Structure of Hadrons

quarks

Flavor	A	t	t_0	S	C	B	T	$Q(e)$	Mc^2 (GeV)
u (up)	$\frac{1}{3}$	$\frac{1}{2}$	$-\frac{1}{2}$	0	0	0	0	$+\frac{2}{3}$	0.002–0.008
d (down)	$\frac{1}{3}$	$\frac{1}{2}$	$+\frac{1}{2}$	0	0	0	0	$-\frac{1}{3}$	0.005–0.015
s (strange)	$\frac{1}{3}$	0	0	-1	0	0	0	$-\frac{1}{3}$	0.1–0.3
c (charm)	$\frac{1}{3}$	0	0	0	1	0	0	$+\frac{2}{3}$	1.0–1.6
b (bottom)	$\frac{1}{3}$	0	0	0	0	-1	0	$-\frac{1}{3}$	4.1–4.5
t (top)	$\frac{1}{3}$	0	0	0	0	0	1	$+\frac{2}{3}$	180 ± 12

estimates based on properties of hadrons!

- the least massive are u- and d-quarks (hence the lightest baryons and mesons must be made exclusively of these two quarks)
- strange quark carries a quantum number called strangeness S. Strange particles (such as kaons) carry this quark
- there are also six antiquarks
- they are fermions; they carry half-integer spins

Hadrons

baryons

fermions
made up of three quarks
must have half-integer spins

mesons

bosons
made up of quark-antiquarks*
must have integer spins

* Bosons can be annihilated;
strong interactions conserve the # of quarks

Structure of Hadrons

antiparticles

$$p + \bar{p} \rightarrow \gamma + \gamma$$

particle-antiparticle annihilation

- particle and antiparticle have opposite charges, baryon numbers, etc.
- they must have opposite intrinsic parities
- they must have opposite isospins

charge number

$$Q = -t_0 + \frac{A}{2}$$

baryon number

third component of isospin

Charge conjugation transforms particles into antiparticles

$$|p\rangle = a^{+}_{1/2,-1/2}|0\rangle \qquad |n\rangle = a^{+}_{1/2,+1/2}|0\rangle$$

particles

$$|\bar{n}\rangle = b^{+}_{1/2,-1/2}|0\rangle \qquad |\bar{p}\rangle = b^{+}_{1/2,+1/2}|0\rangle$$

antiparticles

Particles into antiparticles are not independent of each other; they transform into each other through charge conjugation!

$$b^{+}_{t,t_0} = (-1)^{t-t_0}\, a_{t,-t_0}$$

$$|p\rangle \xrightarrow{C} |\bar{p}\rangle \qquad |n\rangle \xrightarrow{C} -|\bar{n}\rangle$$

the same relation holds for quarks

$$|u\rangle \xrightarrow{C} |\bar{u}\rangle \qquad |d\rangle \xrightarrow{C} -|\bar{d}\rangle$$

Structure of Hadrons

isospin of quarks

$$|p\rangle = |uud\rangle \qquad |n\rangle = |udd\rangle$$

d- and u-quarks form an isospin doublet

$$\tau_{\pm}(\text{nucleon}) = \sum_{i=1}^{3} \tau_{\pm}(q_i)$$

$$\tau_{+}|u\rangle = |d\rangle \qquad \tau_{-}|d\rangle = |u\rangle$$

quark wave functions of pions

Consider π^- (t=1 and t_0=1). The only possible combination is

$$|\pi^-\rangle = |\bar{u}d\rangle$$

In general, it is possible to find several linearly independent components corresponding to the same t and t_0. The appropriate combination is given by isospin coupling rules. Furthermore, the wave function must be antisymmetric among the quarks

$$|\pi^0\rangle = \frac{1}{\sqrt{2}}\tau_{-}|\pi^-\rangle = \frac{1}{\sqrt{2}}\left(|u\bar{u}\rangle - |d\bar{d}\rangle\right)$$

$$|\pi^+\rangle = -|u\bar{d}\rangle$$

T=1 triplet

What about the symmetric combination?

$$|\eta_0\rangle = \frac{1}{\sqrt{2}}\left(|u\bar{u}\rangle + |d\bar{d}\rangle\right)$$

T=0 singlet

To produce heavier mesons we have to introduce excitations in the quark-antiquark system or invoke s- and other more massive quarks

Structure of Hadrons

strange mesons

The lightest strange mesons are kaons or K-mesons. They come in two doublets with *t*=1/2:

$$\left\{K^{+}(u\bar{s}), K^{0}(d\bar{s})\right\}, \quad \left\{K^{-}(\bar{u}s), \bar{K}^{0}(\bar{d}s)\right\}$$

This means that s-quark has zero isospin (no strange mesons with *t*=3/2 have been seen),

$$Q = t_0 + \frac{1}{2}\left(A + S + C + B + T\right)$$

generalization

$$\pi^{-}(\bar{u}d) + p(uud) \rightarrow K^{0}(d\bar{s}) + \Lambda(uds)$$

strangeness is conserved!

Strong interactions conserve the total number of each type of quarks. However, quarks can be transformed from one flavor to another through weak interactions (CKM matrix!).

Structure of Hadrons

color

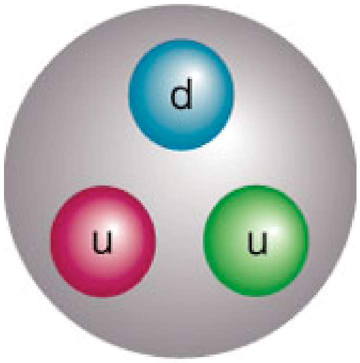
Consider a Δ-particle. It has isospin 3/2 with four different charge states:

$$\Delta^{-}, \Delta^{0}, \Delta^{+}, \Delta^{++}$$

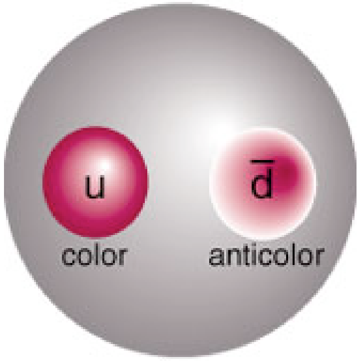
Since it is nonstrange baryon, is has to be made of u- and d-quarks. Δ^{++} has Q=2 and the only possiblity is a (uuu) combination. The intrinsic parity of Δ^{++} is known to be positive, its intrinsic spin is 3/2, and isospin is also 3/2. How to get the wave function that is untisymmetric with respect to a permutation of any two of the three quarks? **Is Pauli wrong?**

Color saves the day. It is a property of quarks. There are three colors: R (red), G (green) and B (blue). The wave function has to be antisymmetric in the color degree of freedom: **the net color in hadron must vanish!**

Since color is an unobserved property, all hadrons must be colorless oblects!



proton



pion

Structure of Hadrons

static quark model of hadrons

pseudoscalar mesons

$$\vec{J} = \vec{\ell} + \vec{S}, \quad \vec{S} = \vec{s}_q + \vec{s}_{\bar{q}}$$

$\vec{J}$ total angular momentum
$\vec{\ell}$ orbital angular momentum
$\vec{S}$ total spin

S can be either 0 or 1. The mesons with the relative zero orbital angular momentum are lower in energy. For the pion, S=0, hence J=0. Consequently, pions are “scalar” particles. But what about their parity? The parity of the pion is a product of intrinsic parities of the quark (+1), antiquark (-1) and the parity of the spatial wave function $(-1)^l$=+1. Hance the pion has negative parity. It is a **pseudoscalar meson.**

With (u,d,s) quarks, one can construct 9 pseudoscalar mesons:

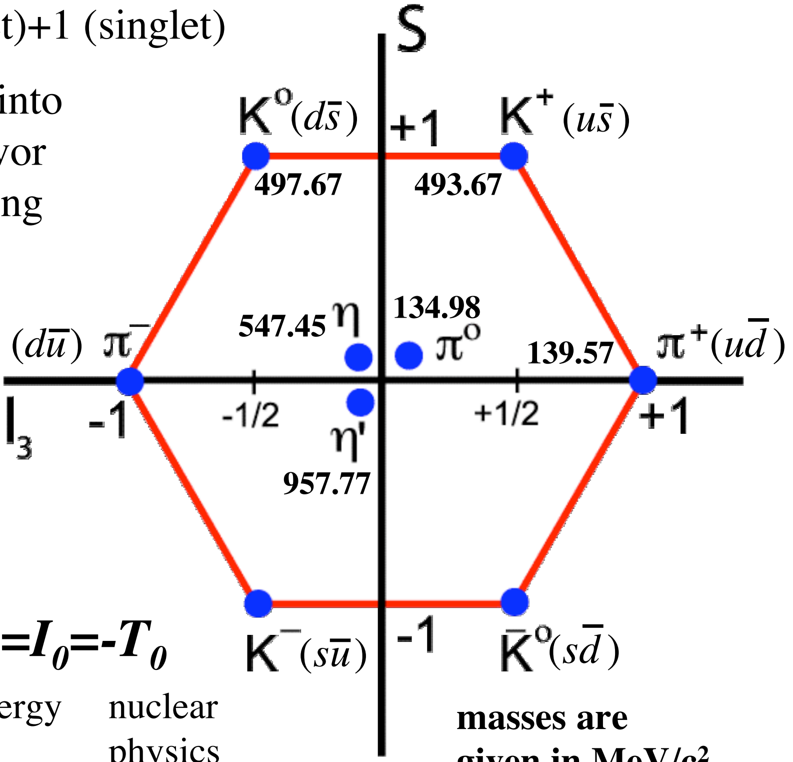
9=8 (octet)+1 (singlet)

Members of the octet transform into each other under rotations in flavor space (SU_3 group!). The remaining meson, $\square_0$, forms a 1-dim irrep.

$$|\square^0\rangle = \frac{1}{\sqrt{2}}\left(|u\bar{u}\rangle \square |d\bar{d}\rangle\right)$$

$$|\square_8\rangle = \frac{1}{\sqrt{6}}\left(|u\bar{u}\rangle + |d\bar{d}\rangle \square 2|s\bar{s}\rangle\right)$$

$$|\square_0\rangle = \frac{1}{\sqrt{3}}\left(|u\bar{u}\rangle + |d\bar{d}\rangle + |s\bar{s}\rangle\right)$$



$I_3 = I_0 = -T_0$

High-energy physics

nuclear physics

masses are given in MeV/c²