy \vee \sim Bxy)]$	11, DeM
13	$(\forall x)(\sim \sim Ax \& (\exists y) \sim Bxy)$	12, Idem
14	$(\forall x)(Ax \& (\exists y) \sim Bxy)$	13, DN
15	$(\forall x)(Ax \& (\exists y) \sim Bxy) \equiv \sim (\exists x)[\sim Ax \vee (\forall y)(Bxy \& Bxy)]$	1-7, 8-14, \equiv I

17. $\{(\forall x)[(Ax \& Bx) \supset Cx], \sim (\forall x)(Ax \supset Cx)\} \vdash \sim (\forall x)Bx$

1	$(\forall x)[(Ax \& Bx) \supset Cx]$	Assumption
2	$\sim (\forall x)(Ax \supset Cx)$	Assumption
3	$(\exists x) \sim (Ax \supset Cx)$	2, QN
4	$\sim (Aa \supset Ca)$	Assumption
5	$\sim (\sim Aa \vee Ca)$	4, Imp
6	$\sim \sim Aa \& \sim Ca$	5, DeM
7	$\sim Ca$	6, &E
8	$(Aa \& Ba) \supset Ca$	1, \forall E
9	$\sim (Aa \& Ba)$	7, 8, MT
10	$\sim Aa \vee \sim Ba$	9, DeM
11	$\sim \sim Aa$	6, &E
12	$\sim Ba$	10, 11, \vee E
13	$(\exists x) \sim Bx$	12, \exists I
14	$(\exists x) \sim Bx$	3, 4-13, \exists E
15	$\sim (\forall x)Bx$	14, QN

18. $\{(\forall x)(\exists y)Axy \supset (\forall x)(\exists y)Bxy, (\exists x)(\forall y) \sim Bxy\} \vdash \sim (\exists x)(\forall y)Axy$	<table border="0"> <tr><td>1</td><td>$(\forall x)(\exists y)Axy \supset (\forall x)(\exists y)Bxy$</td><td>Assumption</td></tr> <tr><td>2</td><td>$(\exists x)(\forall y) \sim Bxy$</td><td>Assumption</td></tr> <tr><td>3</td><td>$(\exists x) \sim (\exists y)Bxy$</td><td>2, QN</td></tr> <tr><td>4</td><td>$\sim (\forall x)(\exists y)Bxy$</td><td>3, QN</td></tr> <tr><td>5</td><td>$\sim (\forall x)(\exists y)Axy$</td><td>1, 4, MT</td></tr> <tr><td>6</td><td>$(\exists x) \sim (\exists y)Axy$</td><td>5, QN</td></tr> <tr><td>7</td><td>$(\exists x)(\forall y) \sim Axy$</td><td>6, QN</td></tr> </table>	1	$(\forall x)(\exists y)Axy \supset (\forall x)(\exists y)Bxy$	Assumption	2	$(\exists x)(\forall y) \sim Bxy$	Assumption	3	$(\exists x) \sim (\exists y)Bxy$	2, QN	4	$\sim (\forall x)(\exists y)Bxy$	3, QN	5	$\sim (\forall x)(\exists y)Axy$	1, 4, MT	6	$(\exists x) \sim (\exists y)Axy$	5, QN	7	$(\exists x)(\forall y) \sim Axy$	6, QN	20. $\{(\exists x)[Ax \& (\forall y)(By \supset Cxy)], (\exists x)Ax \supset Bj\} \vdash (\exists x)Cxy$	<table border="0"> <tr><td>1</td><td>$(\exists x)[Ax \& (\forall y)(By \supset Cxy)]$</td><td>Assumption</td></tr> <tr><td>2</td><td>$(\exists x)Ax \supset Bj$</td><td>Assumption</td></tr> <tr><td>3</td><td>$Aa \& (\forall y)(By \supset Cay)$</td><td>Assumption</td></tr> <tr><td>4</td><td>Aa</td><td>3, &E</td></tr> <tr><td>5</td><td>$(\exists x)Ax$</td><td>4, \existsI</td></tr> <tr><td>6</td><td>Bj</td><td>2, 5, \supsetE</td></tr> <tr><td>7</td><td>$(\forall y)(By \supset Cay)$</td><td>3, &E</td></tr> <tr><td>8</td><td>$Bj \supset Caj$</td><td>7, \forallE</td></tr> <tr><td>9</td><td>Caj</td><td>6, 8, \supsetE</td></tr> <tr><td>10</td><td>$(\exists x)Cxy$</td><td>9, \existsI</td></tr> <tr><td>11</td><td>$(\exists x)Cxy$</td><td>1, 3-10, \existsE</td></tr> </table>	1	$(\exists x)[Ax \& (\forall y)(By \supset Cxy)]$	Assumption	2	$(\exists x)Ax \supset Bj$	Assumption	3	$Aa \& (\forall y)(By \supset Cay)$	Assumption	4	Aa	3, &E	5	$(\exists x)Ax$	4, \exists I	6	Bj	2, 5, \supset E	7	$(\forall y)(By \supset Cay)$	3, &E	8	$Bj \supset Caj$	7, \forall E	9	Caj	6, 8, \supset E	10	$(\exists x)Cxy$	9, \exists I	11	$(\exists x)Cxy$	1, 3-10, \exists E																																				
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19. $\{(\exists x)[Ax \& (\forall y)(By \supset Cxy)]\} \vdash (\forall x)[Bx \supset (\exists y)(Ay \& Cyx)]$	<table border="0"> <tr><td>1</td><td>$(\exists x)[Ax \& (\forall y)(By \supset Cxy)]$</td><td>Assumption</td></tr> <tr><td>2</td><td>$[Aa \& (\forall y)(By \supset Cay)]$</td><td>Assumption</td></tr> <tr><td>3</td><td>Bb</td><td>Assumption</td></tr> <tr><td>4</td><td>$(\forall y)(By \supset Cay)$</td><td>2, &E</td></tr> <tr><td>5</td><td>$Bb \supset Cab$</td><td>4, \forallE</td></tr> <tr><td>6</td><td>Cab</td><td>3, 5, \supsetE</td></tr> <tr><td>7</td><td>Aa</td><td>2, &E</td></tr> <tr><td>8</td><td>$Aa \& Cab$</td><td>6, 7, &I</td></tr> <tr><td>9</td><td>$(\exists y)(Ay \& Cyb)$</td><td>8, \existsI</td></tr> <tr><td>10</td><td>$Bb \supset (\exists y)(Ay \& Cyb)$</td><td>3-9, \supsetI</td></tr> <tr><td>11</td><td>$Bb \supset (\exists y)(Ay \& Cyb)$</td><td>1, 2-10, \existsE</td></tr> <tr><td>12</td><td>$(\forall x)(Bx \supset (\exists y)(Ay \& Cyx))$</td><td>11, \forallI</td></tr> </table>	1	$(\exists x)[Ax \& (\forall y)(By \supset Cxy)]$	Assumption	2	$[Aa \& (\forall y)(By \supset Cay)]$	Assumption	3	Bb	Assumption	4	$(\forall y)(By \supset Cay)$	2, &E	5	$Bb \supset Cab$	4, \forall E	6	Cab	3, 5, \supset E	7	Aa	2, &E	8	$Aa \& Cab$	6, 7, &I	9	$(\exists y)(Ay \& Cyb)$	8, \exists I	10	$Bb \supset (\exists y)(Ay \& Cyb)$	3-9, \supset I	11	$Bb \supset (\exists y)(Ay \& Cyb)$	1, 2-10, \exists E	12	$(\forall x)(Bx \supset (\exists y)(Ay \& Cyx))$	11, \forall I	21. $\{(\forall x)[(\exists y)Rxy \supset (\exists z) \sim Wz], (\exists y)(\exists z)(Ryz \& Hz), (\forall x)(\sim Hx \supset Wx)\} \vdash (\exists z)(\sim Wz \& Hz)$	<table border="0"> <tr><td>1</td><td>$(\forall x)[(\exists y)Rxy \supset (\exists z) \sim Wz]$</td><td>Assumption</td></tr> <tr><td>2</td><td>$(\exists y)(\exists z)(Ryz \& Hz)$</td><td>Assumption</td></tr> <tr><td>3</td><td>$(\forall x)(\sim Hx \supset Wx)$</td><td>Assumption</td></tr> <tr><td>4</td><td>$(\exists z)(Raz \& Hz)$</td><td>Assumption</td></tr> <tr><td>5</td><td>$(Rab \& Hb)$</td><td>Assumption</td></tr> <tr><td>6</td><td>Rab</td><td>5, &E</td></tr> <tr><td>7</td><td>$[(\exists y)Ray \supset (\exists z) \sim Wz]$</td><td>1, \forallE</td></tr> <tr><td>8</td><td>$(\exists y)Ray$</td><td>6, \existsI</td></tr> <tr><td>9</td><td>$(\exists z) \sim Wz$</td><td>7, 8, \supsetE</td></tr> <tr><td>10</td><td>$\sim Wc$</td><td>Assumption</td></tr> <tr><td>11</td><td>$\sim Hc \supset Wc$</td><td>3, \forallE</td></tr> <tr><td>12</td><td>$\sim \sim Hc$</td><td>10, 11, MT</td></tr> <tr><td>13</td><td>Hc</td><td>12, DN</td></tr> <tr><td>14</td><td>$\sim Wc \& Hc$</td><td>10, 13, &I</td></tr> <tr><td>15</td><td>$(\exists z)(\sim Wz \& Hz)$</td><td>14, \existsI</td></tr> <tr><td>16</td><td>$(\exists z)(\sim Wz \& Hz)$</td><td>9, 10-15, \existsE</td></tr> <tr><td>17</td><td>$(\exists z)(\sim Wz \& Hz)$</td><td>4, 5-16, \existsE</td></tr> <tr><td>18</td><td>$(\exists z)(\sim Wz \& Hz)$</td><td>2, 4-17, \existsE</td></tr> </table>	1	$(\forall x)[(\exists y)Rxy \supset (\exists z) \sim Wz]$	Assumption	2	$(\exists y)(\exists z)(Ryz \& Hz)$	Assumption	3	$(\forall x)(\sim Hx \supset Wx)$	Assumption	4	$(\exists z)(Raz \& Hz)$	Assumption	5	$(Rab \& Hb)$	Assumption	6	Rab	5, &E	7	$[(\exists y)Ray \supset (\exists z) \sim Wz]$	1, \forall E	8	$(\exists y)Ray$	6, \exists I	9	$(\exists z) \sim Wz$	7, 8, \supset E	10	$\sim Wc$	Assumption	11	$\sim Hc \supset Wc$	3, \forall E	12	$\sim \sim Hc$	10, 11, MT	13	Hc	12, DN	14	$\sim Wc \& Hc$	10, 13, &I	15	$(\exists z)(\sim Wz \& Hz)$	14, \exists I	16	$(\exists z)(\sim Wz \& Hz)$	9, 10-15, \exists E	17	$(\exists z)(\sim Wz \& Hz)$	4, 5-16, \exists E	18	$(\exists z)(\sim Wz \& Hz)$	2, 4-17, \exists E
1	$(\exists x)[Ax \& (\forall y)(By \supset Cxy)]$	Assumption																																																																																											
2	$[Aa \& (\forall y)(By \supset Cay)]$	Assumption																																																																																											
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22. $\vdash [(\exists x)Ax \supset ((\exists y)By \supset (\forall z)Cz)] \supset (\forall x)(\forall y)(\forall z)[(Ax \& By) \supset Cz]$ 23. $\{(\exists z)Qz \supset (\forall w)(Lww \supset \sim Hw), (\exists x)Bx \supset (\forall y)(Ay \supset Hy)\}$
 $\vdash (\exists w)(Qw \& Bw) \supset (\forall y)(Lyy \supset \sim Ay)$

1	$[(\exists x)Ax \supset ((\exists y)By \supset (\forall z)Cz)]$	Assumption	1	$(\exists z)Qz \supset (\forall w)(Lww \supset \sim Hw)$	Assumption
2	$(Aa \& Bb)$	Assumption	2	$(\exists x)Bx \supset (\forall y)(Ay \supset Hy)$	Assumption
3	Aa	Assumption	3	$(\exists w)(Qw \& Bw)$	Assumption
4	$(\exists x)Ax$	3, $\exists I$	4	$Qa \& Ba$	Assumption
5	$(\exists y)By \supset (\forall z)Cz$	1, 4, $\supset E$	5	Qa	4, $\&E$
6	Bb	2, $\&E$	6	$(\exists z)Qz$	5, $\exists I$
7	$(\exists y)By$	6, $\exists I$	7	$(\forall w)(Lww \supset \sim Hw)$	1, 6, $\supset E$
8	$(\forall z)Cz$	5, 7, $\supset E$	8	Lbb	Assumption
9	Cc	8, $\forall E$	9	$Lbb \supset \sim Hb$	7, $\forall E$
10	$(Aa \& Bb) \supset Cc$	2-9, $\supset I$	10	$\sim Hb$	8, 9, $\supset E$
11	$(\forall z)[(Aa \& Bb) \supset Cz]$	10, $\forall I$	11	Ba	4, $\&E$
12	$(\forall y)(\forall z)[(Aa \& By) \supset Cz]$	11, $\forall I$	12	$(\exists x)Bx$	11, $\exists I$
13	$(\forall x)(\forall y)(\forall z)[(Ax \& By) \supset Cz]$	12, $\forall I$	13	$(\forall y)(Ay \supset Hy)$	2, 12, $\supset E$
14	$[(\exists x)Ax \supset ((\exists y)By \supset (\forall z)Cz)] \supset$		14	$(Ab \supset Hb)$	13, $\forall E$
15	$(\forall x)(\forall y)(\forall z)[(Ax \& By) \supset Cz]$	1-13, $\supset I$	15	$\sim Ab$	10, 14, MT
			16	$Lbb \supset \sim Ab$	8-15, $\supset I$
			17	$(\forall y)(Lyy \supset \sim Ay)$	16, $\forall I$
			18	$(\forall y)(Lyy \supset \sim Ay)$	3, 4-17, $\exists E$
			19	$(\exists w)(Qw \& Bw) \supset (\forall y)(Lyy \supset \sim Ay)$	3-18, $\supset I$

24. $\{(\forall x)(\forall y)((Ax \& By) \supset Cxy), (\exists y)(Ey \& (\forall w)(Hw \supset Cyw)), (\forall x)(\forall y)(\forall z)[(Cxy \& Cyz) \supset Cxz], (\forall w)(Ew \supset Bw)\} \vdash (\forall z)(\forall w)[(Az \& Hw) \supset Czw]$

1	$(\forall x)(\forall y)((Ax \& By) \supset Cxy)$	Assumption
2	$(\exists y)(Ey \& (\forall w)(Hw \supset Cyw))$	Assumption
3	$(\forall x)(\forall y)(\forall z)[(Cxy \& Cyz) \supset Cxz]$	Assumption
4	$(\forall w)(Ew \supset Bw)$	Assumption
5	<u>$Aa \& Hb$</u>	Assumption
6	<u>$Ec \& (\forall w)(Hw \supset Ccw)$</u>	Assumption
7	Ec	6, &E
8	$Ec \supset Bc$	4, $\forall E$
9	Bc	7, 8, $\supset E$
10	Aa	5, &E
11	$Aa \& Bc$	9, 10, &I
12	$(\forall y)((Aa \& By) \supset Cay)$	1, $\forall E$
13	$((Aa \& Bc) \supset Cac)$	12, $\forall E$
14	Cac	11, 13, $\supset E$
15	$(\forall w)(Hw \supset Ccw)$	6, &E
16	Hb	5, &E
17	$Hb \supset Ccb$	15, $\forall E$
18	Ccb	16, 17, $\supset E$
19	$Cac \& Ccb$	14, 18, &I
20	$(\forall y)(\forall z)[(Cay \& Cyz) \supset Caz]$	2, $\forall E$
21	$(\forall z)[(Cac \& Ccz) \supset Caz]$	20, $\forall E$
22	$[(Cac \& Ccb) \supset Cab]$	21, $\forall E$
23	Cab	19, 22, $\supset E$
24	Cab	2, 6-23, $\exists E$
25	$[(Aa \& Hb) \supset Cab]$	5-24, $\supset I$
26	$(\forall w)[(Aa \& Hw) \supset Caw]$	25, $\forall I$
27	$(\forall z)(\forall w)[(Az \& Hw) \supset Czw]$	26, $\forall I$