Investigative Physics
Module 2: Activity Units for Physics 102

Emory F. Bunn,[1] Mirela Fetea,[1] Gerard P. Gilfoyle,[1] Henry Nebel,[1]
Philip D. Rubin,[2] and Michael F. Vineyard[3]

[1]Department of Physics, University of Richmond, VA

[2]Department of Physics, George Mason University, Fairfax, VA

[3]Department of Physics, Union College, Schenectady, NY

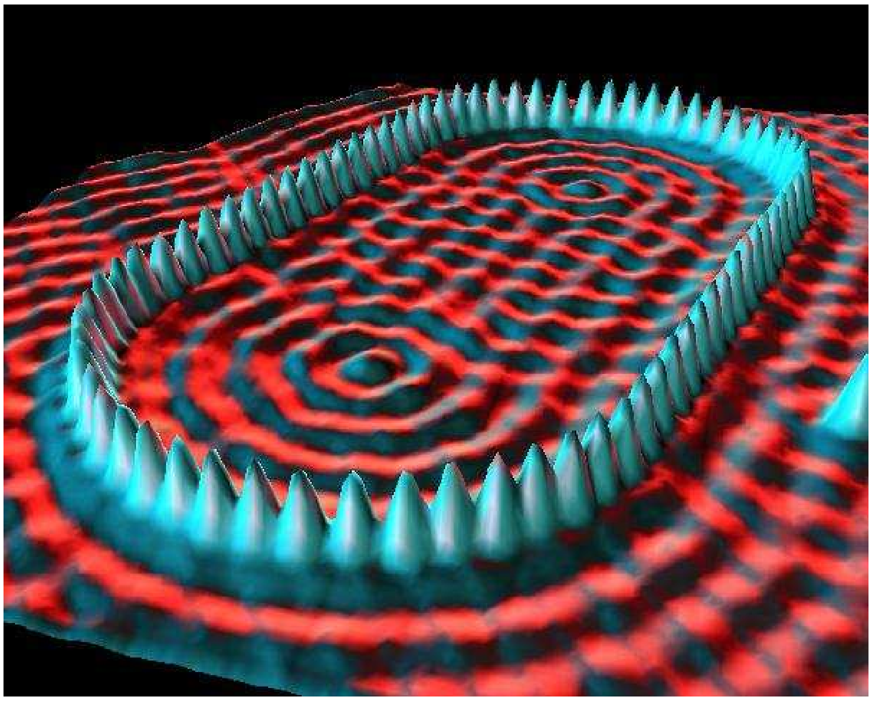
19th November 2006

Abstract

The exercises in this manual have been developed to support an investigative physics course that emphasizes active learning. Some of these units have been taken from the Workshop Physics project at Dickinson College and the Tools for Scientific Thinking project at Tufts University and modified for use at the University of Richmond. Others have been developed locally.

The units are made up of activities designed to guide your investigations in the laboratory. The written work will consist primarily of documenting your class activities by filling in the entries in each unit. The entries consist of observations, derivations, calculations, and answers to questions. Although you may use the same data and graphs as your partner(s) and discuss concepts with your classmates, all entries should reflect your own understanding of the concepts and the meaning of the data and graphs you are presenting. Thus, each entry should be written in your own words. Indeed, it is very important to your success in this course that your entries reflect a sound understanding of the phenomena you are observing and analyzing.

We acknowledge the support of the University of Richmond and the Instrumentation and Laboratory Improvement program of the National Science Foundation. Also, we would like to thank our laboratory directors for their invaluable technical assistance.



Cover art: Scanning tunnelling microscope (STM) picture of a stadium-shaped 'quantum corral' made by positioning iron atoms on a copper surface. This structure was designed for studying what happens when surface electron waves are in a confined region. Courtesy, Don Eigler, IBM and the American Institute of Physics.

Contents

1 Heat, Temperature, and Internal Energy

Name ______________________ Section ____________ Date ____________

Objective

To investigate the relationship between heat and temperature.

Apparatus

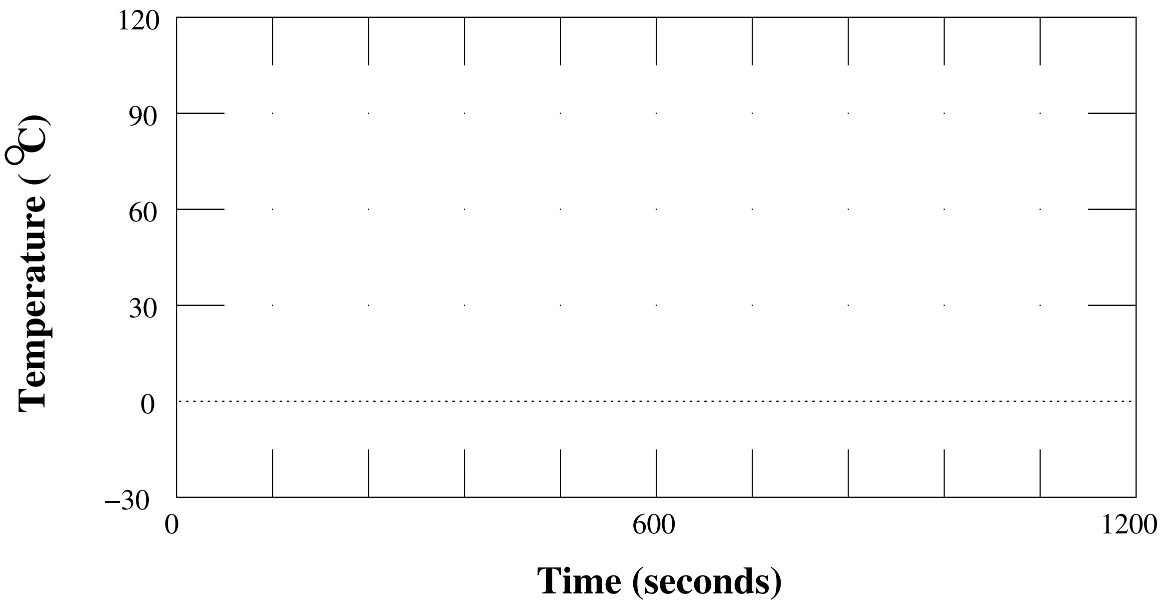
- Glass beaker
- Hot plate
- Ice
- Data Studio software and temperature probe
- Clamp and stand

Temperature of a Substance as a Function of Heat Transfer

As part of our quest to understand heat energy transfer, temperature, and internal energy of a substance, let's consider the temperature change as ice is changed to water and then to steam.

Activity 1: Predicting T vs. t for Water

Suppose you were to add heat at a constant rate to a container of ice water at 0°C until the water begins to boil. Sketch the predicted shape of the heating curve on the graph below using a dashed line. Mark the points at which the ice has melted and the water begins to boil.



Activity 2: Measuring T vs. t for Water

(a) To test your prediction:

1. Fill the glass beaker at least half full of ice water and set it on top of the hot plate.

2. Suspend the temperature probe so that the end is submerged in the ice water but not touching the side or bottom of the beaker. You will need to use the clamp and stand to do this.
3. Open the *Heat, Temp, & Internal Energy* application in the 132 Workshop folder on the **Start** menu.
4. Turn on the hot plate and click the **Start** button on the monitor to begin recording data. The temperature of the water will be recorded on the graph shown on the monitor. While there is still ice, stir gently.
5. After the water begins to boil, turn off the hot plate and stop collecting data using the **Stop** button on the monitor.
6. Sketch the shape of the measured heating curve on the above graph using a solid line. Ignore small variations due to noise and uneven heating. Mark the points at which the ice has melted and the water begins to boil.

(b) Does your prediction agree with the measured heating curve? If not, what are the differences?

(c) What is the relationship between the temperature and the added heat while the ice is melting?

(d) What is the relationship between the temperature and the added heat after the ice has melted, but before the water begins to boil?

(e) What is the relationship between the temperature and the added heat while the water is boiling?

(f) If there are regions of the heating curve in which the temperature is not changing, what do you think is happening to the added heat in these regions?

2 Calorimetry

Name ______________________ Section ____________ Date ____________

Purpose

- To learn to use a method for measuring heat called calorimetry.
- To use calorimetry to determine the specific heat of aluminum and the heat of fusion of ice.

Apparatus

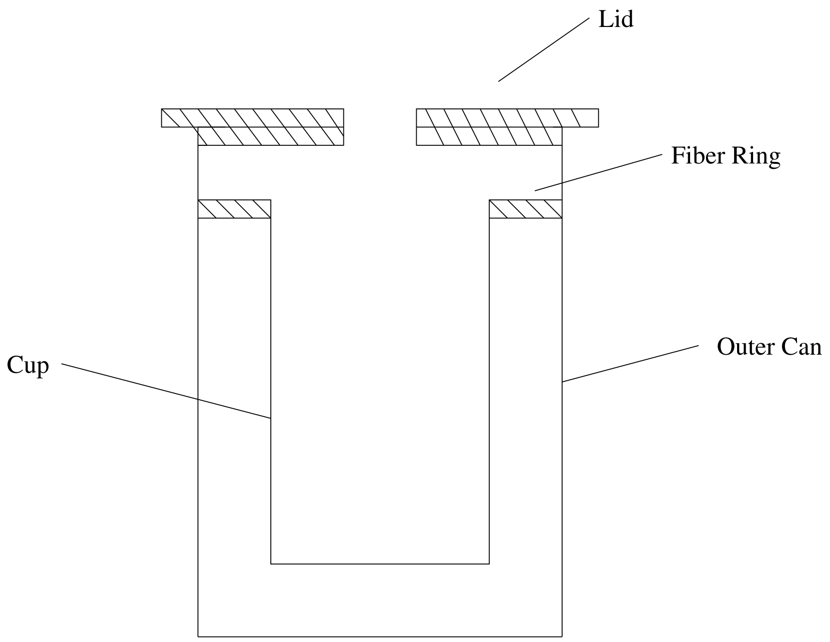
- Hypsometer and stand
- Hot plate
- Aluminum pellets
- Ice
- Triple-beam balance
- 100° C thermometer or temperature meter
- Temperature probe
- Safety goggles
- *DataStudio* software (Calorimetry)

Introduction

Calorimetry is a method for measuring heat. As applied in this experiment, the method involves the mixing together of substances initially at two different temperatures. The substances at the higher temperature lose heat and the substances at the lower temperature gain heat until thermal equilibrium is reached.

Activity 1: Statement of Conservation of Energy

If no heat is transferred to the surroundings, what is the relationship between the heat lost by the substances initially at high temperature and the heat gained by the substances initially at low temperature? Note: This is simply a statement of conservation of energy.



Experimental Equipment

A calorimeter, shown in the above figure, is used in this experiment to minimize the exchange of heat between the system and the surroundings. The inner calorimeter cup is thermally insulated from the surroundings by suspending it on a ring of material with low heat conductivity and surrounding it with a layer of air. Also the cup is shiny to minimize radiation loss. Hence, if the mixture of substances is placed inside the calorimeter cup, the heat lost to or gained from the surroundings can be ignored, and the above relationship can be used. The only part of the calorimeter which is involved in the calculation is the inner calorimeter cup which contains water and in which an exchange of heat between the hot and cold bodies takes place. The cup will undergo the same temperature change as the contained water. Of course, an instrument will have to be introduced to measure the temperature of the system, but the heat gained or lost by the instrument is small and can be ignored.

Activity 2: Specific Heat of Aluminum

(a) Fill the hypsometer (boiler) at least half full of water and start heating the water.

(b) Determine and record the mass of the hypsometer cup, m_h. Then fill it about half full with dry aluminum pellets. Determine and record the mass of the cup and pellets, m_{hp}, and calculate the mass of the pellets, m_p. Record the measurements in the space below.

(c) Place the cup in the top of the hypsometer and put the temperature probe into the middle of the pellets.

(d) Determine and record the mass of the calorimeter cup, m_c. Fill this cup about half full of cold tap water. Determine and record the mass of the cup and water, m_{cw}, and calculate the mass of the water, m_w. Then place the calorimeter cup in the outer can and put the lid on.

(e) Fill the plastic beaker with ice water. Open the *Calorimetry* application in the 132 Workshop folder in the **Start** menu and start collecting data. To make sure the thermometer is working properly place the temperature probe in the ice water and check that it is reading approximately 0°C. If not, then consult your instructor. Next, insert the temperature probe through the hole in the lid of the calorimeter and into the water.

(f) When the temperature of the pellets becomes constant, at or near 100°C, stir the water in the calorimeter GENTLY with the thermocouple and record the temperature of the water as T_w. Also record the temperature of the pellets as T_p. Then remove the thermocouple from the calorimeter.

(g) Now, quickly but carefully, pour the pellets into the water in the calorimeter cup. Put the thermocouple back into the calorimeter cup. Stir the water occasionally with the thermocouple and monitor the temperature of the mixture. When the temperature levels off, record this value as T.

(h) Write the complete heat equation and solve for the unknown specific heat of the metal.

(i) Look up the accepted value for the specific heat of aluminum and calculate the percent difference between this value and the one you determined above. Do the two values agree within experimental uncertainties? Comment on possible sources of error.

Activity 3: Specific Heat of Metals

(a) Repeat steps 2.a-2.i with pellets of a different metal besides aluminum. Record the the type of metal, the mass of the pellets, the temperature of the pellets just before you pour them in the cold water, and the temperature of the combined pellets, water, and cup.

(b) Write the complete heat equation and solve for the unknown specific heat of the new metal. Note the calorimeter cup is made of aluminum.

(c) Look up the accepted value for the specific heat of your new metal and calculate the percent difference between this value and the one you determined above.

(d) Consult the other lab groups in class and record their values of the specific heat of aluminum and the second metal below. Calculate the average and standard deviation for each metal. Can you spot any trends in your data?

(e) The specific heats you measured above were in units of $J/kg - K$. It is more illuminating to express the the specific heat in units of $J/mole - K$; proportional to the specific heat per atom. Do this for each of the averages and standard deviations you obtained in part 3.d by multiplying the result for each metal by it molar mass. Record the results below. Can you spot any trends in your data now? What effect do the standard deviations have on your conclusion?

Activity 4: Heat of Fusion of Ice

(a) The heat of fusion of ice is found experimentally as follows: A known mass of warm water is placed in the calorimeter cup and its temperature recorded. A known mass of ice at 0°C (with no water) is added to the water and allowed to melt. The final temperature of the mixture after the ice has melted is recorded. Perform the experiment and record the data in the space below.

(b) Write the complete heat equation and solve for the unknown heat of fusion of ice.

3 Boyle's Law

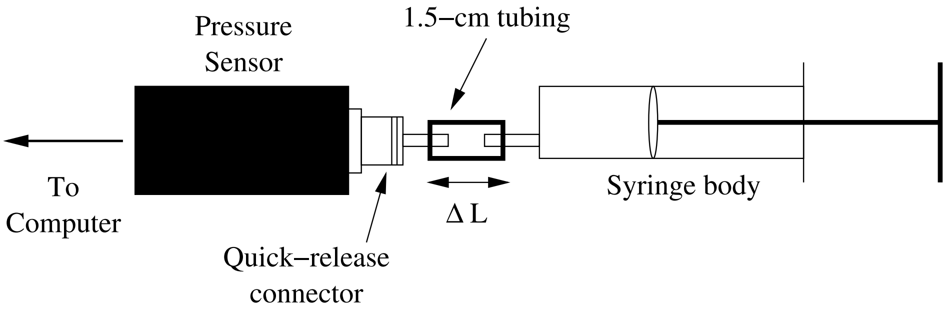
Name ______________ Section ______________ Date ______________

Objective

To investigate the relationship between the pressure and volume of a gas.

Apparatus

- DataStudio 750 Interface
- Pasco Pressure Sensor
- Syringe
- Tubing



Apparatus for Boyle's Law.

Introduction

The behavior of a gas can be described in terms of the macroscopic quantities: temperature (T), pressure (P), and volume (V). The relationship between these quantities is given by the equation of state of the gas. A real gas behaves approximately as an ideal gas if it is far from liquefaction. In that case, the equation of state of an ideal gas can be used to describe a real gas. For a given mass of a gas, if one of the quantities P, T, or V is changed, a change in the other two quantities probably will result. However, if one of the quantities is kept constant, the relationship between the other two can be studied. The relationship between pressure and volume of an ideal gas is called Boyle's law.

The experimental apparatus is shown in the figure above. The gas is air contained in a syringe that has marking on its side to measure the volume of the syringe. A short tube connects the syringe with a pressure sensor that measures the pressure in the tube and converts that measurement into a signal that can be read by the DataStudio interface.

Activity 1: Relationship Between P and V of a Gas

(a) Check that there are no leaks in the apparatus by trying to compressing the syringe from the 20.0 ml position to the 10.0 ml position. It should become increasingly difficult to push the plunger as the volume decreases. If this is not the case, check the couplings for fit. If no problem is obvious, then consult your instructor.

(b) The initial volume of air in the syringe should be set at 20.0 ml. If your syringe is set to some other value, disconnect the quick release connector from the sensor by gently rotating it in the counter-clockwise direction as you look from the syringe toward the pressure sensor. Next, move the piston to the 20.0 ml position, and then re-connect the quick release connector to the pressure sensor.

(c) **Data Recording**. Open the Boyle's Law activity located in the 132 Workshop Folder under the **Start** menu. Click on the window labeled *Volume and Pressure Table*. This is where your data will be displayed as you record it. This table display will show the values of the gas volume in the syringe which you will set by moving the piston to the appropriate marking on the syringe. You will record the pressure at each of these settings with the pressure sensor. To begin recording data, make sure the piston is at the 20-ml setting, and click the Start button. The Start button will change to a Keep button and the table display will show the value of the pressure next to the first volume value (20 ml) in the table. The reading in the pressure column should be colored red. Click the Keep button to record this pressure (notice the reading in the Pressure column beside the 20-ml entry changes from red to black). The next setting for the volume (18 ml) will appear in the Volume column of the data table display.

NOTE: For the first pressure reading at 20 ml, the air in the syringe will be in thermal equilibrium with the environment. This will not be the case immediately after compressing the syringe for the next reading. Therefore, you must allow one second for the system to return to thermal equilibrium after you compress the syringe and before clicking on Keep to record pressure values.

(d) Compress the syringe to the next value of the volume as listed in the data display table (i.e., the window labeled *Volume and Pressure Table*) and wait one second for the system to reach thermal equilibrium. Once thermal equilibrium is reached, click Keep to record the pressure. The data table display will automatically change to show the next value of the volume at which the pressure will be measured.

(e) Repeat step (d) for the remaining values of the volume listed in the table display. In other words, continue taking pressure measurements at the prescribed volume values in the data table display by moving the piston to the prescribed value and clicking on Keep after thermal equilibrium is reached. After you record the pressure for the last volume (8 ml), click the small, red box next to the Keep button (this is the stop button) to end data recording.

(f) **Analysis.** Click on the GraphDisplay to examine the plots of Syringe Volume Reading vs. Pressure, and the Volume to Pressure ratio (as a function of measuring time). Print the GraphDisplay and attach it to the unit. What happened to the pressure when the volume was reduced from 20 ml to 8 ml?

(g) From looking at the data, do the pressure and volume seem to be directly or inversely proportional? Explain.

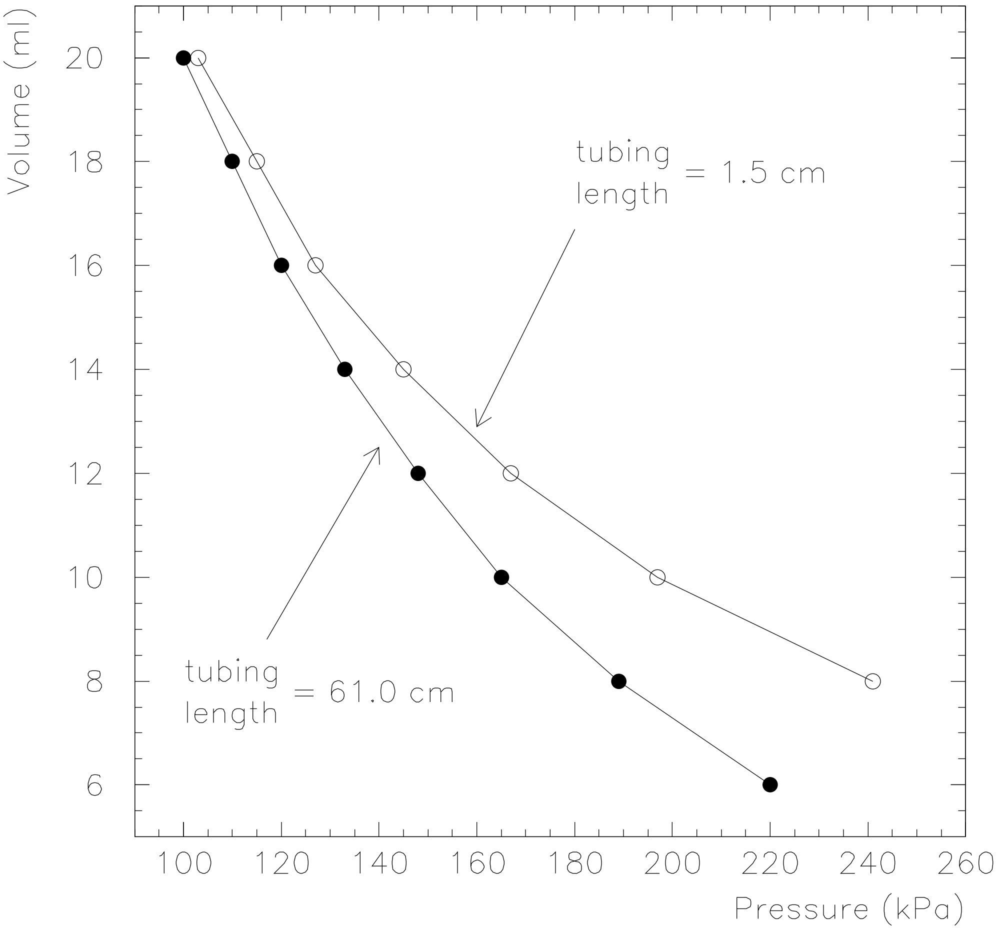
(h) Copy your data into a spreadsheet and plot pressure versus volume. Next, fit your data with some power of the volume. Record the result here. What should you get for the power? Why?

(i) If pressure and volume are inversely proportional, then what can you say about the product of pressure and volume? Explain.

(j) Construct a table in the space below with the column headings: V (ml), P (kPa), and PV. What are the units of the last column? Add your answer to the heading for that column. Enter the results for P and V in this new table and calculate PV for each set of readings. Determine the mean value and the standard deviation σ for PV. Record the results in the form PV = Mean $\pm\,\sigma$. What does this result tell you about the product PV? What does it tell you about the relationship between P and V? Explain.

(k) You may have noticed that the syringe plunger does not always return to the 20 ml mark at the end of a data run. Give three reasons why this may happen, and explain which is most likely and why.

(l) Examine the plot below with results from two different data runs. How do you explain the difference between the curves for the different tubing lengths ($\triangle L$ in the diagram on page 11)?



Results of measurement with Boyle's Law apparatus
different values of $\triangle L$, the tubing length.

4 The P-T Relationship of a Gas

Name ____________________ Section __________ Date __________+

Objectives

To investigate the relationship between pressure and temperature for a constant mass of gas at constant volume and determine the value of absolute zero.

Apparatus

- Pressure sensor
- Temperature sensor
- Air chamber and tubing
- Hot plate
- Glass beaker
- Clamp and stand

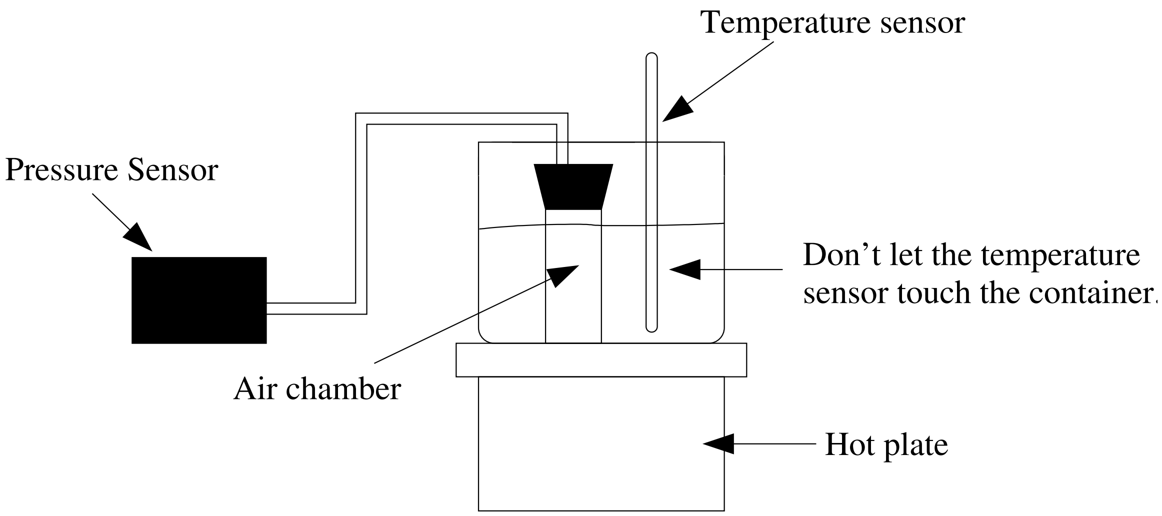


Figure 1: P-T apparatus.

Introduction

The behavior of a gas can be described in terms of the macroscopic quantities: temperature (T), pressure (P), and volume (V). The relationship between these quantities is given by the equation of state of the gas. A real gas behaves approximately as an ideal gas if it is far from liquefaction. In that case, the equation of state of an ideal gas can be used to describe a real gas. For a given mass of a gas, if one of the quantities P, T, or V is changed, a change in the other two quantities probably will result. However, if one of the quantities is kept constant, the relationship between the other two can be studied. The relationship between temperature and pressure of an ideal gas has no particular name associated with it.

The experimental apparatus is shown in the figure above and consists of an air chamber containing dry air. The volume of the gas is fixed.

Activity 1: P-T Relationship for a Gas

(a) Fill the beaker 3/4 full with cold tap water and place it on the hot plate. Immerse the air chamber in the water so that most of the volume of the air chamber is submerged. The air chamber will have to be held in place with a clamp and stand or it will float to the top. Set the temperature sensor in the water in such a way that it is not touching the side or bottom of the beaker.

(b) Open the *P-T* activity in the 132 Workshop Folder under the **Start** menu. Click on the window labeled *Temperature and Pressure Table.* This is where your data will be displayed as you record it. This table display will show the values of the gas pressure in the air chamber and the temperature of the heat bath. To begin recording data click the **Start** button on the *DataStudio* interface. The **Start** button will change to a **Keep** button and the table display will show the values of the temperature and pressure. Click the **Keep** button to record this temperature and pressure.

(c) Turn the hot plate on high. As the temperature rises, click the **Keep** button when the temperature is 5-7° above its first value. Continue recording the temperature and pressure at 5-7° intervals (by clicking the **Keep** button) until the water is close to boiling. You can monitor the temperature on the temperature versus time plot to the right (on the monitor) or by watching the temperature in the *Temperature and Pressure Table.* After your last reading, click the small red box next to the **Keep** button (this is the stop button) to end data recording.

(d) How are the pressure and temperature related? Print your data table, enter the data in *Excel* and plot pressure vs temperature on a linear graph, showing the equation of the graph. Print this graph and add it to this unit.

Activity 2: Absolute Zero and the Kelvin Scale

(a) The absolute zero of temperature can be defined as the temperature at which the pressure of an ideal gas is zero. Determine absolute zero from the equation of your graph by setting P = 0 and solving for T.

(b) Determine the percent difference between your value of absolute zero and the accepted value of -273°C. Are you happy or sad?

(c) Record the results from the other groups in class. Obtain an average and standard deviation and record it here. Are your results consistent with the class average? Explain.

5 Impulse, Momentum, and Interactions[1]

Name ______________________ Section ____________ Date ____________

Objectives

- To verify the relationship between impulse and momentum experimentally.
- To study the forces between objects that undergo collisions and other types of interactions in a short time period.

Apparatus

- Dynamics cart with flag and track
- Force transducer
- Photogate
- *Science Workshop 750 Interface*
- *DataStudio* software (Impulse-Momentum application)

The Impulse-Momentum Theorem

Real collisions, like those between eggs and hands, a Nerfball and a wall, or a falling ball and a table top are tricky to study because Δt is so small and the collision forces are not really constant over the time the colliding objects are in contact. Thus, we cannot calculate the impulse as $F\,\Delta t$. Before we study more realistic collision processes, let's redo the theory for a variable force. In a collision, according to Newton's second law, the force exerted on a falling ball by the table top at any infinitesimally small instant in time is given by

$$\mathbf{F} = \frac{d\mathbf{p}}{dt} \qquad [Eq.\,1]$$

To describe a general collision that takes place between an initial time t_i and a final time t_f , we must take the integral of both sides of the equation with respect to time. This gives

$$\int_{t_i}^{t_f} \mathbf{F} dt = \int_{t_i}^{t_f} \frac{d\mathbf{p}}{dt} dt = (\mathbf{p}_f - \mathbf{p}_i) = \Delta\mathbf{p} \qquad [Eq.\,2]$$

Impulse is a vector quantity defined by the equation

$$\mathbf{I} = \int_{t_i}^{t_f} \mathbf{F}\, dt \qquad [Eq.\,3]$$

By combining equations [2] and [3] we can formulate the impulse-momentum theorem in which

$$\mathbf{I} = \Delta\mathbf{p} \qquad [Eq.\,4]$$

If you are not used to mathematical integrals and how to solve them yet, don't panic. If you have a fairly smooth graph of how the force F varies as a function of time, the impulse integral can be calculated as the area under the F-t curve.

[1] 1990-93 Dept. of Physics and Astronomy, Dickinson College. Supported by FIPSE (U.S. Dept. of Ed.) and NSF. Portions of this material may have been modified locally and may not have been classroom tested at Dickinson College.

Let's see qualitatively what an impulse curve might look like in a real collision in which the forces change over time during the collision. In particular, let's consider the collision of a Nerfball with a wall as shown below.

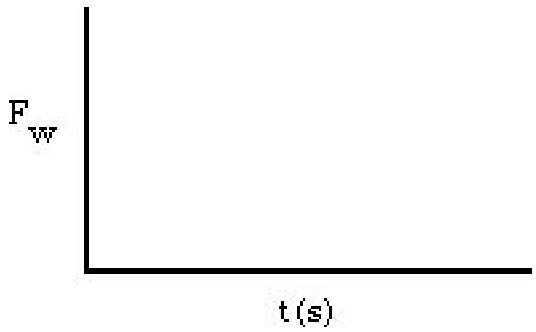
Activity 1: Predicting Collision Forces That Change

(a) Suppose a Nerfball is barreling toward a wall and collides with it. If friction is neglected, what is the net force exerted on the object just before it starts to collide?

(b) When will the magnitude of the force on the ball be a maximum?

(c) Roughly how long does the collision process take? Half a second? Less? Several seconds?

(d) Attempt a rough sketch of the shape of the force the wall exerts on a moving object during a collision.

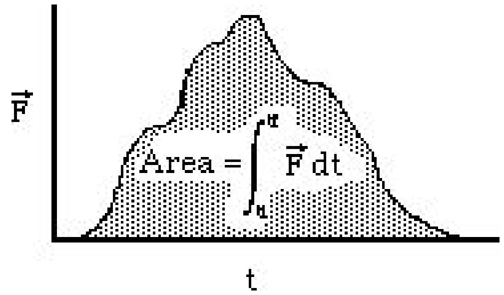


Verification of the Impulse-Momentum Theorem

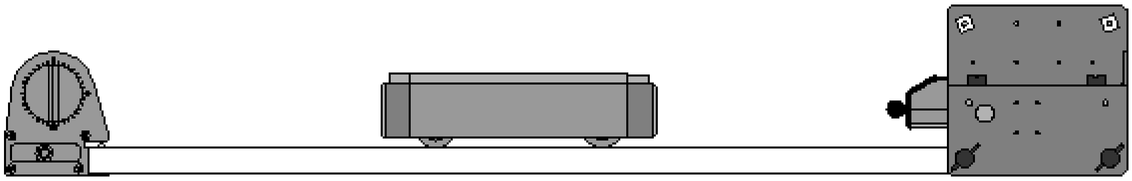
To verify the impulse-momentum theorem experimentally we must show that for an actual collision involving a single force on an object the equation

$$\int_{t_i}^{t_f} \mathbf{F}\,dt = \Delta \mathbf{p}$$

holds, where the impulse integral can be calculated by finding the area under the curve of a graph of F vs. t.



In this experiment you will investigate this theorem by measuring the impulse and the change in momentum of a cart undergoing a one-dimensional collision. The experimental setup is shown in the figure below. The end of the track with the motion detector should be raised about 1.5 cm so that, when released, the cart will collide with the force probe. The force probe will measure the force as a function of time during the collision. The motion detector is used to measure the velocity of the glider before and after the collision. You will use the Impulse-Momentum application to make these measurements.



Activity 2: Verification of the Impulse-Momentum Theorem

(a) Measure and record the mass of the cart, m.

(b) Construct a data table in the space below with the column headings Trial #, Area, v_i, and v_f. Make enough room to record 10 trials.

(c) Open the Impulse-Momentum application. Hold the cart about half-way up the track and press the TARE button on the force sensor. Start recording data and release the cart. Stop recording data after the cart collides with the force transducer and bounces back. The computer will then display graphs of velocity and force versus time.

(d) Determine the area under the force vs. time graph and record the value in your data table. See **Appendix B Introduction to DataStudio** for instructions on how to determine the area under a curve.

(e) Use the smart tool to find the velocity just before the collision and the velocity just after the collision from the velocity versus time graph. Record these values in your data table.

(f) Repeat parts (c) through (e) nine times. Be sure to press the TARE button on the force sensor before each run. Print the graphs for one of your trials.

(g) Construct another data table in the space below with the column headings Trial #, I, Δp, and % Diff. For each trial, calculate and record the impulse, I, and the change in momentum, Δp, in kg m/s. Also, determine the % difference between the two for each trial. Also, show a sample calculation of I, Δp, and % Diff for one of your trials.

(h) Do your results verify the impulse-momentum theorem within experimental uncertainty? Explain.

(i) Is there any indication of a systematic uncertainty? What are the possible sources of error?

6 Kinetic Theory of Ideal Gases[2]

Name ______________________ Section ____________ Date ____________

Objective

To derive a relationship between the macroscopic properties of an ideal gas and the microscopic motion of the unseen atoms that make up the gas. To do this activity you will need:

- A computer with an atomic and molecular motion simulation

Introduction

Do you believe in atoms? Our forefathers believed in the reality of witches. In fact, they thought that they had good evidence that witches existed, good enough evidence to accuse some people of being witches. We believe in atoms. Are we truly more scientific than they were?

Activity 1: Why Atoms!?

(a) List reasons why you do or do not believe that matter consists of atoms and molecules, even though you have never seen them with your own eyes.

(b) What happens when heat energy is being transferred into a substance? If you believe that substances are made of atoms and molecules, how would you use their existence to explain the change in volume of a heated gas?

Models of Pressure Exerted by Molecules

So far in physics we have talked about matter as if it were continuous. We didn't need to invent aluminum atoms to understand how a ball rolled down the track. But ever since the time of the fifth century B.C. Greek philosophers Leucippus and Democritus, some thinkers have believed in "atomism", a picture of the universe in which everything is made up of tiny "eternal" and "incorruptible" particles, separated by "the void". Today, we think of these particles as atoms and molecules.

In terms of every day experience molecules and atoms are hypothetical entities. In just the past 40 years or so, scientists have been able to "see" molecules using electron microscopes and field ion microscopes. But long before atoms and molecules could be "seen" experimentally, nineteenth century scientists such as James Clerk Maxwell and Ludwig Boltzmann in Europe and Josiah Willard Gibbs in the United States used these imaginary microscopic entities to construct models that made the description and prediction of the macroscopic behavior of thermodynamic systems possible. Is it possible to describe the behavior of an ideal gas that obeys the first law of thermodynamics as a collection of moving molecules? To answer this question, let's observe the pressure exerted by a hypothetical molecule undergoing elastic collisions with the walls of a 3D box. By using the laws of mechanics we can derive a mathematical expression for the pressure exerted by the molecule as a function of the volume of the box. If we then define temperature as being related to the average kinetic energy of the molecules in an ideal gas, we can show that kinetic theory is compatible with the ideal gas law and the first law of thermodynamics. This compatibility doesn't prove that molecules exist, but allows us to say that the molecular model would enable us to explain the experimentally determined ideal gas law.

[2]1990-93 Dept. of Physics and Astronomy, Dickinson College. Supported by FIPSE (U.S. Dept. of Ed.) and NSF. Portions of this material may have been modified locally and may not have been classroom tested at Dickinson College.

Atomic Motion and Pressure

Consider a spherical gas molecule that has velocity $\overrightarrow{v} = v_x\hat{i}+v_y\hat{j}+v_z\hat{k}$ and makes perfectly elastic collisions with the walls of a three-dimensional, cubical box of length, width, and height l. Start the program called "*Atoms in Motion*" (in "*Physics Applications*"). You will see a screen like the one shown below. Experiment with it for a few moments. The *Run* and *Stop* buttons control the processing of the simulation of the gas atoms while the *Step* button allows you to watch the 'movie' one frame at a time.

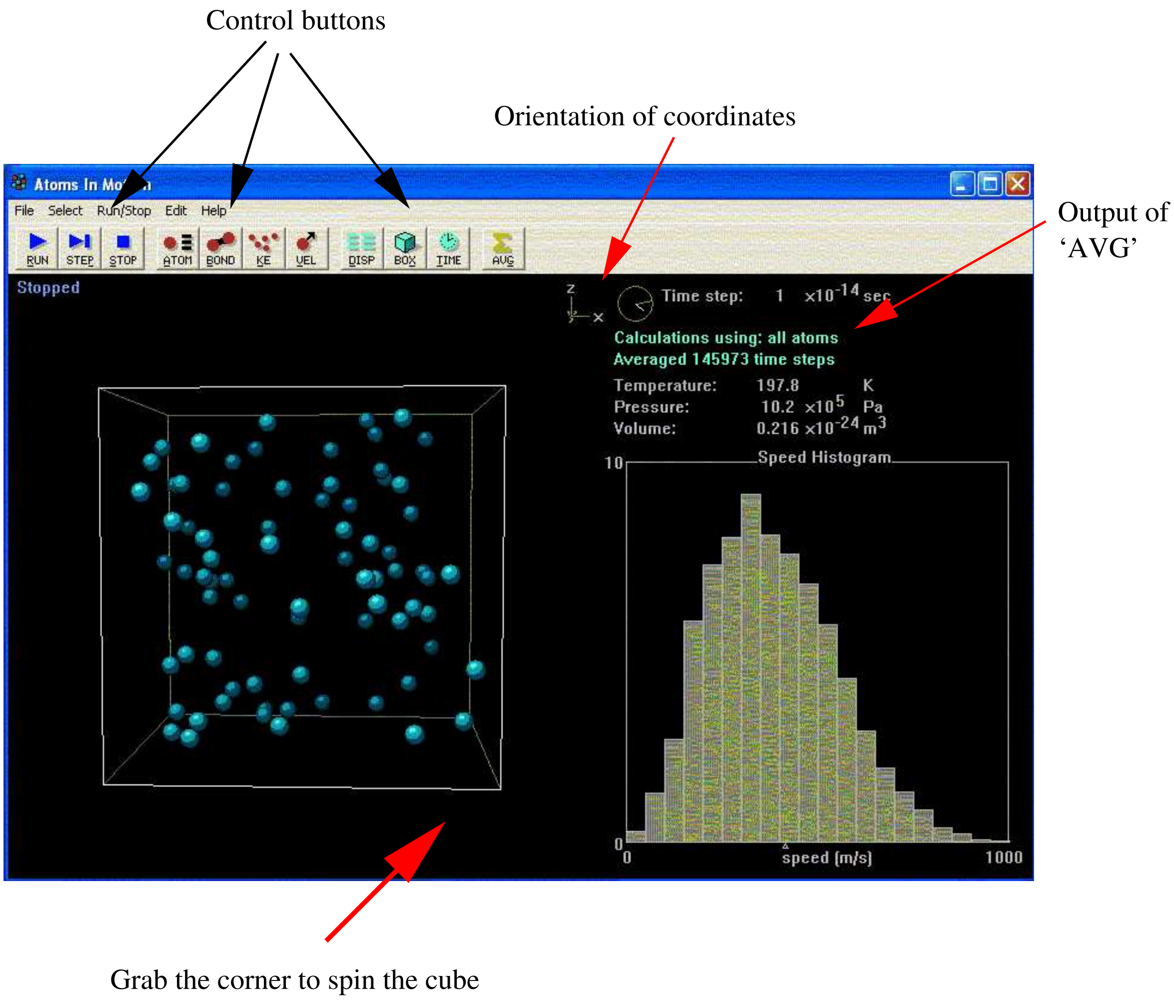


Figure 2: *Atoms in Motion* window.

Can we use the concept of molecules behaving like little billiard balls to explain why the ideal gas law relationship might hold? In the next activity you are to pretend you are looking under a giant microscope at a single spherical molecule as it bounces around in a three-dimensional box by means of elastic collisions and that you can time its motion and measure the distances it moves as a function of time.

If the molecule obeys Newton's Laws, you can calculate how the average pressure that the molecule exerts on the walls of its container is related to the volume of the box. The questions we have to consider are the following. What is the momentum change as the molecule bounces off a wall? How does this relate to the change in the velocity component perpendicular to the wall? How often will our molecule "hit the wall" as a function of its component of velocity perpendicular to the wall and the distance between opposite walls? What happens when the molecule is more energetic and moves even faster? Will the results of your calculations based on mechanics be compatible with the ideal gas law?

Activity 2: The Theory of Atomic Motion

(a) Stop the simulation if it's running and set the number of molecules to one. Do this by clicking on the *ATOM* button and getting a dialog box. Enter '1' for the number of Type A atoms and zero for all the others. Record the mass of the Type A atom. Click *OK* and the cube should now contain a single atom. If not, consult your instructor.

(b) The orientation of coordinates can be seen just above the right-hand corner of the cube (consult Figure 1 also). Suppose the molecule moves a distance $2l$ (across the cube and back)in the x-direction in a time Δt_x. What is the equation needed to calculate its x-component of velocity in terms of l and Δt_x?

(c) Suppose the molecule moves a distance $2l$ in the y-direction in a time Δt_y. What is the equation needed to calculate its y-component of velocity in terms of l and Δt_y?

(d) Suppose the molecule moves a distance $2l$ in the z-direction in a time Δt_z. What is the equation needed to calculate its z-component of velocity in terms of l and Δt_z?

(e) We will now measure the average time Δt_y for one complete round trip from the left side of the cube to the right side and back again. Click *AVG* and you will see some information printed in the color blue on the right-hand side of the *Atoms-in-Motion* window (see also Figure 1). The simulation takes small steps in time and calculates the positions of the atoms at the end of each time step. The number of these time steps taken is shown on the right-hand side and the size of each time step is printed at the top, right-hand-side of the window. Using the *Step* button let the atom in the cube move until it bounces off the left wall of the cube. Record the number of time steps in the space below.

(f) Now run or step the simulation until the atom bounces across the cube, hits the right-hand wall, comes back and strikes the left hand wall. Record the number of time steps. Calculate Δt_y and record it below.

(g) Each side of the cube has a length $l = 50 \times 10^{-10}$ m. Combine this with the previous result to determine v_y and record it.

(h) Repeat the above procedure for the top and bottom walls of the cube to get v_z.

(i) Rotate the cube by clicking and dragging one of the corners of the cube. Spin it until you can see the atom bounces between the walls in the direction of the x coordinate. Measure the x component of the speed of the atom using the same procedure as before.

(j) Write the expression for v_{total} in terms of the x, y, and z components of velocity. (Hint: This is an application of the 3-dimensional Pythagorean theorem.) Determine v_{total} for your atom. We will use these results in a little while to calculate the pressure exerted by our one-atom 'gas'.

(k) Record the value of the pressure and temperature for your 'gas' (as printed on the screen).

(l) We would like to eventually find the average kinetic energy of each atom or molecule in a gas so we now have to think about a gas with many atoms. Since the kinetic energy of a molecule is proportional to the square of its total speed, you need to show that if *on the average* $v_x^2 = v_y^2 = v_z^2$, then $v_{total}^2 = 3v_x^2$.

(m) If the collisions with the wall perpendicular to the x direction are elastic, show that the force exerted on that wall for each collision is just $F_x = 2m\frac{v_x}{\Delta t_x}$ where m is the mass of the particles and Δt_x the mean interval between collisions with the wall. (Hint: Think of the form of Newton's second law in which force is defined in terms of the change in momentum per unit time so that $F = \frac{\Delta p}{\Delta t}$.)

Warning: Physicists too often use the same symbol to stand for more than one quantity. In this case, note that Δp (where "p" is in lower case) indicates the change in *momentum*, not pressure.

(n) Substitute the expression from part (b) for Δt_x to show that

$$F_x = \frac{mv_x^2}{l}$$

(o) We have assumed from the beginning that we have a cubical box of edge length l. Show that the pressure on the wall perpendicular to the x axis caused by the force F_x due to *one* molecule is described by the following expression.

$$P = \frac{mv_x^2}{l^3}$$

(p) Let's say that there are not one but N molecules in the box. What is the pressure on the wall now?

(q) Next, show that if we write the volume of our box as $V = l^3$, and recalling that

$$v_x^2 = \frac{v_{total}^2}{3}$$

we can write the following expression.

$$P = N\frac{mv_{total}^2}{3V}$$

(r) Finally, since the average kinetic energy of a molecule is just

$$< E_{kin} >= \frac{1}{2}mv_{total}^2$$

show that the pressure in the box can be written in the following way.

$$P = \frac{2N < E_{kin} >}{3V}$$

(s) Use the previous result to calculate the pressure using v_{total}, the mass of the atom, N and V for your one-atom gas. Compare your result with the pressure you recorded above from the output of the simulation. Do they agree? Explain any differences.

7 Applying the Kinetic Theory[3]

Name ______________________ Section ____________ Date ____________

Objective

To derive the relationship between temperature and the kinematic properties of the monatomic molecules of an ideal gas. We will also calculate the specific heat per mole of an ideal, monatomic gas at constant volume using the kinetic theory and compare the prediction with data. To do this activity you will need:

- A computer with an atomic and molecular motion simulation

Kinetic Energy, Internal Energy, and Temperature

We have hypothesized the existence of non-interacting molecules to provide the basis for a particle model of ideal gas behavior. We have shown that the pressure of such a gas can be related to the average kinetic energy of each molecule:

$$P = \frac{2N\langle E_{kin}\rangle}{3V} \text{ or } PV = \frac{2}{3}N\langle E_{kin}\rangle$$

Pressure increases with kinetic energy per molecule and decreases with volume. This result makes intuitive sense. The more energetic the motions of the molecules, the more pressure we would expect them to exert on the walls. Increasing the volume of the box decreases the frequency of collisions with the walls, since the molecules will have to travel longer before reaching them, so increasing volume should decrease pressure if $\langle E_{kin}\rangle$ stays the same.

The Molar Specific Heat

The kinetic theory of gases uses the atomic theory to relate the macroscopic properties of gases to the microscopic features of the atoms and molecules that make up the gas. In this laboratory we will extend the calculations that we have made so far to include the molar specific heat of an ideal, monatomic gas. The success of that extension of the theory depends on how well the calculations reproduce the measured heat capacities of a variety of real (not ideal) gases.

Activity 1: Experimenting with the Gas Simulation Program

Open the *Atoms in Motion* program (in *Physics Applications*) in the 132 folder on the Start Menu. We are first going to explore the relationship between pressure and volume in our kinetic theory using the simulation.

(a) According to the ideal gas law PV = nRT = Nk$_B$T. What should happen to the pressure of an ideal gas as its volume increases or decreases?

(b) We now want to run a more realistic simulation. Under the *ATOM* menu set the number of Type A atoms to 50 and set all the others to zero. Click on the *BOX* button and a new dialog box will appear. Check the box beside 'Floor conducts heat' and set the temperature to 200 K. Notice at the top that the box width is $l = 50 \times 10^{-10}$ m. We have now set up a situation where one side of the cube is held at a constant temperature (*e.g.* it's sitting on a stove) so the collisions of the atoms with the floor are no longer elastic. The remaining sides of the cube do not transfer any energy (they're insulated) so elastic collisions still occur at those walls.

Start the simulation and make sure you are averaging the pressure over many time steps. You should see the number of averaged time steps increasing on the right-hand side of the *Atoms-in-Motion* window. If you don't see this information, click on *AVG* and it should appear.

[3]1990-93 Dept. of Physics and Astronomy, Dickinson College. Supported by FIPSE (U.S. Dept. of Ed.) and NSF. Portions of this material may have been modified locally and may not have been classroom tested at Dickinson College.

(c) What happens to the pressure? What happens to the temperature of the gas? You will find that it can take several minutes of computer time for the temperature of the gas to reach equilibrium with the floor. Once the gas temperature is within $8 - 10\ K$ of the floor temperature, we can consider the gas and the floor to be in thermal equilibrium.

(d) Record the pressure and the volume of four more volumes of the cube. Change the volume of the cube using the *BOX* menu and adjusting the box width. The volume is printed on the *Atoms-in-Motion* window. Plot your results and attach it to this unit. Are your results consistent with the ideal gas law and your prediction in part a?

Volume of Box	Average Pressure	Temperature

(e) In the procedure above you should have found the pressure to be inversely proportional to the volume. How could you modify your plot to show the pressure is proportional to 1/V? Make such a plot and fit it. How close is your data to following a straight line? Attach the plot to this unit.

(f) According to the ideal gas law $PV = nRT = Nk_BT$. What should happen to the pressure of an ideal gas as the number of particles increases or decreases? We will explore this idea with the simulation next.

(g) Start off with the gas parameters from the last 'run' of the simulation. Record the number of atoms, temperature, and pressure in the table below. Use the *ATOM* menu to change the number of atoms in the cube. Start the simulation. What happens to the pressure? Record the pressure and the number of molecules for four more values of the number of molecules and plot your results. Attach the plot to this unit. Are your results consistent with the ideal gas law and your prediction in part (f)?

Number of Molecules	Average Pressure	Temperature

Kinetic Theory and the Definition of Temperature

The model of an ideal gas we have just derived requires that

$$PV = \frac{2}{3} N \langle E_{kin} \rangle$$

But we have determined experimentally the ideal gas law:

$$\mathrm{PV} = \mathrm{Nk}_B\mathrm{T}$$

What can we say about the average kinetic energy per molecule for an ideal gas? You can derive a relationship between temperature and the energy of molecules that serves as a microscopic (i.e. molecular) definition of temperature.

Activity 2: Microscopic Definition of T

(a) Derive an expression relating $\langle E_{kin} \rangle$ and T. Show the steps in your derivation.

(b) In general, molecules can store energy by rotating or vibrating, but for an ideal gas of *point* particles, the only possible form of kinetic energy is the motion of the particle. If we can ignore potential energy due to gravity or electrical forces, then the internal energy E_{int} of a gas of N particles is $E_{int} = N \langle E_{kin} \rangle$. Use this to show that for an ideal gas of point particles, E_{int} *depends only on N and T*. Derive the equation that relates E_{int}, N and T. Show the steps.

The microscopic and the macroscopic definitions of temperature are equivalent. The microscopic definition of temperature which you just derived is fundamental to the understanding of all thermodynamics!

Activity 3: Calculating the Molar Specific Heat

In this section we will generate a series of equations that we will then bring together in order to predict the molar specific heat at constant volume.

(a) Consider an ideal gas in a rigid container that has a fixed volume. How is the molar specific heat defined in terms of the heat added Q?

(b) If the gas is heated by an amount Q, then how much work is done against the fixed container? Recall the first law of thermodynamics and incorporate this result into your statement of the first law.

(c) Now use the equations of parts (a) and (b) to relate the change in internal energy ΔE_{int} to the molar specific heat.

(d) Write down an expression for the change in internal energy of the ideal gas in terms of $\langle E_{kin} \rangle$. (Suggestion: see part (b) of Activity 2.) How is $\langle E_{kin} \rangle$ related to the temperature? Incorporate this relationship into your expression for the change in the internal energy. You should find that

$$\Delta E_{int} = \tfrac{3}{2} N k_B \Delta T$$

where k_B is Boltzmann's constant and N is the number of molecules in the gas.

(e) Use the equations is parts (c) and (d) to relate the molar specific heat to the number of particles N and Boltzmann's constant k_B. You should find that

$$nC_V = \tfrac{3}{2} N k_B$$

where n is the number of moles.

(f) How is the number of molecules in the gas N related to the number of moles n and Avogadro's number N_A? Use this expression and the result of part (e) to show

$$C_V = \frac{3}{2} N_A k_B \text{ or } \frac{C_V}{N_A k_B} = \frac{3}{2}$$

Activity 4: Comparing Calculations and Data

We now want to compare our calculation of the molar specific heat of an ideal, monatomic gas with the measured molar specific heats of some real gases. The table below lists some of those measurements.

Molecule	$\frac{C_V}{N_A k_B}$	Molecule	$\frac{C_V}{N_A k_B}$
He	1.50	CO	2.52
Ar	1.50	Cl_2	3.08
Ne	1.51	H_2O	3.25
Kr	1.49	SO_2	3.77
H_2	2.48	CO_2	3.42
N_2	2.51	CH_4	3.25
O_2	2.53		

(a) Has our theoretical calculation been successful at all? Which gases appear to be consistent with our calculation? Which gases are not? How do these two groups of real gases differ?

(b) Can you suggest an explanation for the partial success of the theory? Which one of the original assumptions that went into our kinetic theory might be wrong?

8 Einstein Solid

Name ______________________ Section ____________ Date ____________

Objective

To develop a quantum-mechanical model of an elemental solid (*e.g.* aluminum) and introduce the ideas of statistical mechanics.

Overview of the Model

One of the earliest successful applications of quantum mechanics was done by Albert Einstein in 1907 when he developed a model of an elemental solid (*i.e.* one that consists of a single element from the periodic table like aluminum, lead, *etc.*). We start by assuming that each atom in the solid is bound in a square lattice with each of six neighbors (see Figure 1). Each bond is treated as a simple spring so the mechanical energy for a single atom is

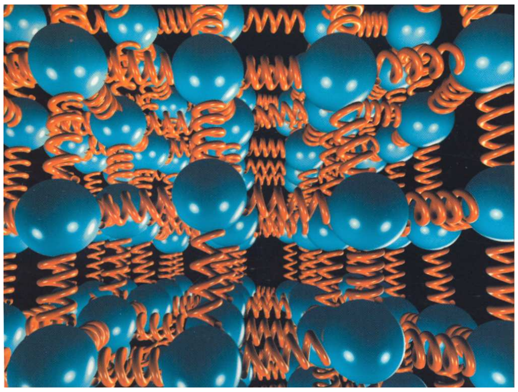


Fig. 1 Einstein solid.

$$E = \frac{1}{2}m(v_x^2 + v_y^2 + v_z^2) + \frac{1}{2}k(x^2 + y^2 + z^2) \tag{1}$$

where k is the spring constant of the bond, the coordinates x, y, and z are relative to the equilibrium position of the atom and v_x, v_y, and v_z are the components of the velocity. Einstein used an idea pioneered by Max Planck in 1901 and guessed the energy in the solid came in discrete pieces or quanta that were all the same size. Adding or removing these quanta heated or cooled the solid. Many years later the quantum mechanical energy E for a mass on a spring was found to be

$$E = (n_x + n_y + n_z + \frac{3}{2})\hbar\omega \tag{2}$$

where $\hbar$ is Planck's constant, ω is related to the spring constant k of the bond mentioned above, and n_x, n_y, and n_z represent the number of quanta associated with each degree of freedom of the spring. The degrees of freedom here correspond to the three possible directions each atom can vibrate. The size of each energy quantum is $\epsilon = \hbar\omega$. The total number of energy quanta in the solid is labeled q_A so the internal energy is $E_{int} = q_A\epsilon$. We then assume that all microstates of the solid have an equal probability of being populated. A microstate is a specific arrangement of the quanta on the atoms in the solid.

Activity 1: The Statistics of Matter

Before you embark on building the model of the Einstein solid consider some ideas from your previous study of gases. You will make some predictions here about the statistical nature of matter that you can refer back to later on in this unit.

(a) Consider a gas in a container. Would it violate Newton's Laws or any other physical law if all the particles in the gas collided in such a way that all of the gas particles ended up in the bottom half of the container leaving the top half empty?

(b) Is such a scenario likely? Explain.

(c) If you started out with all the gas in the bottom half of the container how likely is it to stay there?

The questions you answered above are addressing the notion of irreversibility. Many processes in nature appear to proceed in one 'direction' only. When you add milk to coffee it disperses throughout the coffee. After it is dispersed, the milk never re-concentrates into a blob of milk in the middle of the coffee. These processes go from a more orderly configuration (a concentrated drop of milk) to a disordered state (milk spread throughout the liquid). The reverse never happens. We will return to this notion again in this laboratory.

Activity 2: Calculating the Multiplicity of Some 'Solids'

(a) You will first calculate the configurations of the quanta (the microstates) for a VERY simple solid consisting of a single atom! The number of atoms for solid A is $N_A = 1$ so there are three degrees of freedom $N_a = 3$ because there is one degree of freedom for each spatial direction. The atom's vibration can be decomposed into three components, one for each direction. Let the 'solid' contain two quanta of energy so $q_A = 2$. Make a table with the headings n_1, n_2, and n_3 and in each row enter one arrangement of the two quanta. Each row in the table is a microstate. Make a table with all of the possible microstates. The multiplicity Ω_A of the system is the number of all possible microstates. What is your multiplicity? Record it here.

(b) You can calculate the multiplicity Ω_A using the expression

$$\Omega(N_A, q_A) = \frac{(q_A + 3N_A - 1)!}{q_A!(3N_A - 1)!} \tag{3}$$

Make the calculation for $N_A = 1$ and $q_A = 2$. Does this agree with your result in part 2.a?

(c) Now do the same thing for a different 'solid'. This time for solid B, let $N_B = 2$ (two whole atoms!) and $q_B = 1$. How many degrees of freedom does solid B have? Make a table analogous to the one in part 2.a on the same sheet as before. What is the multiplicity of solid B? Record it here. Use the expression in Activity 2.b to check your calculation.

Activity 3: Putting the 'Solids' Together

When two solids are brought together heat/energy can flow between the two objects. For the model of the Einstein solid you are building this corresponds to energy quanta ($\hbar\omega$) moving from atom to atom and occupying different microstates of the combined system.

(a) Now bring the your solids A and B 'together' into a single system. What is N_{AB} the total number of atoms? What is the number of degrees of freedom of the combined system?

(b) What is the total number of energy quanta q_{AB} for the combined system?

(c) The system is in its initial macrostate. A macrostate is a configuration of the system defined here by the total number of atoms and quanta in each solid. In this case the macrostate is defined by $N_A = 1$, $q_A = 2$, $N_B = 2$, and $q_B = 1$. What is the total multiplicity Ω_{AB} for the combined system with $q_A = 2$ and $q_B = 1$ in its initial macrostate?

(d) Now take the energy quantum in solid B and put it in solid A, *i.e.*, let heat flow from solid B into solid A. This is now a macrostate where $q_A = 3$ and $q_B = 0$. What is the new multiplicity Ω_A for solid A and the multiplicity Ω_B for solid B?

(e) What is the multiplicity Ω_{AB} for the combined system (solids A and B)?

(f) Remember that a macrostate is defined by the combination of N_A, N_B, q_A, and q_B. Which macrostate had the greatest multiplicity, $(q_A = 2,\ q_B = 1)$ or $(q_A = 3,\ q_B = 0)$ (remember that N_A and N_B are the same in each configuration so we don't list those parameters here)?

(g) If the energy quanta can move from atom to atom which macrostate $(q_A = 2,\ q_B = 1)$ or $(q_A = 3,\ q_B = 0)$ is most probable? Why?

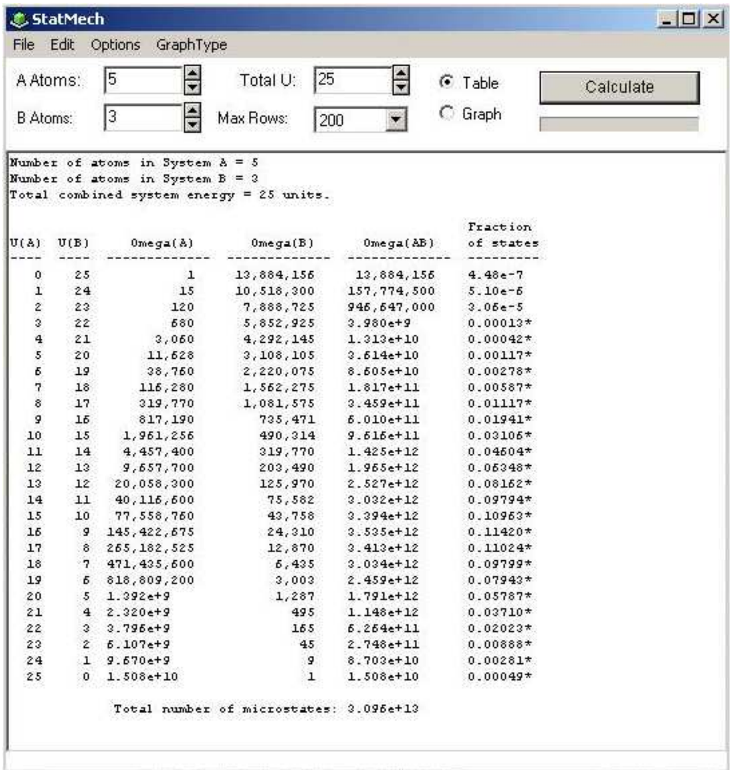
(h) If you started out in the $(q_A = 3,\ q_B = 0)$ macrostate is it more likely that you will remain in that macrostate of evolve to the $(q_A = 2,\ q_B = 1)$ macrostate? Why?

What you have discovered is a version of the irreversibility mentioned earlier, One macrostate $(q_A = 2, q_B = 1)$ is preferred over the other because it has more microstates than the other. This result depends critically on your assumption that all states are equally populated.

Activity 4: Using *StatMech* For More Complex Cases

You should have found in the previous activity that the ($q_A = 2,\ q_B = 1$) macrostate was more likely to occur and the process proposed in part 3.d is relatively unlikely. In other words, it is more likely for energy to be spread evenly throughout the system. This is good news because it means the statistical picture we are painting is consistent with reality. Remember what happens to the blob of milk in the coffee.

(a) You should realize that making the sorts of calculations you did in Activity 3 above would become rather painful for say $N_A = 300$ atoms. In order to push the model further you will use a software packaged called *StatMech* to perform the same calculations. To run the program go to the `Physics Applications` menu and click on `StatMech`. You should see a window like the one below. The top of the window has several entry boxes



Number of atoms in System A = 5
Number of atoms in System B = 3
Total combined system energy = 25 units.

U(A)	U(B)	Omega(A)	Omega(B)	Omega(AB)	Fraction of states
0	25	1	13,884,156	13,884,156	4.48e-7
1	24	15	10,518,300	157,774,500	5.10e-6
2	23	120	7,888,725	946,647,000	3.06e-5
3	22	680	5,852,925	3.980e+9	0.00013*
4	21	3,060	4,292,145	1.313e+10	0.00042*
5	20	11,628	3,108,105	3.614e+10	0.00117*
6	19	38,760	2,220,075	8.605e+10	0.00278*
7	18	116,280	1,562,275	1.817e+11	0.00587*
8	17	319,770	1,081,575	3.459e+11	0.01117*
9	16	817,190	735,471	6.010e+11	0.01941*
10	15	1,961,256	490,314	9.616e+11	0.03106*
11	14	4,457,400	319,770	1.425e+12	0.04604*
12	13	9,657,700	203,490	1.965e+12	0.06348*
13	12	20,058,300	125,970	2.527e+12	0.08162*
14	11	40,116,600	75,582	3.032e+12	0.09794*
15	10	77,558,760	43,758	3.394e+12	0.10963*
16	9	145,422,675	24,310	3.535e+12	0.11420*
17	8	265,182,525	12,870	3.413e+12	0.11024*
18	7	471,435,600	6,435	3.034e+12	0.09799*
19	6	818,809,200	3,003	2.459e+12	0.07943*
20	5	1.392e+9	1,287	1.791e+12	0.05787*
21	4	2.320e+9	495	1.148e+12	0.03710*
22	3	3.796e+9	165	6.264e+11	0.02023*
23	2	6.107e+9	45	2.748e+11	0.00888*
24	1	9.670e+9	9	8.703e+10	0.00281*
25	0	1.508e+10	1	1.508e+10	0.00049*

Total number of microstates: 3.096e+13

Figure 2: The `StatMech` window showing the table of multiplicities for each microstate. Each row corresponds to a different value of q_A.

where you can set the number of atoms (N_A and N_B) and the total number of energy quanta in the system U. The parameter U is the total internal energy $_{int}$ in the system in units of $\epsilon = \hbar\omega$. It is equivalent to the sum $q_{AB} = q_A + q_B$. You can also set the number of rows of microstates to print out or choose to view a graph instead of the table. To test the operation of *StatMech* redo the calculations of the microstates that you did in Activity 3. Make sure your results in Activity 3 agree with the output of *StatMech*. You will also see there are other macrostates that were ignored in Activity 3 for simplicity.

(b) Now run *StatMech* for the case where $N_A = 10$, $N_B = 20$, $U = 500$. What is the value of q_A for the most probable macrostate? Record it here. Click on the button at the top of the *StatMech* window and choose graph. You will see a graph of the table and it should look something like Figure 3. The vertical axis is the probability of a particular macrostate divided by the maximum probability of any macrostate. The horizontal axis is U_A/U_{TOT} where U_A is the energy of solid A in units of ϵ (equivalent to q_A) and U_{TOT} is the total internal energy of the solid in units of ϵ (equivalent to the total number of quanta q_{AB}). What is the value of U_A/U_{TOT} for the most probable state? How is this value related to the value of q_A for the most probable macrostate? Also, explain in words what this plot is showing you.

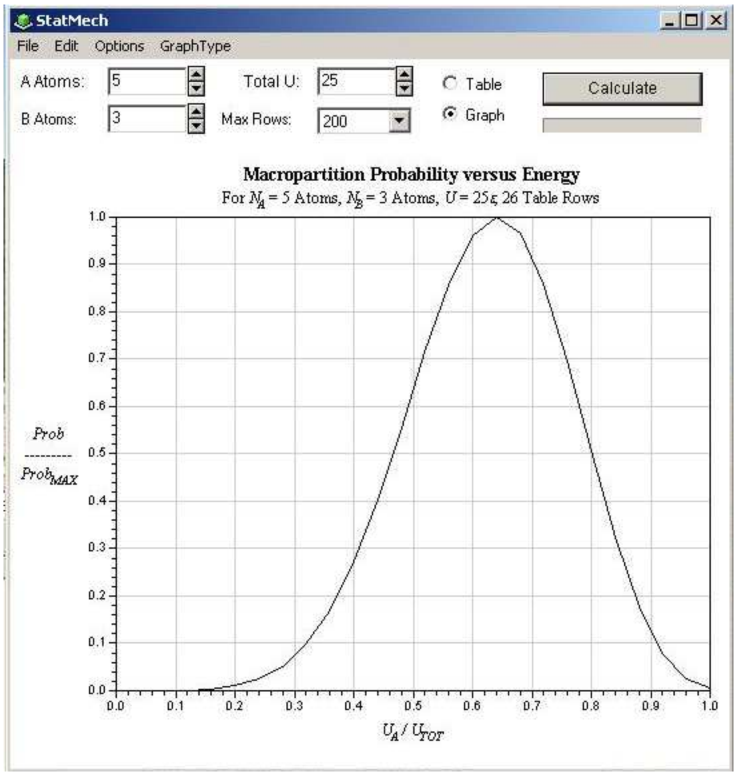


Figure 3: The `StatMech` window showing a graph of the multiplicities as a function of E_A/E_{int} where $E_A = q_A\hbar\omega$ and $E_{int} = q_{AB}\hbar\omega = (q_A + q_B)\hbar\omega$.

(c) How wide is the distribution of microstates? Measure this number by estimating the full-width-half-maximum (FWHM) from your graph. Do this by finding the largest value on the vertical axis, divide that value by two, and find the two points on either side of the peak where the distribution is equal to that half-maximum. Take the difference between these two points along the horizontal and this is the FWHM. Record your result here.

(d) Now repeat steps 4.b-c with $U = 100,000$ and N_A and N_B at their last values. What is the most probable value of U_A/U_{TOT} and the FWHM? How have things changed.

(e) Keeping $U = 100,000$ now repeat steps 4.b-c, but this time double the values of N_A and N_B. Record the most probable macrostate and the FWHM. Repeat this doubling of the number of atoms in each solid while keeping U fixed at least 3-4 times. Record the most probable macrostate and the FWHM each time along with N_A and N_B.

(f) How does the value of q_A for the most probable macrostate change as the number of atoms increases?

(g) How does the FWHM change as the number of atoms increases?

Activity 5: Irreversibility

You will now use the results from the previous Activity to delve into some of the implications of the statistical mechanics of the Einstein solid.

(a) As the number of atoms increases, what happens to the probability for finding the system in a macrostate different from the most probable one? Use the results of your calculations to explain your answer.

(b) When the system is in a macrostate far from the most probable one, what is the most likely thing to happen as energy or heat flows around the system?

(c) For the last calculation in Activity 4 what is the probability of the state with the minimum value of q_A? In other words what is the probability that all of the quanta would end up all in solid B? What is the probability of the most probable macrostate?

(d) Go back to the questions in Activity 1 and look at your answers. Do they still appear to be correct? A situation where all of the gas particles end up in one part of the container is a macrostate of the system analogous to the situation in Activity 5.c where all of the quanta end up in one of the solids and not the other. Answer those questions in Activity 1 again in terms of microstates, macrostates, and probability.

The behavior you are seeing here is for an Einstein solid, but is actually typical for most macroscopic systems. These systems have a large number of atoms or molecules with a variety of different energy states available. They evolve to the most probable macrostate and there is essentially no chance to occupy a state far from the most probable one. When two materials are first put in thermal contact they may be far from the most probable macrostate, but they equilibrate at that most probable one (where the temperatures are equal) and never go back. This is irreversibility.

Activity 6: Homework Problems (E - exercise, P - problem)

1. (E) Consider the following ‘gas’. It consists of four atoms in a cubical box. At any instant, there is a 50% chance of each atom being in the left half of the box (L) or the right half (R). Make a table showing all the microstates of this system. (Hint: There are 16.) How many macrostates are there? How many microstates are in each macrostate?

2. (E) Show that for N gas atoms in a box, the number of possible microstates is 2^N when microstates are defined by whether a given molecule is in the left half of the box or the right half of the box. The volumes of each half are equal.

3. (E) Imagine that we have an ideal gas consisting of 15 molecules. We can flip the signs of each of the three velocity components of a given molecule w without changing its overall energy (and thus without changing the gas's macrostate). How many possible patterns of sign choices are there?

4. (E) Calculate the multiplicity of an Einstein solid with $N = 1$ and $E_{int} = 6\epsilon$ by directly listing and counting the microstates. Check your work by using equation 3.

5. (E) Calculate the multiplicity of an Einstein solid with $N = 1$ and $E_{int} = 5\epsilon$ by directly listing and counting the microstates. Check your work by using equation 3.

6. (E) Use equation 3 to calculate the multiplicity of an Einstein solid with $N = 4$ and $E_{int} = 10\epsilon$.

7. (E) Use equation 3 to calculate the multiplicity of an Einstein solid with $N = 3$ and $E_{int} = 15\epsilon$.

8. (E) How many times more likely is that the combined system of solids described in the table below will be found in macropartition 3:3 than in macropartition 0:6, if the fundamental assumption is true?

9. (E) How many times more likely is it that the combined system of solids describe in the table below will not be found in macropartition 3:3 than it is to be found in macropartition 0:6, if the fundamental assumption is true?

Macropartition	E_A	E_B	Ω_A	Ω_B	Ω_{AB}
0:6	0	6	1	28	28
1:5	1	5	3	21	63
2:4	2	4	6	15	90
3:3	3	3	10	10	100
4:2	4	2	15	6	90
5:1	5	1	21	3	63
6:0	6	9	28	1	28
				Total=	462

Table 1: Possible macropartitions for $N_A = 1$, $N_B = 1$, $E_{int} = 6\epsilon$.

10. (E) Consider the system consisting of a pair of Einstein solids in thermal contact. A certain macropartition has a multiplicity of 3.7×10^{1024}, while the total number of microstates available to the system in all macropartitions is 5.9×10^{1042}. If we look at the system at a given instant of time, what is the probability that we will find it to be in our certain macropartition?

11. (E) Consider the system consisting of a pair of Einstein solids in thermal contact. A certain macropartition has a multiplicity of 1.2×10^{346}, while the total number of microstates available to the system in all macropartitions is 5.9×10^{362}. If we look at the system at a given instant of time, what is the probability that we will find it to be in our certain macropartition?

12. (E) Consider the system consisting of a pair of Einstein solids in thermal contact. Imagine that it is initially in a macropartition that has a multiplicity of 8.8×10^{123}. The adjacent macrostate closer to the equilibrium macrostate has a multiplicity of 4.2×10^{1234}. If we look at the system a short later, how many times more likely is it to have moved to the second macropartition than to have stayed with the first?

13. (E) Consider the system consisting of a pair of Einstein solids in thermal contact. Imagine that it is initially in a macropartition that has a multiplicity of 7.6×10^{3235}. The adjacent macropartition closer to the equilibrium macropartition has a multiplicity of 4.1×10^{3278}. If we look at the system a short time later, how many times more likely is it to have moved to the second macropartition than to have stayed with the first?

14. (P) Suppose you put 100 pennies in a cup, shake it up, and toss them all into the air. (a) After landing, how many different head-tail arrangements (microstates) are possible for the hundred pennies? (b) What is the probability of finding exactly 50 heads? (c) 49 heads? (d) 1 head?

15. (P) You ask your roommate to clean up a mess he or she made in your room. Your roommate refuses, because cleaning up the mess would violate the second law of thermodynamics, and campus security's record of your roommate's legal violation is already excessive. Gently but firmly explain why complying will not put your roommate at risk of such an infraction.

16. (P) The classic statement of Murphy's law reads, 'If something can go wrong, it will.' Explain how this is really a consequence of the second law of thermodynamics. (Hint: What is the entropy of 'wrong' in a given context compared to the entropy of 'right'?)

17. (P) Run the *StatMech* program to answer the questions below.

    (a) For two Einstein solids in contact with $N_A = N_B = 100$ and $E_{int} = 200\epsilon$ answer the following questions. (1) How many times more likely is the system to be found in the center macropartition than in the extreme macropartition where $E_A = 0$ and $E_B = 200\epsilon$ (2) What is the range of values that E_A is likely to have more than 99.98% of the time? (3) if E_A were initially to have the extreme value 0, how many times more likely is it to move to the next macropartition nearer the center than to remain in the extreme one?

    (b) Answer the same question as in (a) for a run where you scale everything up by a factor of 10, so that $N_A = N_B = 1000$ and $E_{int} = 2000\epsilon$.

    (c) Answer the same question as in (a) for a run where $N_A = N_B = 1000$ and $E_{int} = 200\epsilon$. Comment on the effect that increasing just the size of the system by a factor of 10 has on these answers.

    (d) Answer the same question as in (a) for a run where $N_A = N_B = 100$ and $E_{int} = 2000\epsilon$. Comment on the effect that increasing just the energy available to the system by a factor of 10 has on these answers.

18. (P) Consider two Einstein solids in thermal contact. The solids have different values of N but are identical in all other respects. It is plausible, since every atom in the combined system is identical, that in equilibrium the energy will be distributed among the solids in such a way that the average energy per atom is the same. Use *StatMech* to test this hypothesis in the situation where $E_{int} = 1000\epsilon$ and N_A and N_B have various different values such that $N_A + N_B = 1000$. (Set `Max Rows` to 1000 so that you can see every macropartition).

    (a) Is it true in most cases that in the most probable macropartition the solids have energies such that the average energy per atom in each is the same? Is it strictly true in every case? Answer these questions by discussing the values N_A and N_B you tested, and whether the actual most probable macropartition is the same as that predicted by the hypothesis.

    (b) In any case where the hypothesis does not work, does increasing both N_A and N_B by a factor of 10 or 100 (but leaving U alone) yield a result more or less consistent with the hypothesis?

    (c) Speculate as to the value of this hypothesis in the large-N limit.

19. (P) For the following questions, you will find that using *StatMech* is by far the fastest way to calculate the multiplicity.

    (a) What is the entropy of an Einstein solid with 5 atoms and an energy of 15ϵ? Express your answer as a multiple of k_b.

(b) What is the entropy of an Einstein solid with 50 atoms and an energy of 100ϵ? Express your answer as a multiple of k_b.

20. (P) A certain macropartition of two Einstein solids has an entropy of $305.2k_b$. The next macropartition closer to the most probable one has an entropy of $335.5k_b$. If the system is initially in the first macropartition and we check it again later, how many times more likely is it to have moved to the other than to have stayed in the first?

21. (P) My calculator cannot display e^x for $x > 230$. One can calculate e^x for larger values of x as follows. Define y such that $x = y \ln 10$. This means that $e^x = e^{y \ln 10} = (e^{\ln 10})^y = 10^y = 10^{x \ln 10}$. Note that we can calculate 10 raised to a non-integer power (for example, 103.46) as follows: $10^{3.46} = 10^{3+0.46} = 10^3(10^{0.46}) = 2.9 \times 10^3$. Use these techniques to solve the following problem. The entropy of the most probably macropartition for a certain system of Einstein solids is $6025.3k_b$, while the entropy of an extreme macropartition is only $5755.4k_b$. What is the probability of finding the system at a given time in the extreme macropartition compared to that of finding it in the most probable macropartition?

22. (P) In principle, the entropy of a isolated system decreases a little bit whenever random processes cause its macropartition to fluctuate away from the most probable macropartition. We can certainly see this with small systems. But is this really a possibility for a typical macroscopic system? Imagine that we can measure the entropy of a system of two solids to within 2 parts in 1 billion. This means that we could just barely distinguish a system that has an entropy of 4.99999999 J/K (eight 9s!) from one that has 5.00000000 J/K. (This is a reasonable entropy for a macroscopic system).

    (a) Imagine that the entropy of the equilibrium macropartition is 5.00000000 J/K. Show that the approximate probability that at any given time later we will find the system in a macropartition with entropy 4.99999999 J/K (*i.e.*, with an entropy that is only barely measurably smaller) is about 10315,000,000,000,000 times smaller that the probability we will still find it to have entropy 5.00000000 J/K. (Hint: See problem 17.)

    (b) Defend the statement that the entropy of an isolated system in thermal equilibrium never decreases.

9 Entropy and Temperature

Name ______________________ Section ____________ Date ____________

Objective

To explore the connection between the fundamental definition of entropy and temperature.

Overview

Consider the microscopic definition of the entropy of a system

$$S = k_B \ln \Omega \tag{1}$$

where k_B is Boltzmann's constant and Ω is the multiplicity or number of microstates. A microstate is defined by a particular arrangement of energy quanta among the atoms. A macrostate is defined by the total number of energy quanta q and the number of atoms N. We are building a model of an elemental solid (*e.g.*, like aluminum) where the total internal energy in the solid E_{int} is described by

$$E_{int} = q\hbar\omega \tag{2}$$

where $\hbar$ is Planck's constant divided by 2π and ω is a constant that characterizes the strength of the bonds between the atoms. The parameter q is the total number of quanta in the system and is a constant. These quanta are statistically distributed over the N atoms of the solid so all possible states of the system are equally likely and the multiplicity Ω is

$$\Omega = \frac{(q + 3N - 1)!}{q!(3N - 1)!} \quad . \tag{3}$$

This model of an elemental solid is called an Einstein solid.

We want to find a connection between the entropy defined in Equation 1 and the temperature. Recall how temperature is usually defined relative to some properties of matter like the freezing and boiling points of water. You are developing the microscopic picture of entropy, but it won't be successful until you can connect it to the observed behavior of bulk matter and our familiar notions of temperature.

Activity 1: The Entropy of Einstein Solids in Thermal Equilibrium

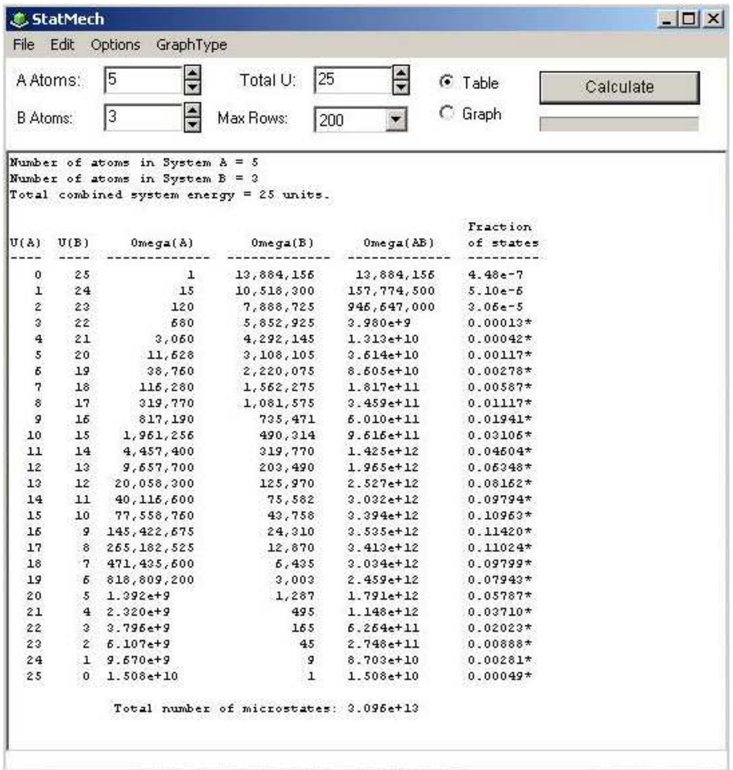
(a) To start connecting the entropy to the temperature you have to study the behavior of the entropy as the energy changes. To do this we will study two Einstein solids (A and B) in thermal equilibrium with each other. Their total internal energy will be

$$E_{int} = q_{AB}\hbar\omega = (q_A + q_B)\hbar\omega \tag{4}$$

where q_A and q_B are the numbers of energy quanta in each solid and q_{AB} is their sum.

Use the program *StatMech* (see the `Physics Applications` menu) for the configuration where you choose $N_A > 100$, $N_B > 80$ and $U > 400$. The label U in the *StatMech* window refers to the total number of energy quanta in the system in units of $\hbar\omega$ and is equivalent to q_{AB} here. An example of the output of *StatMech* is shown in Figure 1. The first two columns in the lower panel of Figure 1 represent $U(A)$ and $U(B)$, the energies in each individual solid (again in units of $\hbar\omega$) and are equivalent to q_A and q_B. After you perform the calculation with *StatMech* scan quickly down the column labeled 'Omega(AB)'. If any of the exponents you see exceed the value 307, then run the calculation again with smaller inputs until no exponent exceeds 307. This limitation is a restriction on MicroSoft *Excel* that you will use later to make plots. Record your values of N_A, N_B, and U.

(b) Now generate plots of $S_{AB} = S_A + S_B$, S_A, and S_B from the *StatMech* table. You can do this with *Excel*, but there are some intermediate steps necessary. Start Microsoft *Word* first. Next, go to the *StatMech* window, highlight the table, copy it (see the `Edit` menu on the *StatMech* window), and paste it into the *Word* document. In *Word* edit out all the commas (',') and asterisks ('*') in the file (use the `Replace` option under the `Edit`



Number of atoms in System A = 5
Number of atoms in System B = 3
Total combined system energy = 25 units.

U(A)	U(B)	Omega(A)	Omega(B)	Omega(AB)	Fraction of states
0	25	1	13,884,156	13,884,156	4.48e-7
1	24	15	10,518,300	157,774,500	5.10e-6
2	23	120	7,888,725	946,647,000	3.06e-5
3	22	680	5,852,925	3.980e+9	0.00013*
4	21	3,060	4,292,145	1.313e+10	0.00042*
5	20	11,628	3,108,105	3.614e+10	0.00117*
6	19	38,760	2,220,075	8.605e+10	0.00278*
7	18	116,280	1,562,275	1.817e+11	0.00587*
8	17	319,770	1,081,575	3.459e+11	0.01117*
9	16	817,190	735,471	6.010e+11	0.01941*
10	15	1,961,256	490,314	9.616e+11	0.03106*
11	14	4,457,400	319,770	1.425e+12	0.04604*
12	13	9,657,700	203,490	1.965e+12	0.06348*
13	12	20,058,300	125,970	2.527e+12	0.08162*
14	11	40,116,600	75,582	3.032e+12	0.09794*
15	10	77,558,760	43,758	3.394e+12	0.10963*
16	9	145,422,675	24,310	3.535e+12	0.11420*
17	8	265,182,525	12,870	3.413e+12	0.11024*
18	7	471,435,600	6,435	3.034e+12	0.09799*
19	6	818,809,200	3,003	2.459e+12	0.07943*
20	5	1.392e+9	1,287	1.791e+12	0.05787*
21	4	2.320e+9	495	1.148e+12	0.03710*
22	3	3.796e+9	165	6.264e+11	0.02023*
23	2	6.107e+9	45	2.748e+11	0.00888*
24	1	9.670e+9	9	8.703e+10	0.00281*
25	0	1.508e+10	1	1.508e+10	0.00049*

Total number of microstates: 3.096e+13

Figure 4: The `StatMech` window showing the table of multiplicities for each microstate. Each row corresponds to a different value of q_A.

menu). Save the Word file, but save it as a plain text ('.txt') file. You can now open the file in *Excel*. When you open the file, *Excel* pops up a `Text Import Wizard` that will guide you through the format of the input file. The defaults usually seem to work. Use *Excel* to calculate and plot on one graph S_{AB}, S_A, and S_B as a function of q_A. Print out your plot and attach it to this unit.

(c) What is q_A for the most probable macrostate? What mathematical condition can you impose on the total entropy S_{AB} to determine the most probable macrostate? How do you think the temperatures of solids A and B are related at the most probable microstate?

(d) How are the slopes of S_A and S_B related to one another at the most probable macrostate?

(e) How is E_A, the energy in solid A related to q_A? How is E_B, the energy in solid B related to q_A? Remember that q_{AB} is a constant and $q_{AB} = q_A + q_B$. Calculate the differentials dE_A and dE_B and rewrite the answer in part 1.d in terms of dS_A/dE_A and dS_B/dE_B.

Activity 2: Relating Entropy and Temperature

(a) Using the spreadsheet you generated in Activity 1, calculate dS_A/dq_A as a function of q_A and plot it. You can do this to an adequate approximation by doing taking the difference between S_A at adjacent values/rows of q_A. To do this suppose your spreadsheet has the values of S_A in column H. The *Excel* syntax for estimating the derivative for the first value of q_A (the first row) is '`=(H2-H1)/1.0`' where `H2` is the value in the second row and `H1` is the value in the first row. The numerator of one is redundant, but it shows you are approximating the derivative using the data from points that differ by 1.0 in q_A, the number of quanta in solid A. The syntax for dS_A/dq_A for the second value of q_A is '`=(H3-H2)/1.0`' and so on. Do the same for dS_B/dq_A and dS_{AB}/dq_A. Does the slope of S_{AB} pass through zero at the correct spot (recall part 1.c)? How are dS_A/dq_A and dS_B/dq_A related at the most probable macrostate. Does you plot agree with that result?

(b) If the energy E_A and q_A of solid A increases what should the temperature of solid A do? If the energy E_A of solid A increases what happens to dS_A/dq_A in your plot? Do the temperature and dS_A/dq_A change in the same way or in a different way as E_A increases?

(c) We want to come up with a relationship between temperature and the entropy. From the results above (parts 1.a-e) you should have found

$$\frac{dS_A}{dE_A} = \frac{dS_B}{dE_B} \tag{5}$$

and

$$T_A = T_B \tag{6}$$

for the most probable macrostate. This means there is some function of the temperature T such that

$$f(T) = \frac{dS}{dE} \tag{7}$$

for each solid that will be equal at equilibrium. We want $f(T)$ to behave like the temperatures we are accustomed to using. In other words, as the energy in the solid increases T should increase. Recall part 2.b and the behavior of dS/dE as T increases. Try to guess a mathematical form of $f(T)$ that acts like 'normal' temperatures and one that doesn't. Explain your reasoning.

Solid	dE/dT per mole	Solid	dE/dT per mole

Table 2: Heat capacities (dE/dT) for several elemental solids.

(d) How would you choose which of the forms you guessed in part 2.c is the correct one?

Activity 3: Determining $f(T)$ and the Heat Capacity

In the previous Activity you should have found that the mathematical form of $f(T)$ has to be something like $1/T^n$ where n is some positive number. This is necessary because your graphs should show that as the energy E_A (and the number of quanta q_A) of the solid increases $f(T) = dS/dE$ goes down. To make sure the temperature T behaves reasonable (and goes up with E_A and q_A) $f(T)$ must be some inverse of function of T. To decide exactly which function is right requires comparing Equation 7 or some result from it to some data.

Recall the table of heat capacities we generated in the laboratory entitled *Calorimetry*. Use those results to fill in Table 1 making sure that you are using molar heat capacities. The heat capacities are constant with respect to temperature and are similar in value to one another. These are the data that will help us determine $f(T)$. To calculate dE/dT we must find a relationship between E and T for the Einstein solid.

(a) Start with Equations 1 and 7 and the chain rule and show the following.

$$\frac{dS}{dE} = k_B \frac{1}{\Omega} \frac{d\Omega}{dE} \tag{8}$$

(b) Use Equation 2 to show

$$dE = \hbar\omega dq \qquad \text{and} \qquad \frac{d\Omega}{dE} = \frac{1}{\hbar\omega} \frac{d\Omega}{dq} \qquad . \tag{9}$$

(c) Starting with Equations 1 and 3 one can show

$$\frac{d\Omega}{dq} = \frac{3N\hbar\omega}{E}\Omega \qquad . \tag{10}$$

Combine this equation (number 10) and the results from 3.a-b to get a relationship for dS/dE for the Einstein solid in terms of N and E. Set that expression equal to $f(T)$, and solve for E the internal energy. It is the derivative of this last equation (dE/dT) that will give you the heat capacity. What function of $f(T)$ will give a result that is independent of temperature when you take the derivative with respect to T of your expression for the internal energy E?

(d) What is the final form of Equation 7 and $f(T)$?

(e) Determine the mean and standard deviation of the heat capacity of the elemental solids in Table 1. Calculate the heat capacity (dE/dT) for the Einstein solid using your results from parts 3.c-d. Is the heat capacity for the model of the Einstein solid consistent with the measured ones?

Activity 4: The Second Law of Thermodynamics

(a) Go back to your plots of the entropy as a function of q_A from part 1.b. Consider two Einstein solids that are brought together at a value of q_A that is higher than the equilibrium one at the most probable macrostate. Choose a value of q_A that is halfway between the most probable value and the maximum. Once the two Einstein solids are in contact, how will the system evolve? What happens to S_A and S_B? Do they go up, down, or stay the same? What happens to the total entropy S_{AB}? In fact, based on your plot from part 2.b, is there any circumstance where S_{AB} will not increase?

(c) To vividly see what happens when Einstein solids come in thermal contact, run the program *equilib.exe* available in the `Physics Applications` menu. This program starts with two Einstein solids with all the energy quanta in solid B. It simulates the evolution of the two solids as they march toward thermal equilibrium. Click `Evolve` to see the simulation run.

What you should have discovered in the previous part is that the entropy of the combined systems always increases regardless of what configuration the Einstein solids are in when they come in contact. The system always evolves to the most probable, most disordered macrostate where the temperatures will be equal and the entropy is a maximum. The energy quanta are most spread out. This result is stated in several different ways, but the most succinct is simply $\Delta S > 0$ for an isolated system. The entropy of an isolated system always increases. This is called the Second Law of Thermodynamics.

Activity 4: Homework Problems

1. (E) An object's entropy is measured to increase by 0.1 J/K when we add 35 J of energy. What is its approximate temperature? (Assume that the object's temperature does not change much when we add the 35 J.)

2. (E) A certain Einstein solid's entropy changes from $305.2k_b$ to $338.1k_b$ when we add 1 unit ϵ of energy. What is the value (and units) of k_bT/ϵ for this solid? If $\epsilon = 1.0\ eV$, what is its temperature T?

3. (E) Does it make sense to talk about the temperature of a vacuum? If so, how could you measure or calculate it? If not, why not?

4. (E) An Einstein solid in a certain macrostate has a multiplicity of 3.8×10^{280}. What is its entropy (expressed as a multiple of k_B)?

5. (E) A pair of Einstein solids in a certain macropartition has multiplicities of 4.2×10^{320} and 8.6×10^{132}. What are the entropies of each solid? What is the total entropy of the system in this macropartition? (Express entropies as multiples of k_b.)

6. (E) Is it really true that the entropy of an isolated system consisting of two Einstein solids never decreases? (Consider a pair of very small solids.) Why is this statement more accurate for large systems than for small systems? Explain in your own words.

7. (P) In lab we argued on fairly fundamental grounds that $dS/dU = f(T)$. In principle, we could define $f(T)$ to be anything that we like: this would amount to defining temperature and its scale. Still, some definitions would violate deeply embedded preconceptions about the nature of temperature. For example, the simplest definition of temperature would be $dS/dU = T_{new}$. Show that this definition

   (a) Would imply that T_{new} has units of K^{-1} and

   (b) Would imply that heat would flow spontaneously from objects with low T_{new} to objects with high T_{new}. This would imply that object with low values of T_{new} are hot, while objects with high values T_{new} are cold (we might want to call T_{new} so defined *coolness* instead of *temperature*). While we could define temperature in this way, it would really fly in the face of convention (if not intuition).

   (c) If we did define coolness T_{new} in this way, what ordinary temperature T would an object with absolutely zero coolness ($T_{new} = 0$) have? What about something that is infinitely cool ($T_{new} = \infty$)?

8. (P) Imagine that the entropy of a certain substance as a function of N and U is given by the formula $S = Nk_b \ln U$. Using the definition of temperature, show that the thermal energy of this substance is related to its temperature by the expression $U = Nk_bT$.

9. (P) Imagine that the multiplicity of a certain substance is given by $\Omega(U, N) = Ne^{\sqrt{NU/\epsilon}}$, where ϵ is some unit of energy. How would the energy of an object made out of this substance depend on its temperature? Would this be a 'normal' substance in our usual sense of temperature.

10. (P) Consider an Einstein solid having $N = 20$ atoms.

(a) What is the solids temperature when it has an energy of 10ϵ, assuming that $\epsilon = h\omega = 0.02eV$? Calculate this directly from the definition of temperature by finding S at 10ϵ and 11ϵ, computing $dS/dU \approx [S(11\epsilon) - S(10\epsilon)]/\epsilon$, and then applying the definition of temperature. (You will find that your work will go faster if you use *StatMech* to tabulate the multiplicities.)

(b) How does this compare with the result from the formula $U = 3Nk_bT$ (which is only accurate if N is large and $U/3N\epsilon > 1$)?

(c) If you have access to *StatMech*, repeat for $N = 200$ and $U = 100\epsilon$. (Hint: If your calculator cannot handle numbers in excess of 10^{100}, use the fact that in $(a \times 10^b) = \ln a + b \ln 10$).

11. (P) A newly-created material has a multiplicity

$$\Omega = \alpha N E$$

where N is the number of atoms in the solid, E is the total, internal energy in the solid, and α is a constant of proportionality.

(a) How does the temperature of the new material depend on the internal energy?

(b) What is the molar heat capacity for this solid?

(c) Could this material really exist? Why or why not?

12. (P) A newly-created material has a multiplicity

$$\Omega = \beta M E^2$$

where N is the number of atoms in the solid, E is the total, internal energy in the solid, and α is a constant of proportionality.

(a) How does the temperature of the new material depend on the internal energy?

(b) What is the molar heat capacity for this solid?

(c) Could this material really exist? Why or why not?

10 Electrostatics

Name ______________________ Section ____________ Date ____________

Objective

- To understand the basic phenomena of electric charges at rest.

Introduction

Atoms consist of a central nucleus made up of protons and neutrons surrounded by one or more electrons. While the nuclei of solids are essentially localized, some of the electrons may be free to move about. A substance which has as many electrons as it has protons is said to be electrically neutral. Dissimilar objects have different affinities for electrons. When two such objects, initially neutral, are rubbed together, the friction may cause electrons to pass from one to the other. After separation, neither object is neutral. Each is said to have been "charged by friction". An isolated, electrified object becomes neutral again if its electron-proton balance is restored. A convenient means for accomplishing this is to connect the object to earth by means of a conductor, through which electrons readily travel. This process is called "grounding the body". Since an electrified object is referred to as "charged", grounding is also referred to as "discharging".

Substances through which electrons do not move easily are called "non-conductors", or "insulators". Experiment has shown that when rubber and wool are rubbed together, electrons pass form the wool to the rubber. The electrons remain on the surface of the rubber–a non-conductor–where they were transferred.

Rubbing a metal rod with a wool cloth can also transfer electrons. This rod, however, is a conductor and electrons pass through it to the experimenter and then to the earth. People, made mostly of salt water, are good conductors, as well. Metal that is isolated, however, can be electrified. This can be demonstrated with an electroscope, which has a metal knob connected to a stem from which a thin metal leaf hangs. An insulator prevents contact of these metal parts with the case, and consequently the earth.

Apparatus

- electroscope
- rubber and glass rods
- wool and silk cloth

Activity 1: Charging by Friction

1. Be sure the electroscope is discharged by touching the knob with your finger. Explain what happened and why you are convinced the electroscope is discharged.

2. **Prediction:** If you rub the knob of an electroscope with a wool cloth, what will be the state of the electroscope when you remove the cloth? Explain.

3. Gently and repeatedly rub the knob of the electroscope for a couple of minutes. Remove the cloth. Note any differences in the electroscope from its appearance before you rubbed.

4. Explain what, if anything, happened.

Activity 2: Charging by Contact

1. Discharge the electroscope as before.
2. Charge the rubber rod by friction with the wool cloth.
3. Does anything occur in the electroscope when you bring the disc close to the knob without touching it?
4. **Prediction:** What will happen to the electroscope if you repeatedly touch its knob with a freshly charged object?
5. Touch the disc to the knob; rub the disc again and again touch it to the knob; repeat this procedure two or three more times. Describe any changes to the electroscope.
6. Repeat the procedure above until the electroscope's leaf is at approximately a twenty degree angle with the stem.

Activity 3: Kinds of Electrification

1. Electrify one end of the rubber rod by wrapping the wool cloth around the rod, squeezing the wool against the rod, twisting the rod vigorously to ensure good contact, and separating the wool from the rod.
2. **Prediction:** What will happen when you bring the electrified end of the rubber rod toward, but not touching, the electroscope's knob? What will happen if you do the same with the wool cloth?
3. Bring the charged end of the rubber rod toward the knob, but do not touch it. Record what happens.
4. Repeat number 3 with the wool cloth.

5. What differences were there between the trial with the rod and the trial with the cloth?

6. How would you account for these differences?

7. **Note:** By definition, the electrical state of the rubber after being rubbed by the wool is negative. That is, an object that has an excess of electrons is said to be negatively charged. Realize that this is only a convention.
8. If the rubber is said to be negatively charged, in what electrical state is the wool cloth?

9. How can an electroscope be used to determine the nature of any charge?

10. Rub the end of the glass rod with the silk cloth and determine the charge of each after they are separated.
11. What is the charge of an electron? Of a proton?

Activity 4: Action of the Electroscope

1. **Discussion:** Two facts explain the rise or fall of the leaves of an electroscope: (a) Like charges repel (unlike charges attract); and (b) Free electrons move about in a conductor when an electric force acts upon them.
2. Discharge the electroscope.
3. **Prediction:** What will happen when you bring the charged rubber rod near the discharged electroscope? What will happen if you do the same with the wool cloth?

4. Test your predictions; record the results; try to explain them.

5. When the wool approaches the knob, which way do the free electrons in the metal of the electroscope move (up toward the knob or down toward the leaf)?

6. Do the leaf and stem now become positive or negative?

7. In Activity 3, the electroscope was negatively charged before either the rod or the wool was brought toward the knob. For the case of the rod, in which direction do the free electrons in the electroscope move? Does the electron displacement increase or decrease the electrostatic force separating the leaf from the stem?

Activity 5: Charging by Induction

1. Discharge the electroscope.
2. **Prediction:** What will be the effect on the electroscope if you perform the following experiment: while grounding the electroscope with your finger, bring an electrified rubber rod near the knob, then take away your finger and then the rod (in that order)?

3. Carry out the experiment and describe the result.

4. Explain the result and why your prediction agreed or disagreed with it.

5. **Prediction:** Note that no electrons moved between the rod and the electroscope. What charge has been induced on the electroscope?

6. Test your prediction with the negatively charged rubber rod and the positively charged wool.
7. Does the test verify or contradict your prediction?

11 The Electrical and Gravitational Forces[4]

Name ______________________ Section ____________ Date ____________

> I began to think of gravity extending to the orb of the moon, and . . . I deduced that the forces which keep the planets in their orbs must be reciprocally as the squares of their distances from the centers about which they revolve: and thereby compared the force requisite to keep the moon in her orb with the force of gravity at the surface of the earth, and found them to answer pretty nearly. All this was in the two plague years of 1665 and 1666, for in those days I was in the prime of my age for invention, and minded mathematics and philosophy more than at any time since. - Isaac Newton

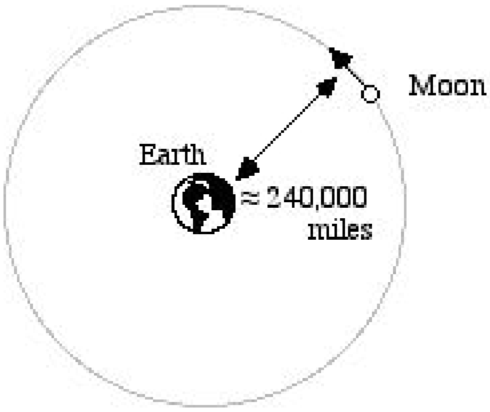
Objective

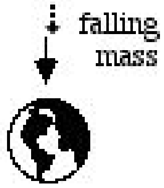
To understand the similarities of the gravitational and electrical forces.

Overview

The enterprise of physics is concerned ultimately with mathematically describing the fundamental forces of nature. Nature offers us several fundamental forces, which include a strong force that holds the nuclei of atoms together, a weak force that helps us describe certain kinds of radioactive decay in the nucleus, the force of gravity, and the electromagnetic force.

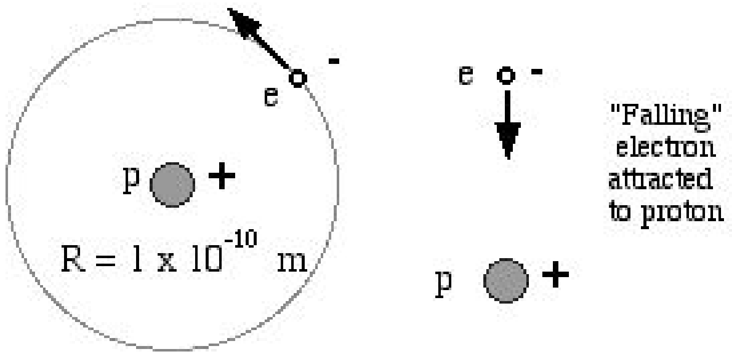
Two kinds of force dominate our everyday reality-the gravitational force acting between masses and the Coulomb force acting between electrical charges. The gravitational force allows us to describe mathematically how objects near the surface of the earth are attracted toward the earth and how the moon revolves around the earth and planets revolve around the sun. The genius of Newton was to realize that objects as diverse as falling apples and revolving planets are both moving under the action of the same gravitational force.





Similarly, the Coulomb force allows us to describe how one charge "falls" toward another or how an electron orbits a proton in a hydrogen atom.

[4]1990-93 Dept. of Physics and Astronomy, Dickinson College. Supported by FIPSE (U.S. Dept. of Ed.) and NSF. Portions of this material have been modified locally and may not have been classroom tested at Dickinson College.

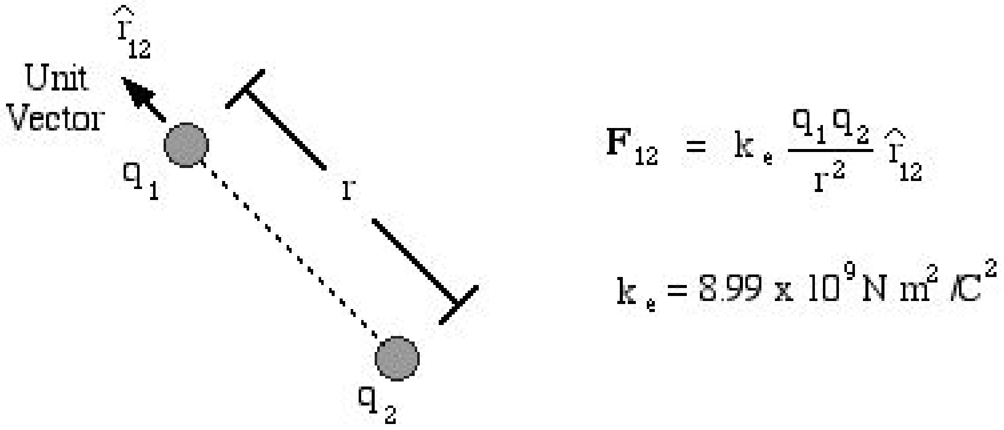


The fact that both the Coulomb and the gravitational forces lead to objects falling and to objects orbiting around each other suggests that these forces might have the same mathematical form.

In this unit we will explore the mathematical symmetry between electrical and gravitational forces for two reasons. First, it is beautiful to behold the unity that nature offers us as we use the same type of mathematics to predict the motion of planets and galaxies, the falling of objects, the flow of electrons in circuits, and the nature of the hydrogen atom and of other chemical elements. Second, what you have already learned about the influence of the gravitational force on a mass can be applied to aid your understanding of the forces on charged particles.

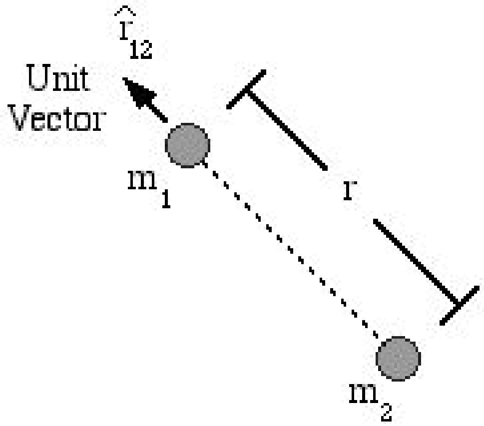
Activity 1: Comparison of Electrical and Gravitational Forces

Let's start our discussion of this comparison with the familiar expression of the Coulomb force exerted on charge 1 by charge 2.



Charles Coulomb did his experimental investigations of this force in the 18th century by exploring the forces between two small charged spheres. Much later, in the 20th century, Coulomb's law enabled scientists to design cyclotrons and other types of accelerators for moving charged particles in circular orbits at high speeds.

Newton's discovery of the universal law of gravitation came the other way around. He thought about orbits first. This was back in the 17th century, long before Coulomb began his studies. A statement of Newton's universal law of gravitation describing the force experienced by mass 1 due to the presence of mass 2 is shown below in modern mathematical notation:



$$\mathbf{F}_{12} = -G \frac{m_1 m_2}{r^2} \hat{r}_{12}$$

$$G = 6.67 \times 10^{-11} N\, m^2 / kg^2$$

About the time that Coulomb did his experiments with electrical charges in the 18th century, one of his contemporaries, Henry Cavendish, did a direct experiment to determine the nature of the gravitational force between two spherical masses in a laboratory. This confirmed Newton's gravitational force law and allowed him to determine the gravitational constant, G. A fact emerges that is quite amazing. Both types of forces, electrical and gravitational, are very similar. Essentially the same mathematics can be used to describe orbital and linear motions due to either electrical or gravitational interactions of the tiniest fundamental particles or the largest galaxies. This statement needs to be qualified a bit when electrons, protons and other fundamental particles are considered. A new field called quantum mechanics was developed in the early part of this century to take into account the wave nature of matter, which we don't actually study in introductory physics. However, even in wave mechanical calculations electrical forces like those shown above are used.

Activity 2: The Electrical vs. the Gravitational Force

Examine the mathematical expression for the two force laws.

(a) What is the same about the two force laws?

(b) What is different? For example, is the force between two like masses attractive or repulsive? How about two like charges? What part of each equation determines whether the like charges or masses are attractive or repulsive?

(c) Do you think negative mass could exist? If there is negative mass, would two negative masses attract or repel?

Which Force is Stronger– Electrical or Gravitational?

Gravitational forces hold the planets in our solar system in orbit and account for the motions of matter in galaxies. Electrical forces serve to hold atoms and molecules together. If we consider two of the most common fundamental particles, the electron and the proton, how do their electrical and gravitational forces compare with each other?

Let's peek into the hydrogen atom and compare the gravitational force on the electron due to interaction of its mass with that of the proton to the electrical force between the two particles as a result of their charge. In order to do the calculation you'll need to use some well known constants.

Electron: m_e = 9.1 x 10^{-31} kg, q_e = - 1.6 x 10^{-19} C

Proton: m_p = 1.7 x 10^{-27} kg, q_p = +1.6 x 10^{-19} C

Distance between the electron and proton: r = 1.0 x 10^{-10} m

Activity 3: The Electrical vs. the Gravitational Force in the Hydrogen Atom

(a) Calculate the magnitude of the electrical force on the electron. Is it attractive or repulsive?

(b) Calculate the magnitude of the gravitational force on the electron. Is it attractive or repulsive?

(c) Which is larger? By what factor (i.e. what is the ratio)?

(d) Which force are you more aware of on a daily basis? If your answer does not agree with that in part (c), explain why.

Activity 4: The Gravitational Force of the Earth

(a) Use Newton's law to show that the magnitude of the acceleration due to gravity on an object of mass m at a height h above the surface of the earth is given by the following expression:

$$\frac{GM_e}{(R_e + h)^2}$$

Hint: Because of the spherical symmetry of the Earth you can treat the mass of the Earth as if it were all concentrated at a point at the Earth's center.

(b) Calculate the acceleration due to gravity of a mass m at the surface of the earth (h=0). The radius of the earth is $R_e \approx 6.38 \times 10^3$ km and its mass $M_e \approx 5.98 \times 10^{24}$ kg. Does the result look familiar? How is this acceleration related to the gravitational acceleration g?

(c) Use the equation you derived in part (a) to calculate the acceleration due to gravity at the ceiling of the room you are now in. How does it differ from the value at the floor? Can you measure the difference in the lab using the devices available?

(d) Suppose you travel halfway to the moon. What is the new value of the acceleration due to gravity (neglecting the effect of the moon's pull)? (Recall that the earth-moon distance is about 384,000 km.)

(e) Is the gravitational acceleration "constant", g, really a constant? Explain.

(f) In part (d) you showed that there is a significant gravitational attraction halfway between the earth and the moon. Why, then, do astronauts experience weightlessness when they are orbiting a mere 120 km above the earth?

12 Electric Fields and Equipotential Lines

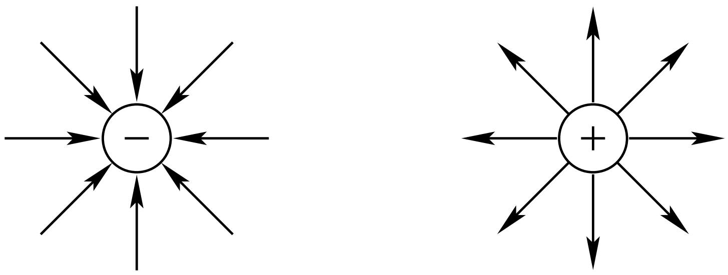
Name ______________________ Section ____________ Date ____________

Objective

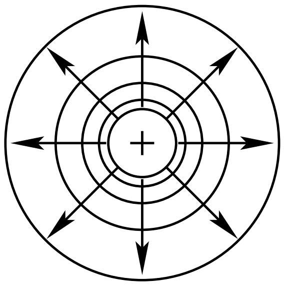
- To learn the shape of electric fields.

Introduction

Charged objects exert an electrical force on other charged objects in proportion to the amount of charge each has, just as massive objects exert a gravitational force on one another in proportion to their masses. The magnitudes of both forces depend, too, on the distance between objects. However, whereas the gravitational force is always attractive, electrical forces may be either attractive or repulsive depending on the sign of the charges. It is convenient in understanding the nature of electrical forces to draw pictures of them. We represent the fields, which provide the magnitude and direction of the forces, as lines. We agree on a convention: the direction of the field is that of the force on an infinitesimal positive test charge. Thus, the lines of force originate on and come out of positive charges and are directed toward and terminate on negative charges (see figure below). The magnitude of the field, and therefore the force, is proportional to the density of the field lines.

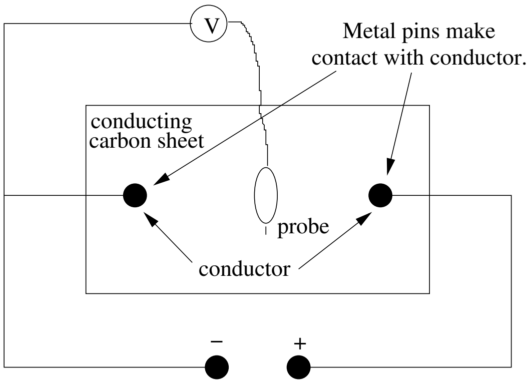


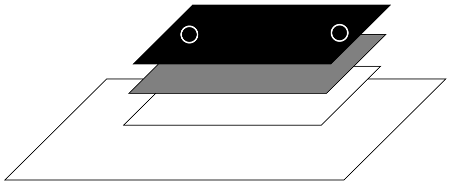
Please note that when the situation is electrostatic, 1) the electric field within a metal is zero, and 2) the electric field just outside the surface of a metal is perpendicular to the surface. If either of these conditions were altered, then there would be an electric current in the metal, which is not an electrostatic situation. Because an electric field exerts a force on a charge, work must be done to move a charged object along any of the field lines. On the other hand, movement perpendicular to the field lines requires no work. Such movement is said to be along an equipotential line.



In the figure above, the electric field for a positive point charge is shown as lines with arrows. The regions of equipotential (equipotential lines) are shown with circles. Notice that the equipotential lines are perpendicular to the electric field lines and that the density of equipotential lines is proportional to the electric field strength.

Electric field lines are difficult to measure directly, but potentials can be measured with a voltmeter. An electric field will arise in the space surrounding two separated charged conductors. With one lead of a voltmeter connected to one of the conductors and the other used as a probe, the potentials can be determined (see figure below).





Apparatus

- Power supply
- Voltmeter
- Conducting sheets
- Carbon and white paper
- Wooden board and pins

Activity 1: Field Lines for Two Point Charges

Prediction: Using the rules given in the introduction and given in the first set of figures, draw the electric lines for two oppositely charged point objects. Sketch the equipotential lines.

1. Find the conducting paper with the two silver circles on the front and lay it over a copy carbon and a sheet of paper on top of the wooden board.
2. Connect the positive output of the power supply to one of the circles and the negative to the other.
3. Connect the negative lead of the voltmeter to the negative conductor and use the positive lead as the probe.
4. With the power supply voltage turned on and set to 10 volts, probe lightly with the voltmeter to find a number of points on the carbon paper registering 8 volts. Push down each time you find a point so that marks will be made on the bottom paper.
5. Repeat for 6 volts, 4 volts, and 2 volts.
6. You should end up with a series of dots on your sheet of paper. Connect those associated with the same potential with smooth lines.

7. Recalling the relationship between electric field lines and equipotential lines, sketch in the electric field lines. (*Other group members can sketch copies of the same results.*)
8. Does your experimental result agree with your prediction? Explain.

Activity 2: Field Lines for Parallel Plates

Prediction: Draw what you think the field lines and equipotential lines between parallel plates will look like.

1. Carry out the instructions from Activity 1 to check your prediction.
2. Does your result agree with your prediction? Explain.

Activity 3: Field Lines Between a Point Charge and a Plate

Prediction: Draw what you think the field lines and equipotential lines between a point charge and a parallel plate will look like.

1. Map the field lines as before.
2. Does your result agree with your prediction? Explain.
3. If the potential is zero, must the electric field be zero as well?
4. What can you say about the electric field along an equipotential line?

Activity 4: Field Lines for a Plate and a Charged Circle

Prediction: Sketch what you think the field and potential lines between a charged circle and a plate look like.

1. Determine the field lines.
2. What is the field strength within a charged, continuous surface?

13 The Electric Potential[5]

Name ______________________ Section ____________ Date ____________

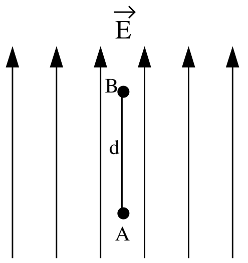
Overview

It takes work to lift an object in the earth's gravitational field. Lowering the object releases the energy that was stored as potential energy when it was lifted. Last semester, we applied the term *conservative* to the gravitational force because it "releases" all of the stored energy. We found experimentally that the work required to move a mass in the gravitational field was path independent. This is an important property of any conservative force. Given the mathematical similarity between the Coulomb force and the gravitational force, it should come as no surprise that experiments confirm that an electric field is also conservative. This means that the work needed to move a charge from point A to point B is independent of the path taken between points. A charge could be moved directly between the two points or looped around and the work expended to take either path would be the same. Work done by an electric field on a test charge q traveling between points A and B is given by

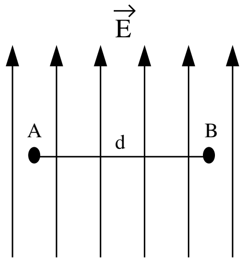
$$W = \int_A^B \vec{F} \cdot d\vec{s} = \int_A^B q\vec{E} \cdot d\vec{s}$$

Activity 1: Work Done on a Charge Traveling in a Uniform Electric Field

(a) A charge q travels a distance d from point A to point B; the path is parallel to a uniform electric field of magnitude E. What is the work done by the field on the charge? How does the form of this equation compare to the work done on a mass m traveling a distance d in the almost uniform gravitational field near the surface of the earth?

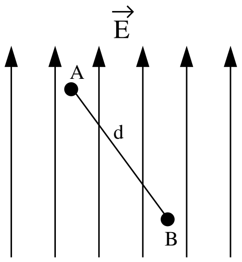


(b) The charge q travels a distance d from point A to point B in a uniform electric field of magnitude E, but this time the path is perpendicular to the field lines. What is the work done by the field on the charge?



(c) The charge q travels a distance d from point A to point B in a uniform electric field of magnitude E. The path lies at a 45° angle to the field lines. What is the work done by the field on the charge?

[5] 1990-93 Dept. of Physics and Astronomy, Dickinson College. Supported by FIPSE (U.S. Dept. of Ed.) and NSF. Portions of this material may have been modified locally and may not have been classroom tested at Dickinson College.



Potential Energy and Potential Difference

Recall that by definition the work done by a conservative force equals the negative of the change in potential energy, so that the change in potential energy of a charge moving from point A to point B under the influence of an electrical force is given by:

$$\Delta E_{pot} = E_{pot,B} - E_{pot,A} = -\int_A^B q\vec{E} \cdot d\vec{s}$$

By analogy to the definition of the electric field, we are interested in defining the *electric potential difference* $\Delta V = V_B - V_A$ as the change in electric potential energy ΔE_{pot} per unit charge. Formally, *the potential difference is defined as the work per unit charge that an external agent must perform to move a test charge from A to B without changing its kinetic energy.* The potential difference has units of joules per coulomb. Since 1 J/C is defined as one *volt*, the potential difference is often referred to as *voltage*.

Activity 2: The Equation for Potential Difference

Write the equation for potential difference as a function of $\vec{E}$, $d\vec{s}$, A, and B.

The Potential Difference for a Point Charge

The simplest charge configuration that can be used to consider how voltage changes between two points in space is a single point charge. We will start by considering a single point charge and then move on to more complicated configurations of charge.

A point charge q produces an electric field that points radially outward in all directions for a positive charge, radially inward for a negative charge. The line integral equation for the potential difference can be evaluated to find the potential difference between any two points in space A and B (a line integral is one that follows a path through space).

It is common to choose the reference point for the determination of voltage to be set at infinity so that we are determining the work per unit charge that is required to bring a test charge from infinity to a certain point in space. Let's choose a coordinate system so that the point charge is conveniently located at the origin. In this case we will be interested in the potential difference between infinity and some point which is a distance r from the point charge. Thus, we can write the equation for the potential difference, or voltage, as

$$\Delta V = V_B - V_A = V_r - V_\infty = -\int_\infty^r \vec{E} \cdot d\vec{s}$$

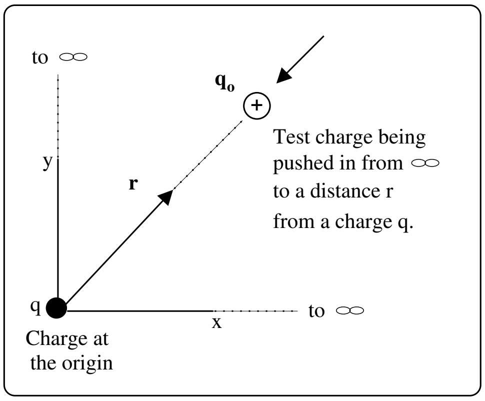
Often, when the reference point for the potential difference is at infinity, this difference is simply referred to as "the potential", and the symbol ΔV is just replaced with the symbol V.

Activity 3: Potential at a Distance r from a Charge

Starting from the expression for the electric field of a point charge, show that, if A is at infinity and B is a distance r from a point charge q, then the potential V is given by the expression

$$V = \frac{kq}{r}$$

where $k = \frac{1}{4\pi\varepsilon_o} = 8.99 \times 10^9$ Nm^2/ C^2.

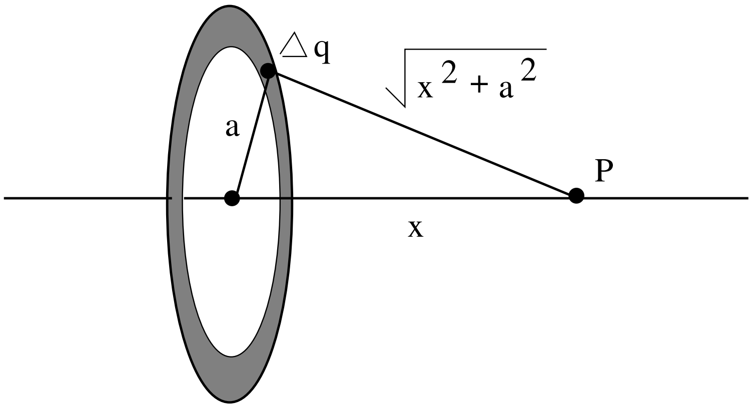


Hint: What is the mathematical expression for an E-field from a point charge?

The Potential Difference Due to Continuous Charge Distributions

The potential from a continuous charge distribution can be calculated several ways. Each method should yield approximately the same result. First, we can use an integral method in which the potential dV from each element of charge dq is integrated mathematically to give a total potential at the location of interest. Second, we can approximate the value of the potential V by summing up several finite elements of charge Δq by using a computer spreadsheet or hand calculations.

Again, let's consider a relatively simple charge distribution. In this case we will look at a ring with charge uniformly distributed on it. We will calculate the potential on the axis passing through the center of the ring as shown in the diagram below. (Later on you could find the potential difference from a disk or a sheet of charge by considering a collection of nested rings).



A ring of charge has a total charge of $Q = 20\mu$C (i.e. 20 x 10^{-6} C). The radius of the ring, a, is 30 cm. We want to find the electric field and the electric potential at a distance x from the ring along an axis that is perpendicular to the ring and passes through its center. Let's begin by calculating the potential in the next activity.

Hints: Since the potential is a scalar and not a vector we can calculate the potential at point P (relative to ∞) for each of the charge elements Δq and add them to each other. This looks like a big deal but it is actually a trivial problem because all the charge elements are the same distance from point P.

Activity 4: Numerical Estimate of the Potential from a Charged Ring

(a) Divide the ring into 20 elements of charge Δq and calculate the total V at a distance of $x = 20$ cm from the center of the ring using a spreadsheet program or by hand calculation. Summarize the result below. Be sure to attach a printout of your spreadsheet results.

Activity 5: Calculation of the Potential from a Charged Ring

By following the steps below, you can use an integral to find a more exact value of the potential.

(a) Show that

$$V = k \int \frac{dq}{r} = k \int \frac{dq}{\sqrt{x^2+a^2}}$$

(b) Show that

$$k \int \frac{dq}{\sqrt{x^2+a^2}} = \frac{k}{\sqrt{x^2+a^2}} \int dq$$

(i.e. show that $\sqrt{x^2 + a^2}$ is a constant and thus can be pulled out of the integral).

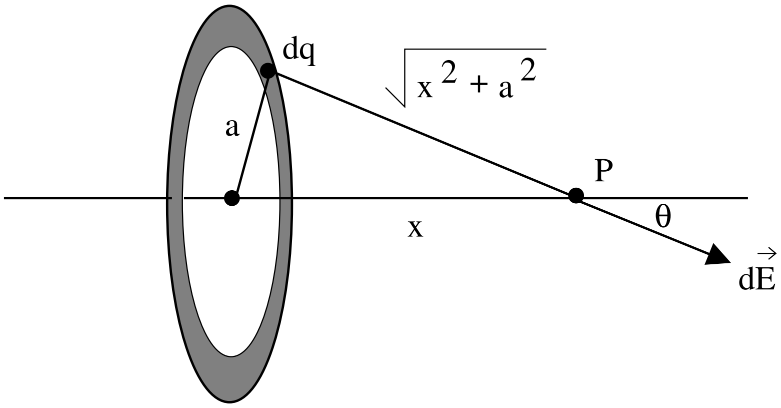
(c) Perform the integration in part (b) above. Then substitute values for a, x, and Q into the resulting expression in order to obtain a more "exact" value for the potential.

(d) How does the "numerical" value that you obtained in Activity 4 compare with the "exact" value you obtained in (c)?

Now let's take a completely different approach to the same problem. If we can find the vector equation for the electric field at point P due to the ring of charge, then we can use the expression

$$\Delta V = V_r - V_\infty = -\int_\infty^r \vec{E} \cdot d\vec{s}$$

as an alternative way to find a general equation for the potential at point P.



Activity 6: ΔV from a Ring Using the E-field Method

(a) Starting from the electric field of a point charge, show that the electric field at point P from the charged ring is given by

$$\vec{E} = \frac{kqx}{(x^2+a^2)^{3/2}}\hat{x}$$

Hints: (1) Why is there no y component of the E-field? (2) $\cos\theta = \frac{x}{\sqrt{x^2+a^2}}$

(b) To find ΔV use the following equation.

$$\Delta V = V_r - V_\infty = -\int_\infty^r \vec{E} \cdot d\vec{s}$$

Hint: You will probably need to consult the integral tables in the appendix of your text. $\cos\theta = \frac{x}{\sqrt{x^2+a^2}}$

(c) How does the result compare to that obtained in Activity 5 (c)?

Equipotential Surfaces

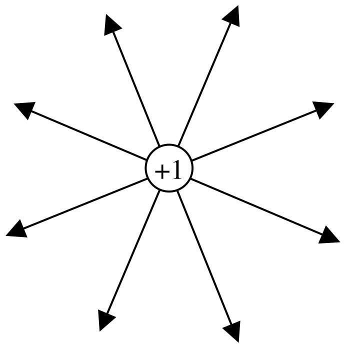
Sometimes it is possible to move along a surface without doing any work. Thus, it is possible to remain at the same potential energy anywhere along such a surface. If an electric charge can travel along a surface without doing any work, the surface is called an *equipotential surface.*

Consider the three different charge configurations shown below. Where are the equipotential surfaces? What shapes do they have?

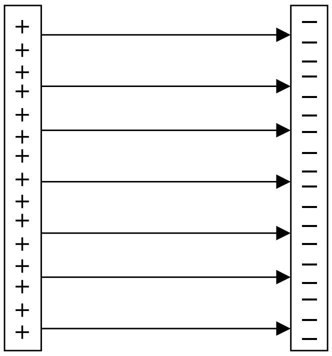
Hint: If you have any computer simulations available to you for drawing equipotential lines associated with electrical charges, you may want to check your guesses against the patterns drawn in one or more of the simulations.

Activity 7: Sketches of Electric Field Lines and Equipotentials

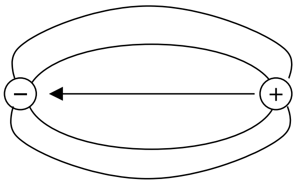
(a) Suppose that you are a test charge and you start moving at some distance from the charge below (such as 4 cm). What path could you move along without doing any work, i.e. $\vec{E} \cdot d\vec{s}$ is always zero? What is the shape of the equipotential surface? Remember that in general you can move in *three* dimensions.



(b) Find some equipotential surfaces for the charge configuration shown below, which consists of two charged metal plates placed parallel to each other. What is the shape of the equipotential surfaces?



(c) Find some equipotential surfaces for the electric dipole charge configuration shown below.



(d) In general, what is the relationship between the direction of the equipotential lines you have drawn (representing that part of the equipotential surface that lies in the plane of the paper) and the direction of the electric field lines?

14 The Electric Field and the Electric Potential I

Name ______________ Section ______________ Date ______________

Objective

- To investigate the electric field and potential of a point charge.

Apparatus

- Electric field and potential simulation entitled *EM Field 6*.

Introduction

The direction of an electric field is the direction of the force on a tiny positive test charge placed in the region of space where the field is to be measured. If the magnitude of this test charge is infinitesimally small, so small that it will not displace or disturb the charges that are the source of the field, we can use the test charge to determine quantitatively the strength of the electric field. The strength of the electric field is taken to be the electric force, F, on the test charge divided by the magnitude of the test charge, q_t: $E = \frac{F}{q_t}$. The force (Coulomb's Law) between two charges, q_1 and q_2, is $F = k\frac{q_1 q_2}{r^2}$, where k = 9 x 10^9 $\mathrm{Nm}^2/\mathrm{C}^2$. The units of E are newtons per coulomb, so another way of describing the field strength is to say it is the force experienced by a unit positive test charge.

Recall from a previous laboratory exercise that the potential difference between two points A and B, V_B - V_A, is the work done carrying a unit positive charge from point A to point B. Also, the lines of force (the electric field lines) are always perpendicular to the equipotential lines, lines on which all points are at the same potential. In a static electric field, the electric potential difference between two points is a constant and does not depend on the path used for its computation. The absolute potential, as opposed to the potential difference, is the amount of work done in carrying a unit charge from infinity to point B. The magnitude of the absolute potential, then, is computed as the integral from infinity to the point B of the electric field.

Investigation 1: A Single Charge

Activity 1: The Electric Field

(a) Start the program *EM Field 6* from the 132 menu. Click on the screen and you will see a screen with a set of instructions. Go to the **Sources** menu and select **3D Point Charges**. A blank 'table top' with a set of menu buttons at the top and bottom will appear. See Figure 1 below.

(b) Go to the Display menu and set *EM Field 6* to **Show Grid** and **Constrain to Grid**. These choices will make the following investigation a bit easier to perform.

(c) Select the charge labeled "+4" from the available set by clicking on it and dragging it to the center of the table top.

(d) **Prediction**: You will take measurements of the field at different distances from the charge. You know the relative size of the charge (+4), but you don't know the size of the charge in coulombs. Generate an expression for the magnitude of the field from an unknown charge with appropriate numerical constants and units. The only unknown in your result should be the charge in coulombs. How does the electric field depend on r, the distance from the point charge?

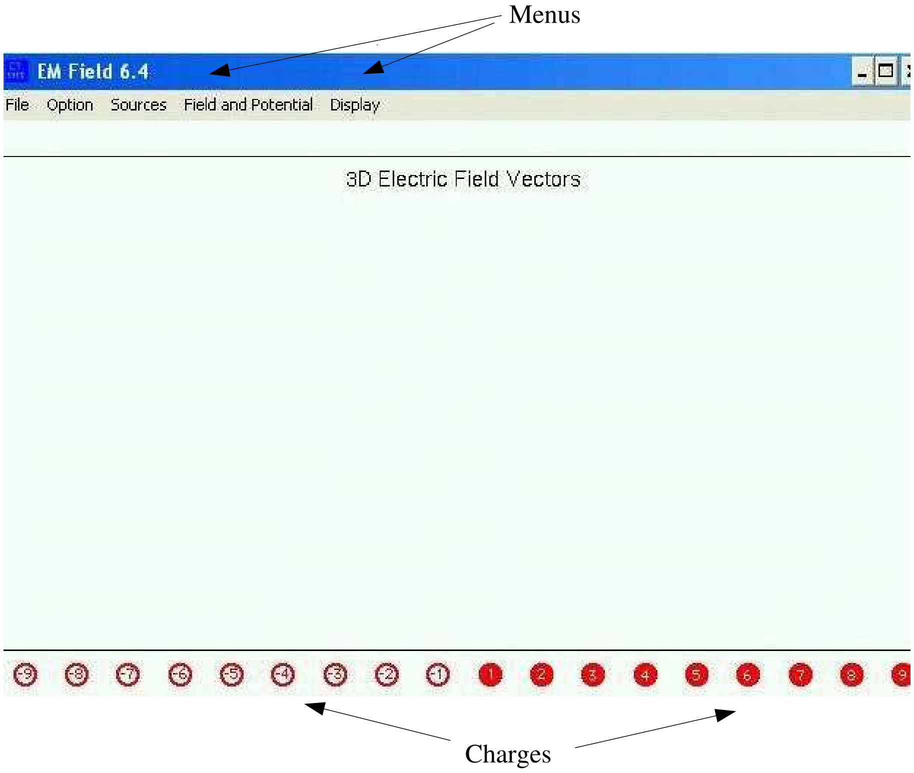


Figure 5: Table top for *EM Field 6.*

(e) Click anywhere in the table top and you will see an arrow drawn. The size and direction of the arrow represent the magnitude and direction of the electric field at that point due to the '+4' charge. In what direction does the arrow point? Click on the opposite side of the table top. In what direction does this arrow point? How is it related to the first arrow?

(f) Click on many points so that you get a wide range of magnitudes from large (barely fits on the table top) to small (barely bigger than a dot).

(g) Print the table top and use a ruler to measure the lengths of each of the arrows on your plot. Enter this data in the table below. Use the scale at the bottom of the table top to convert the length of each arrow into an electric field magnitude. The units of the scale electric field vector are 1.0 N/C.

Distance from Charge (m)	Arrow Length (m)	Measured E (N/C)	Charge (C)

(h) Use the results in column 3 of your table to determine the unknown charge for each electric field measurement and enter the results in the table. Calculate the average and standard deviation of the values of the charge. Are your results consistent? Explain.

(i) Did the measured values agree with your predictions? If they didn't, can you explain why not?

(j) **Prediction**: From Coulomb's Law, we expect the spatial variation of the field strength to obey a power law: $|E| = Ar^n$, where A and n are constants. What do you predict the value of n to be?

(k) Graph field strength as a function of r. Using the power fitting function, determine the power of the function, n, and record it here. Attach the plot to this unit.

(l) Does your result agree with your prediction? Explain any discrepancy.

Activity 2: The Electric Potential

(a) Under the **Display** menu click on **Clean up Screen** to erase the electric field vectors.

(b) **Prediction**: You will now take measurements of the potential. How do you expect the electric potential to change with distance from the point charge?

(c) Click on the **Potential** option under the **Field and Potential** menu. Click on the table top and a marker will be placed at that point and labeled with the value of the potential there. Click on many spots on the table top from very close to the point charge to far away. When you are finished print the table top.

(d) Measure and record in the table the values of the distance from the point charge and the potential.

Distance (m)	Measured V (volts)	Predicted V (volts)

(e) Calculate the value of the electric potential at each of these points from the distance you measured from the point charge and the value of the charge from the previous activity. Fill in the appropriate columns of the table with the distance and predicted potential. Show a sample calculation in the space below.

(f) Did the measured values agree with your calculations? If they didn't, can you explain why not?

(g) **Prediction**: From Coulomb's Law and the definition of the electric potential, we expect the spatial variation of the potential to obey a power law: $\Delta V = Br^m$, where B and m are constants. What do you predict the value of m to be?

(h) Graph the voltage as a function of r. Using the power fitting function, determine the power of the function, m, and record it here.

(i) Does your result agree with your prediction? Explain any discrepancy.

15 The Electric Field and the Electric Potential II

Name ______________________ Section ____________ Date ____________

Objective

- To investigate the electric field and potential of a charge distribution.

Apparatus

- Electric field and potential simulation entitled *EM Field 6*.

Introduction

In the previous unit (which we will refer to as Investigation 1) we studied the dependence of the electric field and the electric potential on r, the distance from a single charge. Now we will study the same ideas for a different charge distribution.

Investigation 2: Four Symmetrically Arranged Charges

Activity 1: The Electric Field

(a) Start the program *EM Field 6* from the 132 menu or (if it's already running) use the options under the **Display** menu to clear the table top and delete any charges. Go to the **Sources** menu and select **3D Point Charges**. A blank 'table top' with a set of menu buttons at the top and bottom will appear (see Figure 1 in Investigation 1, the previous unit).

(b) Go to the Display menu and set *EM Field 6* to **Show Grid** and **Constrain to Grid** if they are not already set. These choices will make the following investigation a bit easier to perform.

(c) Under **Sources**, click on **3D Point Charges**. Select the charge labeled "+1" from the available set by clicking on it. Add four individual charges, arranging them symmetrically within about 1 cm of the central point where the "+4" charge was located in Investigation 1 (the previous unit).

(d) **Prediction:** How will the electric field be oriented within the region of the four charges? How will the field be oriented outside the region of the four charges? How will the field depend on r, the distance from the center of the four charges, at large r?

(e) Click on the **Field** option under the **Field and Potential** menu. Click anywhere in the table top and you will see an arrow drawn. The size and direction of the arrow represent the magnitude and direction of the electric field at that point due to the four charges. In what direction does the arrow point? Click on the opposite side of the table top. In what direction does this arrow point? How is it related to the first arrow?

(f) Click on many points so that you get a wide range of magnitudes from large (barely fits on the table top) to small (barely bigger than a dot).

(g) Print the table top and use a ruler to measure the lengths of each of the arrows on your plot. Enter this data in the table below. Use the scale at the bottom of the table top to convert the length of each arrow into an electric field magnitude. The units of the scale electric field vector are 1.0 N/C.

Distance from Charge Center (m)	Arrow Length (m)	Measured E (N/C)

(h) **Prediction**: From Coulomb's Law, we expect the spatial variation of the field strength to obey a power law: $|E| = Ar^n$, where A and n are constants. What do you predict the value of n to be?

(i) Graph your results. Using the power fitting function, determine the power of the function, n, and record it here. Attach the plot to this unit.

(j) Does your result agree with your prediction? Explain any discrepancy.

Activity 2: The Electric Potential

(a) Under the **Display** menu click on **Clean up Screen** to erase the electric field vectors.

(b) **Prediction**: You will now take measurements of the potential. How do you expect the electric potential to change with distance from the center of the four charges?

(c) Click on the **Potential** option under the **Fields and Potentials** menu. Click on the table top and a marker will be placed at that point and labeled with the value of the potential there. Click on many spots on the table top from very close to the charges to far away. When you are finished print the table top.

(d) Measure and record in the table below the values of the distance from the point charge and the potential.

Distance (m)	Measured V (volts)

(e) **Prediction**: From Coulomb's Law and the definition of the electric potential, we expect the spatial variation of the potential to obey a power law: $\Delta V = Br^m$, where **B** and m are constants. What do you predict the value of m to be?

(f) Graph your results. Using the power fitting function, determine the power of the function, m, and record it here.

(g) Does your result agree with your prediction? Explain any discrepancy.

(h) How do your results for the power constants, n and m, of the four symmetrically-arranged charges compare with the power constants you determined in Investigation 1 (the previous unit) for the single point charge?

(i) What can you conclude about the field and potential effects due to a distribution of charge outside the region of the distribution (in relation to a single point charge)?

16 The Electric Field and the Electric Potential III

Name ______________________ Section ____________ Date ____________

Objective

- To investigate the electric field and potential of an electric dipole.

Apparatus

- Electric field and potential simulation entitled *EM Field 6*.

Introduction

In the previous units (which we will refer to as Investigation 1 and 2) we studied the dependence of the electric field and the electric potential on r, the distance from a charge distribution. Now we will study the same ideas for a charge distribution commonly found in physics and chemistry using the same methods we used before.

Investigation 3: An Electric Dipole

Activity 1: The Electric Field

(a) Start the program *EM Field 6* from the 132 menu or use the options under the **Display** menu to clear the table top and delete any charges. Go to the **Sources** menu and select **3D Point Charges**. A blank 'table top' with a set of menu buttons at the top and bottom will appear.

(b) Go to the Display menu and set *EM Field 6* to **Show Grid** and **Constrain to Grid** if they are not already set. These choices will make the following investigation a bit easier to perform.

(c) Clear the table top and build an electric dipole by placing two magnitude "4" charges of opposite sign a distance 2 cm apart near the left side of the table top.

(d) **Prediction** How will the electric field be oriented between the two charges? How will the field be oriented outside the region of the two charges? How will the field depend on r, the distance from the midpoint of a line joining the two charges, at large r?

(e) Click on the **Field** option under the **Fields and Potentials** menu. Click along a line perpendicular to the midpoint of a line joining the two charges. The size and direction of the arrow represent the magnitude and direction of the electric field at that point due to the dipole. In what direction does the arrow point?

(f) Click on many points along the same line so that you get a wide range of magnitudes from large (barely fits on the table top) to small (barely bigger than a dot).

(g) Print the table top and use a ruler to measure the lengths of each of the arrows on your plot and the distance from the midpoint of a line joining the two charges. Enter this data in the table below. Use the scale at the bottom of the table top to convert the length of each arrow into an electric field magnitude. The units of the scale electric field vector are 1.0 N/C.

Distance from Charge Center (m)	Arrow Length (m)	Measured E (N/C)

(h) **Prediction**: From Coulomb's Law, we expect the spatial variation of the field strength to obey a power law: $|E| = Ar^r$, where A and n are constants. What do you predict the value of n to be?

(i) Graph your results. Using the power fitting function, determine the power of the function, n, and record it here. Attach the plot to this unit.

(j) Does your result agree with your prediction? Explain any discrepancy.

(k) How do your results compare with the power law constants you found in Investigations 1 and 2? Explain.

Activity 2: The Electric Potential

(a) Under the **Display** menu click on **Clean up Screen** to erase the electric field vectors.

(b) **Prediction**: You will now take measurements of the potential. How do you expect the electric potential to change with distance from the electric dipole along the line perpendicular to the midpoint of a line joining the two charges?

(c) Click on the **Potential** option under the **Field and Potential** menu. Click on the table top and a marker will be placed at that point and labeled with the value of the potential there. Click on many spots on the table top along the line perpendicular to the midpoint of the line joining the two charges. When you are finished print the table top.

(d) Measure and record in the table the values of the distance from the midpoint of a line joining the two charges and the potential.

Distance (m)	Measured ΔV (volts)

(e) **Prediction**: From Coulomb's Law and the definition of the electric potential, we expect the spatial variation of the potential to obey a power law: $\Delta V = Br^m$, where **B** and m are constants. What do you predict the value of m to be?

(f) Graph your results. Using the power fitting function, determine the power of the function, m, and record it here.

(g) Does your result agree with your prediction? Explain any discrepancy.

(h) How do your results compare with the power law constants you found in Investigations 1 and 2? Explain.

Activity 3: Equipotential Lines and Field Lines

(a) Under the **Field and Potential** menu, drag down to **Equipotentials**. Click the mouse on the table top and a dotted line will be drawn representing the equipotential line with a label representing the value of the electric potential. [If the curve does not close (i.e., the last point drawn doesn't match up with the starting point), consult the instructor.] Map out the equipotential lines by moving the cursor across the table top away from each charge and clicking the mouse at regular intervals.

(b) What do these curves represent?

(c) Under the **Field and Potential** menu, click on **Field Lines–Auto**. The field lines of the charge distribution will be drawn. [If *EM Field 6* takes a long time to draw one of the field lines, consult your instructor.] Print the result and attach it to this unit.

(d) How are the field lines and the equipotential lines related to one another at the points where they cross?

17 Ohm's Law

Name ______________________ Section ____________ Date ____________

Objectives

- To investigate the most important principle in electronics.
- To determine how resistors in series and parallel add.

Introduction

The rate at which electric charge flows through a conductor is called the electric current. In order to have a current, a potential difference, or voltage is necessary. We first want to determine the relationship between the potential difference at two ends of a conductor and the current flowing through it.

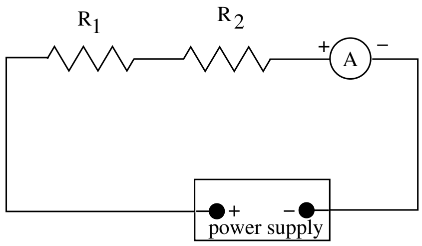
Note: Do not turn on a power supply until you are sure your circuit is correct. If you are at all unsure, please ask your instructor to approve your setup. Ammeters can be instantly and permanently ruined by an improper connection. Be sure to turn off the power supply before making any changes to the circuit.

Apparatus

- power supply
- 2 rheostats
- ammeter
- voltmeter

Activity 1: Ohm's Law

- Connect two rheostats (or variable resistors) in series as shown in the figure below. Set R_1 at about the halfway point and R_2 at the maximum. Connect an ammeter as shown. Also, connect a voltmeter across (that is, connect a probe to each side of) R_1.



- When sure of your circuit, turn on the power supply, and turn the voltage up all the way..
- Record the current through the circuit and the voltage across R_1.

- Reduce the resistance of R_2 and record the current and voltage three more times by turning down the rheostat in approximately equal steps so that for the last time R_2 is turned completely down.

- Turn off the power supply.
- Plot your four pairs of readings with the voltage on the vertical axis and the current on the horizontal axis.
- Fit a straight line to the points, using the origin as a fifth point.
- Is a straight line a good fit to the data? What does that say about the relationship between voltage and current?

- What are the value and meaning of the slope of this line? Write the equation of the line.

- Remove R_1 from the rest of the circuit and use the ohmmeter option on the multimeter to measure the resistance of R_1. Does it agree with the slope you found? What is the percent difference? Replace R_1.

- What is the general relationship between voltage, current, and resistance? This is Ohm's Law.

- Why is the origin a legitimate point on the curve?

Activity 2: Resistors in Series

- Turn rheostat R_2 to its maximum setting. Connect the multimeter across this resistor, being sure to set it for reading voltages.
- When you are sure the circuit is set, turn on the power supply and record the current and voltage. Turn off the power supply.

- **Prediction**: Based on your measurements, predict the resistance of R_2.

- Remove and measure the resistance of R_2. Record the percent difference between your prediction and measurement. Replace R_2.

- Was the current this time different from the first reading in Activity 1?

- What can you conclude about the current through two resistors in series?

- Connect the multimeter across both resistors, being sure to switch to voltage readout.
- When you are sure the circuit is correct, turn on the power supply and record the current and voltage. Turn off the power supply.

- Has the current changed?

- Has your previous conclusion been substantiated or refuted?

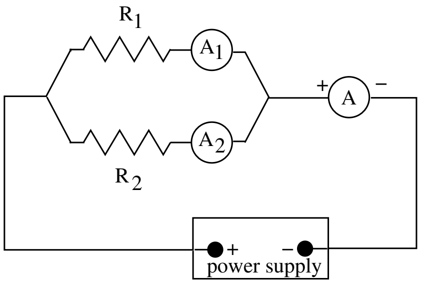
- How is the voltage just measured related to the first voltage measurements in Activities 1 and 2?

- What can you conclude about the voltage across resistors in series?

- Using your conclusions concerning voltages across and current through resistors in series and your formulation of Ohm's law, what can you conclude about the total resistance in a circuit having two resistors in series?

Activity 3: Resistors in Parallel

- Connect the two rheostats in parallel as shown in the figure below, with the ammeter at the point marked A and the voltmeter across the two rheostats. Set the rheostats at about their halfway settings.



- When you are sure the circuit is set up correctly, turn on the power supply and record the total current through the circuit and the voltage drop across the parallel resistance combination. Turn off the power supply.

- Connect the ammeter to the point marked A_1 without disturbing the rest of the circuit; apply power and record the current through R_1 and the voltage reading. Turn off the power supply.

- Repeat the above measurements for R_2, connecting the ammeter at A_2 instead of A_1.

- Using Ohm's Law, calculate the two resistances of the parallel connection and also the total resistance of the circuit. Check with the ohmmeter and determine the percent differences.

- What is the relationship between the total current and the current in each of the branches of the parallel circuit?

- What is the relationship between the total resistance of the parallel circuit and the resistance of each of the branches (you may want to look up in a reference what the correct relationship should be and see if your result agrees with it)?

- Determine, using Ohm's law, what the voltage was in each branch of the parallel circuit. Did it make any difference that you didn't reposition the voltmeter during this activity? On the basis of Ohm's law, does the result make sense?

- Can the total resistance of a series combination ever be less than the resistance of the largest resistor? Explain.

- Can the total resistance of a parallel combination ever be greater than the resistance of the smallest resistor? Explain.

18 Magnetism I

Name ____________________ Section __________ Date __________

Objective

- To investigate the magnetic field around a permanent magnet.

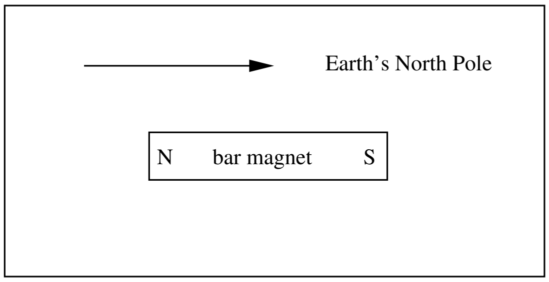
Introduction

The magnetic field characterizes magnetic forces in much the same way that the electric field characterizes electric forces. At a given point in the region around a magnet, the strength of the field, similar to that of an electric field, is the force per unit north pole (one positive unit of magnetism), and the direction is indicated by the orientation of the north pole of a compass located at the point. On earth, the field mapped out around the magnet is actually the resultant of the field due to the magnet and the field due to the earth.

Apparatus

- 2 bar magnets
- small compass
- white paper and tape

Activity 1: A Single Bar Magnet



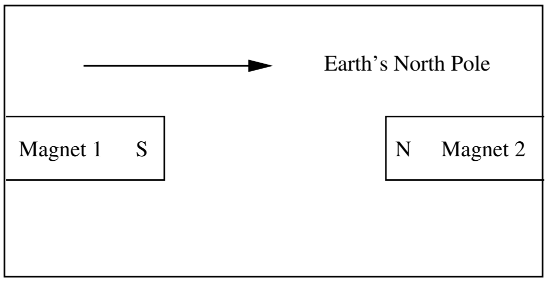
1. Center and tape a bar magnet on a piece of paper and orient it so that the magnet's south pole points to the earth's geographic north pole. Indicate the magnet's polarity and the direction of the earth's field.
2. Place the small compass near the north pole of the magnet and make a dot at each end of the needle using a pencil not encased in metal.
3. Move the compass forward until its south pole points at the previous north pole dot, and make a new dot at the north pole.
4. Repeat 3 until the series of dots reaches the south pole of the magnet or the edge of the paper.
5. In a similar manner, trace enough lines to map the magnetic field over the entire paper. Take points about 0.5 cm apart near the poles and about 2 cm apart near the middle of the magnet.
6. There are two points, called neutral points, near each end of the magnet where the magnet's field and the earth's field are equal and opposite and so cancel. At these points, the compass will align in no particular direction. Try to locate these points by tracing very carefully the lines of force in the neighborhood of the poles.

7. Do lines of force ever cross?

8. Where are the magnetic forces strongest? Weakest? How do the force lines indicate this? Does a line of force represent a constant force along its entire length?

9. Do the lines intersect the magnet at a particular angle (like the electric field lines near a conductor)? What does this imply about the source of a magnetic field as opposed to the surface charge of a conductor as the source of an electric field?

Activity 2: Two Bar Magnets–Unlike Poles Facing One Another



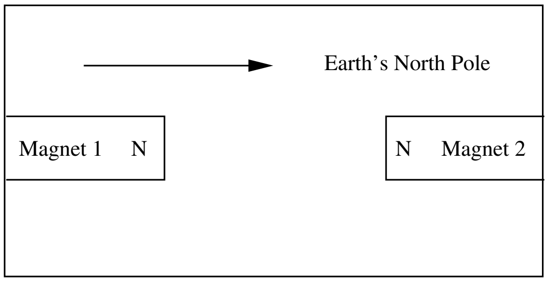
1. Set up two bar magnets on a sheet of paper as shown in the figure above. The magnets should be 8-10 cm apart.

2. Repeat steps 2 through 5 from the previous activity.

3. What sort of charge configuration produces an electric field that looks similar to the magnetic field you just identified?

4. What differences can you recognize?

Activity 3: Two Bar Magnets–Like Poles Facing One Another



1. Set up two bar magnets on a sheet of paper as shown in the figure above. The magnets should be 8-10 cm apart.
2. Repeat steps 2 through 5 from Activity 1 of this investigation.
3. Try to identify on your map a point at which the magnetic field is zero. Explain what causes this effect.

4. What sort of electric charge configuration would produce a similar field map?

19 Magnetism II

Name ______________________ Section ____________ Date ____________

Objectives

To investigate:

- The effect of magnetic fields on moving charges.
- The effect of moving charges (currents) on magnets.

Apparatus

- Bar magnet
- Oscilloscope
- Tangent galvanometer
- Compass
- Power supply

Activity 1: Magnetic Forces on Moving Charges

1. Turn on the oscilloscope by pressing the power button. Turn the TIME/DIV knob completely counterclockwise. Adjust the INTEN (intensity) and FOCUS knobs so that a small bright spot is formed on the oscilloscope screen by the beam of electrons traveling toward you. Adjust the ILLUM (illumination) knob so that the grid on the screen can be seen clearly. Use the horizontal and vertical POSITION controls to center the spot on the screen.
2. **Note**: An oscilloscope is built around the principle of the cathode ray tube. It emits electrons from its back end. These are accelerated by a series of electrodes and focused to strike a fluorescent screen at its front. The result is a visible trace identifying voltage as a function of time.
3. **Predictions**: What, if anything, will happen to the spot on the screen if the north pole of a magnet is brought near the left side of the oscilloscope? What will happen if you do the same with the south pole? What about when each of the poles are brought near to the top? [Please do not touch the oscilloscope with the magnet.]
4. Bring the N-pole of a horizontal bar magnet near, but not touching, the left side of the oscilloscope case at the height of the spot. Record the direction of any deflection. Repeat with the S-pole.

5. Bring the N-pole of a vertical bar magnet near, but not touching, the top of the oscilloscope cast just above the spot. Record the direction of any deflection. Repeat with the S-pole.

6. Turn off the oscilloscope.

7. Did the directions of deflections meet your expectations? Explain.

Activity 2: The Effect of Moving Charges (Currents) on Magnets

In this investigation we will use a device known as a tangent galvanometer to make a qualitative study of the effect of current (moving charges) in a coil of wire on a compass. A sketch of the galvanometer is shown below.

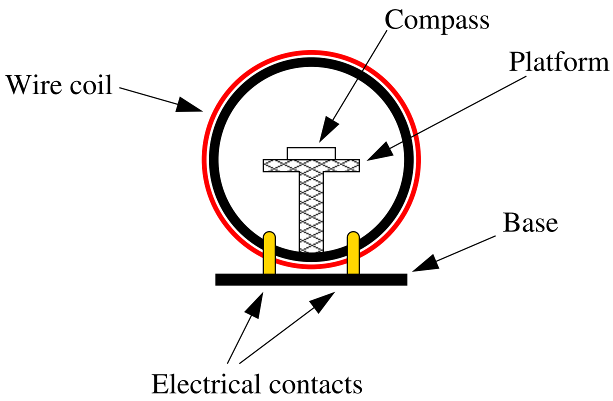


Figure 1. Tangent galvanometer and compass.

1. Connect the positive and negative electrodes on the power supply to the two side screws on the tangent galvanometer.

2. With the power supply off, place the compass on the platform in the center of the tangent galvanometer. Align the compass and the plane of the wire coil of the galvanometer with the contacts of the galvanometer to your right. Turn the power supply on and slowly turn up the voltage. What do you observe? Make a sketch to show the orientation of the compass and tangent galvanometer with the voltage on and off.

3. Turn the voltage on the power supply to zero. Rotate the entire setup (galvanometer, compass, wires) 180° so it faces in the opposite direction with the electric contacts now on your left side. Make sure the plane of the wire coil and the compass are parallel. Slowly turn the voltage back up. What do you observe? Make another sketch to show the orientation of the compass and tangent galvanometer with the voltage on and off.

4. The deflection of the compass when current flows in the tangent galvanometer implies the current creates a magnetic field. From your observations can you tell the direction of the magnetic field? Explain.

5. Reverse the wires on the power supply to reverse the direction of the current in the coil of the tangent galvanometer. We will now repeat the observations from above. With the voltage off, align the compass and the plane of the wire coil of the galvanometer with the contacts of the galvanometer to your right. Slowly turn up the voltage. What do you observe? Make a sketch to show the orientation of the compass and tangent galvanometer with the voltage on and off.

6. Rotate the entire setup (galvanometer, compass, wires) 180° so it faces in the opposite direction with the electric contacts now on your left side. Make sure the plane of the wire coil and the compass are parallel. Slowly turn the voltage back up. What do you observe? Make another sketch.

7. What happens to the magnetic field of the tangent galvanometer when you reverse the direction of the current?

20 Centripetal Force[6]

Name ______________________ Section ____________ Date ____________

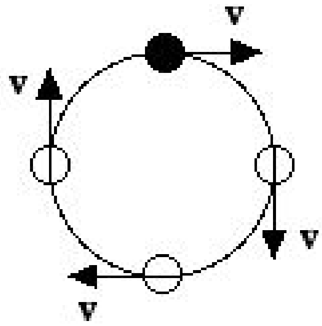
Objective

To explore the phenomenon of uniform circular motion and the accelerations and forces needed to maintain it.

Overview

In a previous unit you began the study of the application of Newton's laws to projectile motion. In this unit we are going to consider the application of Newton's laws to another phenomenon in two dimensions. Since Newton's laws can be used to predict types of motion or the conditions for no motion, their applications are useful in many endeavors including human body motion, astrophysics, and engineering.

You will explore uniform circular motion, in which an object moves at a constant speed in a circle. In particular, you will develop a mathematical description of centripetal acceleration and the force needed to keep an object moving in a circle.



Apparatus

- An airplane.
- A video analysis system (VideoPoint).
- A spring scale.
- Graphing software (Excel).

Moving in a Circle at a Constant Speed

When a race car speeds around a circular track, or when David twirled a stone at the end of a rope to clobber Goliath, or when a planet like Venus orbits the sun, they undergo uniform circular motion. Understanding the forces which govern orbital motion has been vital to astronomers in their quest to understand the laws of gravitation.

But we are getting ahead of ourselves, for as we have done in the case of linear and projectile motion we will begin our study by considering situations involving external applied forces that lead to circular motion in the absence of friction. We will then use our belief in Newton's laws to see how the circular motions of the planets can be used to help astronomers discover the laws of gravitation.

[6]1990-93 Dept. of Physics and Astronomy, Dickinson College. Supported by FIPSE (U.S. Dept. of Ed.) and NSF. Portions of this material have been modified locally and may not have been classroom tested at Dickinson College.

Let's begin our study with some very simple considerations. Suppose an astronaut goes into outer space, ties a ball to the end of a rope, and spins the ball so that it moves at a constant speed.

Activity 1: Uniform Circular Motion

(a) Consider the figure above. What is the speed of a ball that moves in a circle of radius $r = 2.5$ m if it takes 0.50 s to complete one revolution?

(b) The speed of the ball is constant! Would you say that this is accelerated motion?

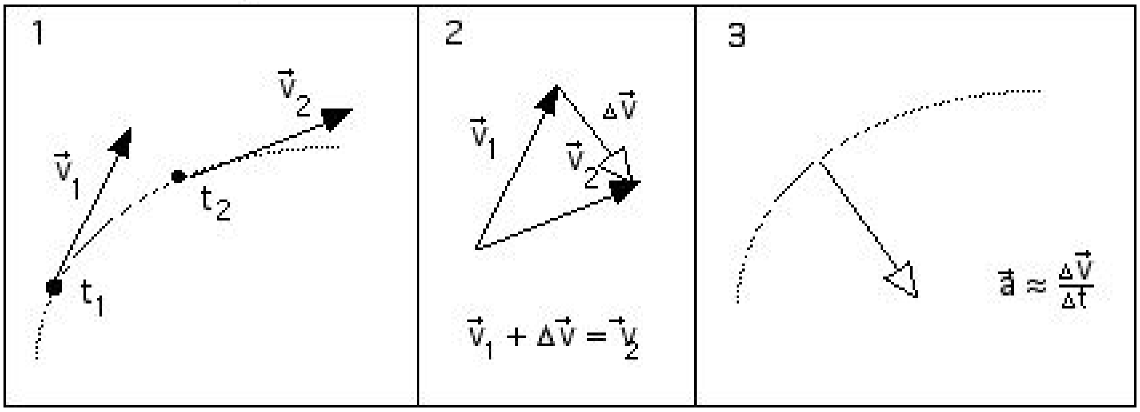
(c) What is the definition of acceleration? (Remember that acceleration is a vector!)

(d) Are velocity and speed the same thing? Is the velocity of the ball constant? (Hint: Velocity is a vector quantity!)

(e) In light of your answers to (c) and (d), would you like to change your answer to part (b)? Explain.

Using Vectors to Diagram How Velocity Changes

By now you should have concluded that since the direction of the motion of an object undergoing uniform circular motion is constantly changing, its velocity is also changing and thus it is accelerating. We would like you to figure out how to calculate the direction of the acceleration and its magnitude as a function of the speed v of an object such as a ball as it revolves and as a function of the radius of the circle in which it revolves. In order to use vectors to find the direction of velocity change in circular motion, let's review some rules for adding velocity vectors.

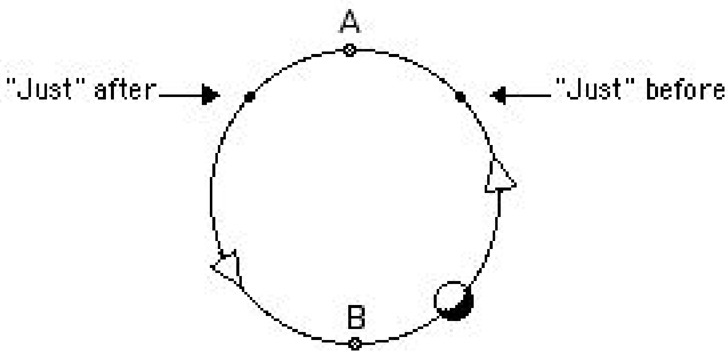


1. To Draw Velocities: Draw an arrow representing the velocity, $\mathbf{v}_1$, of the object at time t_1. Draw another arrow representing the velocity, $\mathbf{v}_2$, of the object at time t_2.
2. To Draw Velocity Change: Find the change in the velocity $\Delta\mathbf{v} = \mathbf{v}_2 - \mathbf{v}_1$ during the time interval described by $\Delta t = t_2 - t_1$. Start by using the rules of vector sums to rearrange the terms so that $\mathbf{v}_1 + \Delta\mathbf{v} = \mathbf{v}_2$. Next place the tails of the two velocity vectors together halfway between the original and final location of the object. The change in velocity is the vector that points from the head of the first velocity vector to the head of the second velocity vector.
3. To Draw Acceleration: The acceleration equals the velocity change $\Delta\mathbf{v}$ divided by the time interval t needed for the change. Thus, $\mathbf{a}$ is in the same direction as $\Delta\mathbf{v}$ but is a different length (unless $\Delta t = 1$). Thus, even if you do not know the time interval, you can still determine the direction of the acceleration because it points in the same direction as $\Delta\mathbf{v}$.

The acceleration associated with uniform circular motion is known as centripetal acceleration. You will use the vector diagram technique described above to find its direction.

Activity 2: The Direction of Centripetal Acceleration

(a) Determine the direction of motion of the ball shown below if it is moving counter-clockwise at a constant speed. Note that the direction of the ball's velocity is always tangential to the circle as it moves around. Draw an arrow representing the direction and magnitude of the ball's velocity as it passes the dot just before it reaches point A. Label this vector $\mathbf{v}_1$.



(b) Next, use the same diagram to draw the arrow representing the velocity of the ball when it is at the dot just after it passes point A. Label this vector $\mathbf{v}_2$.

(c) Find the direction and magnitude of the change in velocity as follows. In the space below, make an exact copy of both vectors, placing the tails of the two vectors together. Next, draw the vector that must be added to vector $\mathbf{v}_1$ to add up to vector $\mathbf{v}_2$; label this vector $\Delta\mathbf{v}$. Be sure that vectors $\mathbf{v}_1$ and $\mathbf{v}_2$ have the same magnitude and direction in this drawing that they had in your drawing in part (a)!

(d) Now, draw an exact copy of $\Delta\mathbf{v}$ on your sketch in part (a). Place the tail of this copy at point A. Again, make sure that your copy has the exact magnitude and direction as the original $\Delta\mathbf{v}$ in part (c).

(e) Now that you know the direction of the change in velocity, what is the direction of the centripetal acceleration, $\mathbf{a}_c$?

(f) If you re did the analysis for point B at the opposite end of the circle, what do you think the direction of the centripetal acceleration, $\mathbf{a}_c$, would be now?

(g) As the ball moves on around the circle, what is the direction of its acceleration?

(h) Use Newton's second law in vector form ($\sum \mathbf{F} = m\mathbf{a}$) to describe the direction of the net force on the ball as it moves around the circle.

(i) If the ball is being twirled around on a string, what is the source of the net force needed to keep it moving in a circle?

Using Mathematics to Derive How Centripetal Acceleration Depends on Radius and Speed

You haven't done any experiments yet to see how centripetal acceleration depends on the radius of the circle and the speed of the object. You can use the rules of mathematics and the definition of acceleration to derive the relationship between speed, radius, and magnitude of centripetal acceleration.

Activity 3: How Does $\mathbf{a}_c$ Depend on v and r?

(a) Do you expect you would need more centripetal acceleration or less centripetal acceleration to cause an object moving at a certain speed to rotate in a smaller circle? In other words, would the magnitude, a_c, have to increase or decrease as r decreases if circular motion is to be maintained? Explain.

(b) Do you expect you would need more centripetal acceleration or less centripetal acceleration to cause an object to rotate at a given radius r if the speed v is increased? In other words, would the magnitude, a_c, have to increase or decrease as v increases if circular motion is to be maintained? Explain.

You should have guessed that it requires more acceleration to move an object of a certain speed in a circle of smaller radius and that it also takes more acceleration to move an object that has a higher speed in a circle of a given radius. Lets use the definition of acceleration in two dimensions and some accepted mathematical relationships to show that the magnitude of centripetal acceleration should actually be given by the equation

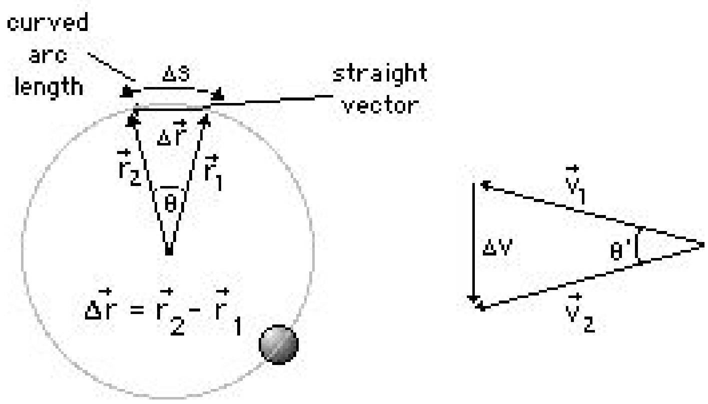
$$a_c = \frac{v^2}{r} \qquad [Eq.\,1]$$

In order to do this derivation you will want to use the following definition for acceleration

$$\langle \mathbf{a} \rangle = \frac{\mathbf{v}_2 - \mathbf{v}_1}{t_2 - t_1} = \frac{\Delta \mathbf{v}}{\Delta t} \qquad [Eq.\,2]$$

Activity 4: Finding the Equation for $\mathbf{a}_c$

(a) Refer to the diagram below. Explain why, at the two points shown on the circle, the angle between the position vectors at times t_1 and t_2 is the same as the angle between the velocity vectors at times t_1 and t_2. Hint: In circular motion, velocity vectors are always perpendicular to their position vectors.



(b) Since the angles are the same and since the magnitudes of the displacements never change (i.e., $r = r_1 = r_2$) and the magnitudes of the velocities never change (i.e., $v = v_1 = v_2$), use the properties of similar triangles to explain why $\frac{\Delta v}{v} = \frac{\Delta r}{r}$.

(c) Now use the equation in part (b) and the definition of $\langle a \rangle$ to show that $\langle a_c \rangle = \frac{\Delta v}{\Delta t} = \frac{(\Delta r)}{(\Delta t)} \frac{v}{r}$.

(d) The speed of the object as it rotates around the circle is given by $v = \frac{\Delta s}{\Delta t}$. Is the change in arc length, Δs, larger or smaller than the magnitude of the change in the position vector, Δr? Explain why the arc length change and the change in the position vector are approximately the same when t is very small (so that the angle θ becomes very small) i.e., why is $\Delta s \simeq \Delta r$?

(e) If $\Delta s \simeq \Delta r$, then what is the equation for the speed in terms of Δr and Δt?

(f) Using the equation in part (c), show that as $\Delta t \to 0$, the instantaneous value of the centripetal acceleration is given by Eq. 1.

(g) If the object has a mass m, what is the equation for the magnitude of the centripetal force needed to keep the object rotating in a circle (in terms of v, r, and m)? In what direction does this force point as the object rotates in its circular orbit?

Experimental Test of the Centripetal Force Equation

The theoretical considerations in the last activity should have led you to the conclusion that, whenever you see an object of mass m moving in a circle of radius r at a constant speed v, it must at all times be experiencing a net centripetal force directed toward the center of the circle that has a magnitude of

$$F_c = ma_c = m\frac{v^2}{r} \qquad [Eq. 3]$$

Let's check this out. Does this rather odd equation really work for an external force?

To test the validity of the derivation we must compare it to experience. We will use a "toy" airplane suspended from a string and flying in a circular path. We will use the video analysis system to measure the properties of the motion and determine the horizontal and vertical components of the force exerted on the airplane using Equation 3. We will compare that result with a direct measurement of the tension in the string.

Activity 5: Verifying the $\mathbf{F}_c$ Equation

(a) Measure the mass of the airplane and the length of the portion of the string that hangs below the horizontal post with the hole in it. Record the values below.

(b) The airplane is suspended from a spring scale. The string should pass straight down from the scale through the small hole in the horizontal post. The camera should be placed about 1 meter above the airplane. Turn on the camera and center the spring scale in the frame by using the pendulum. Is the reading of the spring scale consistent with the mass of the airplane? Mount a ruler somewhere in the camera's field of view to serve as a scaling object. Launch the plane into uniform circular motion. When the motion is steady record a movie of at least one complete revolution and record the reading of the spring scale. See **Appendix D: Video Analysis** for details.

$\mathrm{F}_{scale} =$

(c) Determine the position of the airplane during one complete revolution. To do this task follow the instructions in the second section of **Appendix D: Video Analysis** for recording and calibrating a data file. As you analyze the movie frame by frame, estimate to the nearest fraction of a frame the number of frames for one complete revolution. Record your result. When you calibrate the time and position data, note the number of frames per second of the movie and convert that number to the time interval between frames, Δt. What is the time interval for one complete revolution? Prepare your file for analysis by following the instructions in the last section of **Appendix D**. The file should contain three columns with the values of time, x-position, and y-position.

$frame =$

$\Delta t =$

$t_{rev} =$

(d) Make a graph of the trajectory of the airplane during one full revolution. See **Appendix C: Introduction to Excel** for details on using the graphing software. When you make your plot make sure the x and y axes cover the same size range; otherwise, you will distort the path of the airplane. Print your plot and attach it to your write-up.

1. Is the motion circular? What is your evidence?

2. What is the radius of the motion? $r =$

(e) To test the validity of the expression for F_c we must know the speed. Use the measurements of the radius of the airplane's trajectory and the time for one complete revolution to calculate the average speed.

$v_{ave} =$

(f) Use your results for the mass, the average velocity of the airplane, and the radius of the circular motion to predict the centripetal force exerted by the string.

$F_c =$

(g) We need one more piece of the puzzle to predict the tension in the string, namely, the vertical component of the force exerted on the airplane by the string. Recall that we know r, the radius of the airplane's circular path, and R, the total length of the string that is actually rotating.

1. Using these two distances (r and R), calculate the angle the string makes with the horizontal.

$\theta =$

2. We determined the horizontal component of the force on the airplane in part (f). Now with the angle θ generate an expression for the vertical component exerted by the string and calculate it. Make a vector diagram of the different components. Generate an expression for the total force acting on the airplane due to the string and calculate the result.

   $F_y =$

   $F_{plane} =$

(h) Compare your result for F_{plane} with the measurement of the spring scale F_{scale}. Within experimental uncertainty, how well does your data support the hypothesis that $F_c = mv^2/r$?

(i) Discuss the major sources of uncertainty in this experiment.

21 Weighing an Electron

Name ______________________ Section ____________ Date ____________

Objective

To investigate the force on a charged particle due to a magnetic field and learn how the motion of the particle can be used to weigh it.

Apparatus

- e/m apparatus
- Low- and high-voltage power supplies
- Analog ammeter and digital multi-meter
- Flashlight

Overview

Mass spectroscopy is an experimental technique that is used to determine the mass of atoms, molecules, and sub-atomic particles. The central idea is to make a version of the particle carrying an electric charge (*e.g.*, a bare electron, an ionized molecule, *etc*), accelerate it through an electric potential, and then inject it into a magnetic field. If properly oriented, the magnetic field will bend the object into a curved path. The amount of curvature depends on the mass of the object and the electric charge it is carrying so that different mass particles with the same charge will bend by different amounts. Measuring this bend is equivalent to a mass measurement. This method is widely used to do things like determine the mass of newly discovered particles, hunt for oil, and even validate the authenticity of works of art.

Activity 1: Magnetic Force on a Charged Particle

It has been found by careful measurements that the force $\vec{F}_B$ on a charged particle due to a magnetic field is

$$\vec{F}_B = q\vec{v} \times \vec{B} \tag{1}$$

where q is the electric charge of the particle, $\vec{v}$ is its velocity, and $\vec{B}$ is the local magnetic field. Find the relevant sections in your text for a discussion of the cross product.

(a) The figure below shows the relationship between $\vec{F}_B$, $\vec{v}$, and $\vec{B}$. How is the magnitude of the cross product $\vec{v} \times \vec{B}$ related to the magnitude of v, B, and θ the angle between $\vec{v}$ and $\vec{B}$?

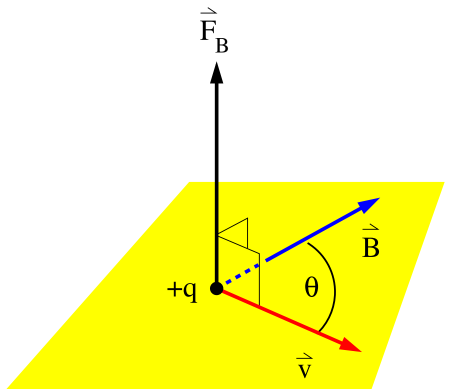


Figure 1: Vectors associated with the magnetic force.

(b) Consider a situation where the magnetic field is uniform in space and points in the z direction $\vec{B} = B\hat{k}$ and the velocity of the charged particle is in the y direction $\vec{v} = v\hat{\jmath}$. What is θ, the angle between the magnetic field and velocity vectors? What is $\vec{F}_B$ in terms of the magnitudes of q, v, and B and the appropriate unit vectors?

(c) Describe the trajectory of the charged particle. What is the z component of its trajectory?

(d) What is the magnitude of the magnetic force as this charged particle moves through the region of the magnetic field? Does this magnitude change as the particle moves through the field?

(e) How are the directions of $\vec{F}_B$ and $\vec{v}$ related?

Activity 2: Relating $\vec{F}_B$ to the Kinematics

At the end of the previous section you should have found that the magnitude of the magnetic force is $|qvB|$ and that $\vec{v}$ and $\vec{B}$ are always perpendicular to one another.

(a) What type of force have we encountered that is perpendicular to the velocity and constant in magnitude? (Hint: Recall some of the applications of Newton's Laws in your text.)

(b) How is centripetal acceleration related to v and r (the radius of the circular motion)? What is the expression for the magnitude of the centripetal force $|\vec{F}_c|$?

(c) Equate the expressions for the magnitudes of $\vec{F}_B$ and $\vec{F}_c$. Why is this ok? Solve for the mass m in terms of the radius r of the particle, $|q|$, v, and B.

(d) Using your result from 2.e answer the following questions. For two particles with the same charge and velocity, but different mass, which one will have the larger radius? How will increasing the velocity change the radius?

(e) Remember that in a mass spectrometer, the particle is first accelerated across an electric potential difference. Typically, we know the kinetic energy K of the charge particle after this acceleration instead of the velocity. Rewrite your result in 2.c in terms of the kinetic energy K instead of the velocity v.

(f) The electron 'falls' across a potential difference created by the accelerating voltage to gain a velocity v before it enters the magnetic field region. Assuming the electron starts from rest, what is the relationship between the accelerating voltage and the kinetic energy K when it leaves the accelerating region and enters the magnetic field? Combine this result with the one from part 2.c to get an expression for the mass of the electron in terms of the accelerating voltage V, the electron charge e, the radius of the electron's path r, and the magnetic field B.

Activity 3: Predictions for a Charged Particle in a Magnetic Field

We now have the mathematical tools to understand the motion of a charged particle in a magnetic field so we can start investigating the physics.

(a) First make some predictions. If $\vec{B} = B\hat{k}$ and $\vec{v} = v\hat{j}$, then what direction is $\vec{v} \times \vec{B}$?

(b) What direction is $\vec{F}_B$ for a positively charge particle if $\vec{B} = B\hat{k}$ and $\vec{v} = v\hat{\jmath}$?

(c) What direction is $\vec{F}_B$ for a negatively charge particle if $\vec{B} = B\hat{k}$ and $\vec{v} = v\hat{\jmath}$?

Activity 4: Measuring a Charged Particle in a Magnetic Field

The last piece of the puzzle before we start measuring things is the magnetic field $\vec{B}$. The apparatus you are using consists of a pair of wire coils (called Helmholtz coils) and an electron gun that produces a beam of electrons. See Figure 2 and identify the Helmholtz coils on your apparatus. The magnitude of the magnetic field along the

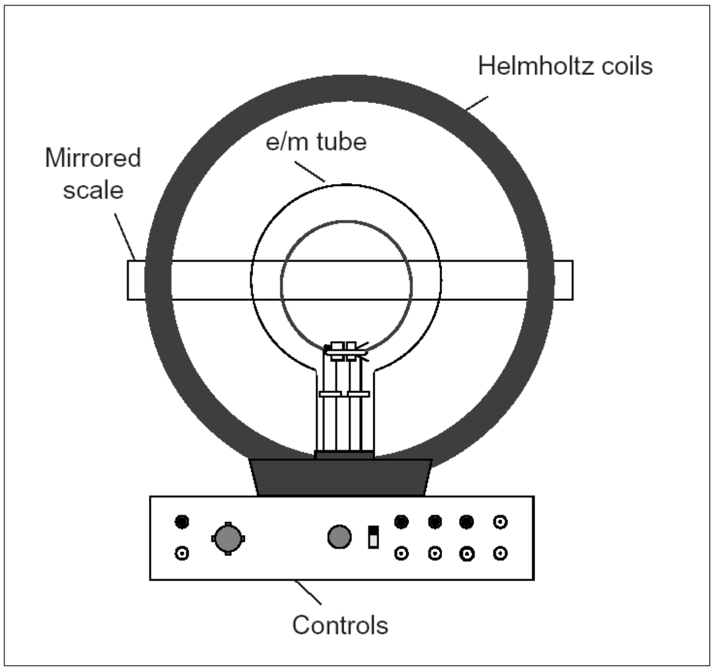


Figure 2: The e/m apparatus.

axis of the pair of coils is

$$B = \left(\frac{4}{5}\right)^{3/2} \frac{N\mu_0 I}{a} \tag{2}$$

where N is the number of turns of wire in each coil, μ_0 is the permeability constant, I is the current in the coils, and a is the radius of the coil. The direction of the field is parallel or anti-parallel to the axis of the pair of coils depending on the direction of the current in the coils. The direction of the current is the same in each coil. The magnetic field in the region between the coils is approximately equal to the field along the axis of the coils so we will use the expression above for our magnetic field in the equation you generated in Activity 2.f.

(a) Measure the radius of each coil of wire of the Helmholtz coils, average your results, and record them below. The number of turns of wire in each coil is $N = 130$.

(b) Identify each item on the front panel below the Helmholtz coils using Figure 3 as a guide. Check that all the connections are correct using Figure 3 as your guide.

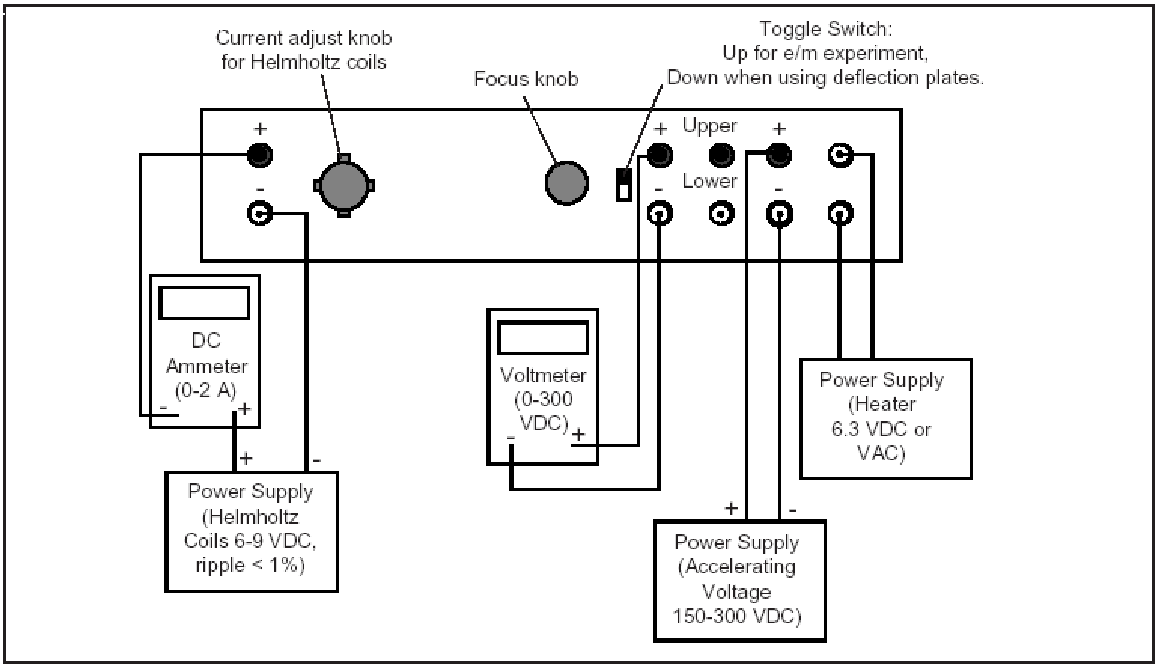


Figure 3: Instrument connections for the e/m experiment.

(c) Place the hood over the e/m apparatus and flip the toggle switch up to the e/m MEASURE position. Turn the current adjust knob for the Helmholtz coils fully counterclockwise to the OFF position.

(d) Identify the ammeter used to measure the current in the Helmholtz coils using Figure 3 as a guide. Next, turn on the low-voltage power supply for the Helmholtz coils. Now watch the current in the ammeter (it should not exceed 2 A under any circumstances) and turn up the voltage on the Helmholtz coils into the range of $6 - 9\ V$.

(e) As you watch the ammeter slowly turn the current adjust knob for the Helmholtz coils clockwise. Take care that the current in the ammeter does not exceed 2 A. Set the current adjust knob for the Helmholtz coils in the $1.5 - 2.0\ A$ range.

(f) Identify the power supply that runs the heater for the electron gun using Figure 3 as a guide and turn on the power supply. Turn the voltage up to 6 volts. **CAUTION:** Do not exceed 6 volts or you may destroy the e/m tube.

(g) Turn on the power supply for the accelerating voltage for the electron gun. Turn the voltage up so that it is in the range of $150 - 300\ V$.

(h) Wait several minutes for the cathode to heat up. When it does, you will see the electron beam emerge from the electron gun and it will be curved by the field from the Helmholtz coils. Check that the electron beam is parallel to the Helmholtz coils. If it is not, consult your instructor. Carefully read the current in the Helmholtz coils from your ammeter and the accelerating voltage from your voltmeter. Record the values below.

(i) Carefully measure the radius of the electron beam. Look through the tube at the electron beam. To avoid parallax errors, move your head to align the electron beam with the reflection of the beam that you can see on the mirrored scale. Measure the radius of the beam as you see it on both sides of the scale, then average the results. Record your results below.

(j) Slowly turn the current adjust knob for the Helmholtz coils either up or down as you watch the ammeter and take care that the current does not exceed 2 A. What parameter does this action effect, the magnetic field or the accelerating voltage? What happens to the path of the electrons as the current in the Helmholtz coils changes? Set the current adjust knob to some value and repeat parts 4.h-i to get a second measurement. Record the current in the Helmholtz coils, the accelerating voltage, and the average radius of the electron.

(k) Slowly change the accelerating voltage either up or down. What parameter does this voltage effect? Consider your result for part 2.f. What happens to the electron's path? Set the accelerating voltage to some value and repeat parts 4.h-i to get a third measurement. Record the current in the Helmholtz coils, the accelerating voltage, and the average radius of the electron.

(l) When you are done, turn down to zero the accelerating voltage, the heater voltage, the Helmholtz coil voltage, and the current adjust knob.

Activity 5: Extracting the Electron Mass From Your Data.

(a) You should now have three separate measurements of the radius of the electron's path. Use Equation 2 to calculate the magnitude of the magnetic field B for each measurement.

(b) Almost there! Now calculate the electron mass for each of your three measurements using the values of the average r, V, and B. Record your results below.

(c) Can you spot any trends in your results for the electron mass? Try averaging the three results and determine the standard deviation. Is your average and uncertainty consistent with the accepted value of the electron mass? Is this averaging of your results acceptable in this situation? Why? What are the possible sources of uncertainty in this experiment?

22 Nuclear Decay and Radiocarbon Dating

Name ______________________ Section ____________ Date ____________

Objective

To develop an understanding of the use of the radioactive decay of atomic nuclei to date objects like the Shroud of Turin.

Apparatus

- Radioactive sources.
- Radiation counter.
- Jack.
- Isotope generator.
- Surgical gloves, eye protection, and lab coat for handling radioactive liquids.
- Lead and plastic sheets.

Introduction

Atoms can be broken down into light, negatively-charged, electrons, and a small, dense, positively-charged nucleus. These atomic nuclei can spontaneously break apart into smaller nuclei in a process called radioactive decay. By measuring the rate of this decay under the appropriate circumstances one can develop a "clock" that can be used to determine how long ago in the past an event occurred. In this laboratory we will apply this notion to a particular object, the Shroud of Turin which is purported to be the burial cloth of Jesus Christ.

Activity 1: Nuclear Terminology

Atomic nuclei can be constructed from protons and neutrons. The number of protons in a nucleus is called the atomic number and is represented by the letter Z while the number of neutrons is represented by the letter N. The protons carry a charge of +e while the neutrons are electrically neutral. The sum of these two quantities is the mass number A.

$$\mathrm{A} = \mathrm{N} + \mathrm{Z}$$

Protons and neutrons are often referred to as nucleons.

Nuclei are represented using their chemical symbol (determined by the atomic number) and the mass number. For example, the most common form of carbon has six protons and six neutrons in its nucleus and is written as ^{12}C. If another neutron is added to this nucleus, then one has an isotope of carbon, ^{13}C. Isotopes of a chemical element have the same atomic number(Z), but have a different numbers of neutrons (N) and a different mass number (A). The difference is reflected in the value of the superscript on the chemical symbol.

(a) Consider the following list of the number of protons and neutrons that combine to form a particular nucleus. In the third column enter the chemical symbol and mass number as shown above (e.g., ^{12}C). Use the periodic chart at the end of this unit to determine what to enter in the third column.

Number of Protons	Number of Neutrons	Nucleus
7	8	
79	118	
26	30	

(b) Consider the following list of atomic nuclei. In the second and third columns enter the number of protons and neutrons in each nucleus.

Nucleus	Number of Protons	Number of Neutrons
^{4}He		
^{235}U		
^{108}Ag		

Some isotopes can spontaneously decay into other nuclei. In many of these decays the number of nucleons is conserved. This means that the number of protons and neutrons added together in the parent nucleus before the decay must be the same in the final products. Electric charge is always conserved.

(c) In the table below a nuclear decay is shown in the first column. In most cases the original nucleus (often referred to as the parent) produces two smaller nuclei (called daughters). Only one of the daughter nuclei is listed. In the adjacent column list the missing nucleus. Notice there are two emitted particles that we have not mentioned before. The e^- which is an electron and is often called a beta particle. It has almost zero mass compared to a nucleon. The other is γ and is called a "gamma" particle. This is photon or a particle of light or electromagnetic energy. The gamma has no mass or charge, but does carry energy and momentum.

Decay	Unknown
$^{190}\mathrm{Po} \rightarrow {}^{4}\mathrm{He} + ?$	
$^{210}\mathrm{Th} \rightarrow {}^{4}\mathrm{He} + ?$	
$^{16}\mathrm{Ne} \rightarrow \mathrm{p} + ?$	
$^{90}\mathrm{Sr} \rightarrow e^- + ?$	
$^{60}\mathrm{Co} \rightarrow \gamma + ?$	

Activity 2: The Properties of Radiation

An essential attribute of radiation is its ability to penetrate matter. Here we will study how three different types of radiation (alpha, beta, and gamma radiation) penetrate matter. We will do this by using a radiation counter and samples of three nuclei. The heart of the radiation counter is a gas-filled cylinder with a wire at high voltage running down its center. This cylinder is called a Geiger-Muller or G-M tube. Sub-atomic particles flying through the counter ionize atoms in the gas which are collected at the center wire producing a voltage pulse that can be measured. NOTE: Before going any further read the appendix on nuclear safety.

(a) Compare the figure below with your setup. Familiarize yourself with the different components. DO NOT TOUCH THE FACE OF THE DETECTOR INSIDE THE SNOUT UNDER ANY CIRCUMSTANCES.

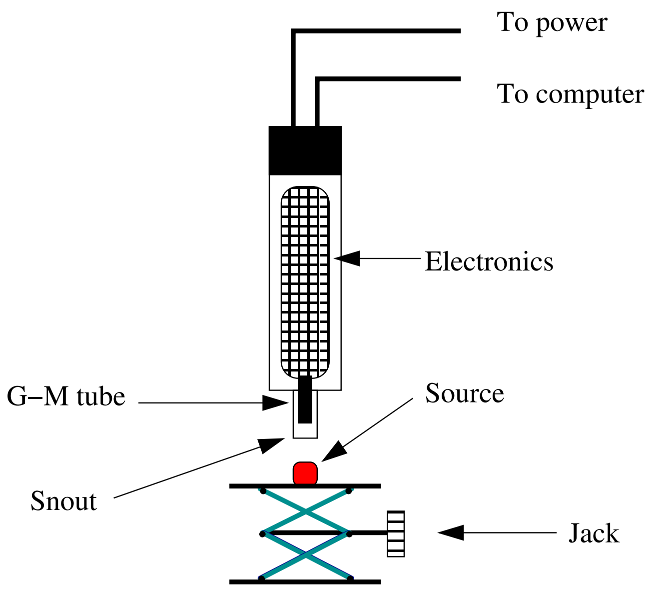


Figure 1. Schematic drawing of radiation counter and setup.

(b) Carefully remove the red, plastic cover on the snout of the radiation counter. DO NOT TOUCH THE FACE OF THE DETECTOR UNDER ANY CIRCUMSTANCES. This would likely break the window and destroy the counter.

(c) Obtain a set of radioactive sources from your instructor. Pick one of the sources, place it on the jack, and position the source about 1 cm below the snout of the radiation counter. Plug the power cord into a standard electric outlet and make sure the data cable is plugged into digital channel 1 on the *DataStudio* 750 interface. You should see the power light come on near the base of the counter (which is actually at the top). Another light near the snout of the counter blinks whenever a particle is detected. If you don't see either light, consult your instructor.

(d) Open the *Radiation Counter* activity in the **132 Workshop** menu and click the **Start** button on the *DataStudio* interface. Nothing will seem to be happening, but after 30 seconds (watch the clock at the top of the *DataStudio* interface) the number of radioactive decays detected by the radiation counter will appear. Record this result in the appropriate place in the table below.

(e) Now carefully place a piece of plastic on top of the source and run the counter for another 30 seconds. Record the result below.

(f) Remove the plastic and place a small sheet of lead on top of the source. Run the counter and record the result.

(g) Repeat steps d-f for the other two radioactive sources.

Radiation	Air	Plastic	Lead
γ			
β			
α			

(h) Which type of radiation is most penetrating? Why?

(i) Which type of radiation is least penetrating? Why?

(j) What material provides the best shielding of radioactivity? Why?

Activity 3: Background Radiation

(a) Return the radioactive sources to the instructor's table.

(b) With no radioactive sources nearby, make several runs with the radiation counter. The counts you observe in the detector are due to cosmic rays, radioactive decay in the building materials surrounding you, and even your own body. Record the counts and calculate the average and standard deviation of this background radiation.

Activity 4: Nuclear Decay

To understand the "clock" we will use to date the Shroud of Turin we will investigate how the clock "ticks". In this activity you will use a sample of radioactive material and a nuclear to detector to measure the behavior of the material as a function of time. We will then build a mathematical model of the time dependence of nuclear decay. We will apply this model to analyze the results of ^{14}C measurements on the Shroud.

To obtain the radioactive material we will use a procedure known as "milking the cow". We start with a liquid that contains the radioactive isotope ^{137}Cs or cesium-137. This isotope decays very slowly; it would take about 30 years for half of a sample to decay (a bit long for an introductory physics experiment). However, when ^{137}Cs does decay it usually does so in the following way.

$$^{137}\mathrm{Cs} \rightarrow \mathrm{e}^- + ^{137}\mathrm{Ba}(0.662)$$

Notice the additional number "0.662" beside the Ba-137. This number means there is still some energy (0.662 million electron-volts or MeV) stored in the Ba-137 nucleus and it has not yet reached its lowest-energy or ground state. This "excited" state of Ba-137 then emits a high-energy photon or gamma ray to reach the stable ground state of ^{137}Ba. A diagram of the process is below.

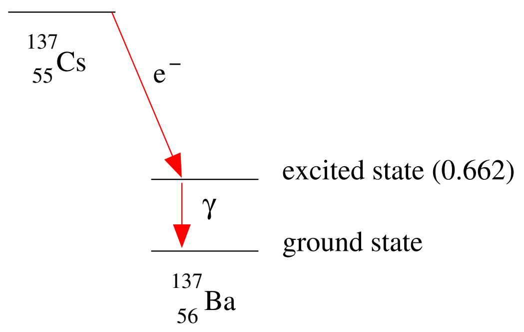


Figure 2. Decay scheme of cesium-137.

This intermediate state (labeled "0.662") decays quickly to the ground state and it is the one we will study.

We will prepare a sample of Ba-137 in its excited, 0.662-MeV state by starting with a Cs-137 "generator". The Cs-137 produces Ba-137 at a steady rate. We remove the Ba-137 from the "generator" by passing a hydrochloric-acid-saline solution through the generator (your instructor will do this). This is called eluting which means separate by washing. The generator is commonly referred to as the "cow" and the Ba-137 is "milked" from the cow. The Ba-137 is eluted from the generator and can then be used to study its decay.

(a) First, make a prediction of the count rate as a function of time. Sketch your prediction in the space below.

(b) What is the mathematical form of your prediction? Why did you choose it?

(c) Open the *Nuclear Decay* application in the **132 Workshop** menu. When you click **Start** it will plot the count rate in intervals of 10 seconds. Get the radioactive sources from your instructor and try this out with one of them to make sure you know how to use the hardware and software. Return the sources to your instructor when you are finished with this test.

(d) Read the rest of this procedure carefully. If you have to redo the procedure it may take a long time for the "generator" to produce enough Ba-137 for you to use.

(e) You have a small, metal disk called a planchette that sits on the jack which will be positioned close to the snout of the radiation counter. This will hold the radioactive material. Put the empty planchette in place and do a "dry run".

(f) One team member should be responsible for positioning the planchette. That person should put on the surgical gloves, eye protection, and a lab coat. The other team member can run the data acquisition.

(g) When you are ready, alert the instructor. He or she will come over and place a few drops of the eluate containing the Ba-137 on the planchette. Immediately place this under the Geiger counter and click **Start** on the *DataStudio* interface.

Caution: Care should be taken in handling the sample. If any portion of the sample touches your skin immediately wash off in the sink.

(h) Let the data acquisition run for about fifteen minutes or so and then click **Stop**. Dispose of the planchette according to the guidance from the instructor.

(i) Make a plot of your results using the data in the *Counts versus Time Table*. Notice that if you click on the title of the table, all of the data will be selected. You can then paste the data into *Excel*. Make sure you subtract the background radiation from your results.

(j) Make a fit to your data. What is the best choice of function for fitting your data? How did you make your choice? Attach a copy of your plot with the fit to this unit. Record the fit equation below. Do NOT close your spreadsheet. We may use it later.

Activity 5: Analyzing Nuclear Decay

Observation of a sample of radioactive material reveals that the decay of the atomic nuclei in the sample is determined by statistical processes. In other words, the number of nuclei N_{nuc} that decay per unit time is proportional to the number of nuclei in the sample.

$$\frac{dN_{nuc}}{dt} \propto N_{nuc}$$

This expression can be turned into an equality by inserting a constant of proportionality λ so

$$\frac{dN_{nuc}}{dt} = -\lambda N_{nuc}$$

where the minus sign is needed because the number of nuclei N_{nuc} decreases with time. The decay constant λ is a characteristic of each atomic nucleus.

(a) In the previous activity, you used a particular function to fit your data. Try to prove that you made the right choice by taking derivatives and seeing if they will satisfy the original differential equation above. Did it work?

(b) It is claimed the solution of the differential equation above describing nuclear decay is the following expression.

$$N_{nuc}(t) = N_0 e^{-\lambda t}$$

Prove this statement by taking the derivative of $N_{nuc}(t)$ and showing it satisfies the original differential equation. Make a sketch of the function and describe it in words. How did your fit function do?

(c) The decay of atomic nuclei is often characterized by a quantity known as the half-life τ. The half-life is the period of time for one-half of the original sample to disappear via radioactive decay. This statement can be expressed mathematically in the following way.

$$N_{nuc}(t = \tau) = \frac{N_0}{2}$$

Starting with the above expression show that the decay constant λ and the half-life are related by the following equation.

$$\tau = \frac{\ln 2}{\lambda}$$

(d) Now return to the results of your experiment. Does your count rate fall off exponentially? Did you fit your data with an exponential? If not, go back and do so. Record the decay constant λ.

(e) What is the half-life of Ba-137? Compare this with the accepted value of 2.552 minutes.

(f) Consider the following example as a warm-up. A sample of the isotope of iodine ^{131}I has an initial decay rate of 1.8 x 10^5 decays/s. This isotope has a half-life of 8.04 days. It is shipped to a medical diagnostic laboratory where it will be used as a radioactive tracer. When the shipment arrives at the lab the decay rate has fallen to 1.4 x 10^5 decays/s. How long did it take for the shipment to reach the laboratory?

Activity 6: Dating the Shroud of Turin

The previous example shows how one can use the measured decay rate of an atomic nucleus as a "clock" to determine the passage of time. The same concept is used in radiocarbon dating. Carbon on the planet Earth consists largely of three isotopes with A = 12, 13, and 14. The most common form is ^{12}C and only a very small fraction of the carbon is ^{14}C. However, ^{14}C decays via

$$^{14}\mathrm{C} \rightarrow {}^{14}\mathrm{N} + \beta^- + \overline{\nu}$$

where β^- is an electron and $\overline{\nu}$ is a particle known as a neutrino. Notice this decay does NOT preserve the number of protons and neutrons in the original nucleus. The ratio R of ^{14}C to ^{12}C on the Earth is 1.30 x 10^{-12} and is roughly constant despite the fact that the ^{14}C constantly disappears. The ratio is constant because the supply of ^{14}C in the atmosphere is replenished by the reaction of cosmic rays from outer space with the nitrogen in the upper atmosphere.

Living organisms contain large quantities of carbon and are constantly exchanging carbon with their surroundings. They contain the same proportion of ^{14}C to ^{12}C as observed in the atmosphere. However, this proportion begins to change after the organism dies. The ^{12}C remains in the dead body, but the ^{14}C turns into gaseous ^{14}N (see decay above) and leaves the body. Hence, the proportion of ^{14}C decreases with time, and one has a "clock" that can be used to determine when an organism died.

The Shroud of Turin is a piece of cloth that bears the image of a man who appears to have been crucified. It was first displayed in France in the fourteenth century and has been kept at the Royal Chapel of Turin Cathedral in a special shrine since 1694. Many believe the image on the Shroud is of Christ and the cloth is his burial wrap. In 1989, three laboratories in Arizona in the USA, Oxford in the UK, and Zurich in Switzerland used advanced methods of radiocarbon dating in an attempt to determine the age of the Shroud[1]. The Shroud is woven of cloth made from plants. Like a living organism that has died, the ^{14}C in the Shroud began to gradually disappear after the plants used to make it were harvested.

(a) The half-life of ^{14}C is 5730 years. What is the decay constant λ?

(b) The three laboratories obtained the following results for the ratio R of ^{14}C to ^{12}C. The ratio of ^{14}C to ^{12}C in the atmosphere is $R_i = 1.30 \times 10^{-12}$. What is the implied age of the Shroud for each measurement? Use the space below for your calculations and enter the results in the table.

Laboratory	R_f	Age (years)
Arizona	1.20×10^{-12}	
Oxford	1.18×10^{-12}	
Zurich	1.19×10^{-12}	

(c) What is the average age of the Shroud?

(d) The typical uncertainty in these measurements is a standard deviation of ±40 years. Are the results of the three laboratories consistent?

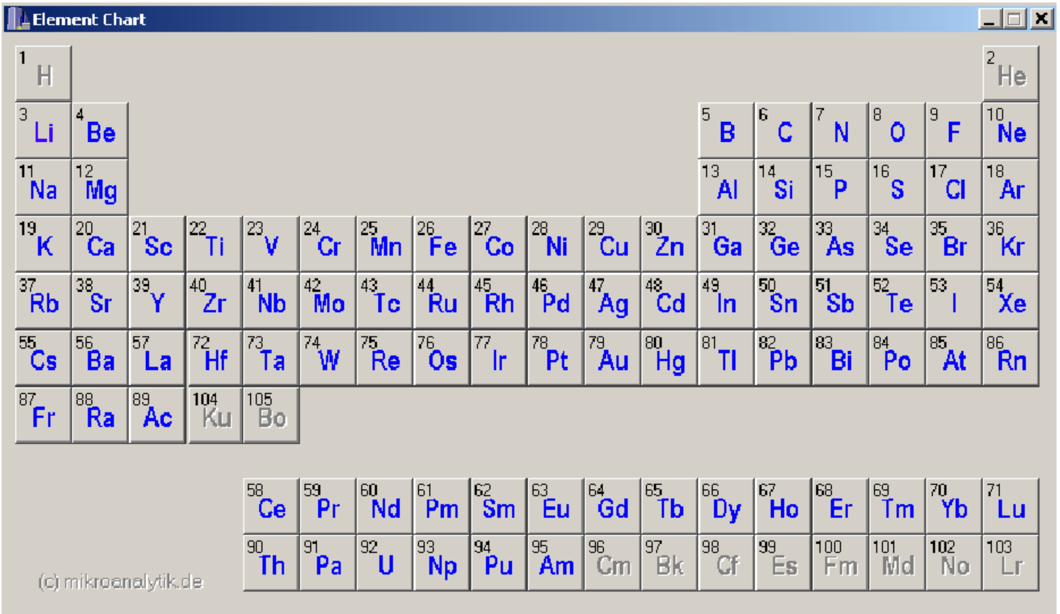
(e) Is the age of the Shroud consistent with it being the burial wrap of Christ?

(f) Are there any reasons to doubt these results?

Homework

1. The half-life of a particular radioactive isotope is 6.5 h. If there are initially 48 x 10^{19} atoms of this isotope, how many atoms of this isotope remain after 26 h?
2. A radioactive isotope of mercury, ^{197}Hg, decays into gold, ^{197}Au, with a disintegration constant of 0.0108 h^{-1}. (a) What is its half-life? (b) What fraction of the original amount will remain after three half-lives? (c) What fraction will remain after 10.0 days?
3. The radionuclide ^{64}Cu has a half-life of 12.7 h. How much of an initially pure, 5.50-g sample of ^{64}Cu will decay during the 2.0-h period beginning 14.0 h later?

The Periodic Chart



References

1. P.E.Damon, D.J.Donahue, B.H.Gore, A.L.Hathaway, A.J.T.Jull, T.W.Linick, P.J.Sercel, L.J.Toolin, C.R.Bronk, E.T.Hall, R.E.M.Hedges, R.Housley, I.A.Law, C.Perry, G.Bonani, S.Trumbore, W.Woelfli, J.C.Ambers, S.G.E.Bowman, M.N.Leese, and M.S.Tite, *Nature*, Vol. **337**, No. 16, P. 611-615.

23 Electromagnetic Induction

Name ______________________ Section ____________ Date ____________

Objectives

To investigate:

- The effect of changing magnetic fields on charge and current.

Introduction

A charged object moving through a magnetic field experiences a force which is proportional to the magnitude of its charge and to its speed perpendicular to the field: $F = qvB_{\perp}$. Changing the number of magnetic field lines–the flux–through a coil of wire results in a current in the wire. The direction of this current is such that the magnetic field it produces opposes the change in the external field. Similarly, varying the current in one coil (the primary) produces a current in another nearby coil (the secondary). The current in the second coil, too, will flow in a direction that creates a magnetic field which opposes the change in the field of the first coil. These relationships between changing fields and currents are known collectively as electromagnetic induction.

Apparatus

- one small wire coil
- bar magnet
- Pasco 750 Interface

Activity 1: A Moving Magnet and a Coil

1. Turn on the computer and launch *EM Induction* in the 132 Workshop in the **Start** menu.
2. Place a bar magnet vertically along the axis of the small coil with the N-pole touching the coil.
3. Start recording data and lift the bar magnet quickly straight up.
4. At the end of the data taking interval, the computer should display a value for the electromotive force (emf) induced in the small coil. Several trials may be required to get the correct timing between starting the data acquisition and removing the magnet. Note and record the sign of the induced emf.
5. **Prediction**: If you lower the magnet, N-pole down, quickly toward the coil, what will be the sign of the emf?
6. Carry out the experiment, starting the data acquisition, then lowering the magnet. Record the sign of the induced emf.
7. Did your result confirm or refute your prediction?

8. **Prediction**: What will happen to the emf if you perform the same pair of experiments with the S-pole toward the coil?

9. Perform the two experiments, lifting and lowering the magnet, with the S-pole down. Record the sign of the induced emf in each case.

10. How did the results compare with your predictions?

Activity 2: Predictions About Making Waves Electromagnetically

Consider what we have observed about electricity and magnetism. A static, unchanging magnetic field does not do much to our coil of wire. A varying magnetic field creates, across space, a current in our coil. The changing magnetic field must be creating an electric field or else the electrons in the coil would not feel a force. We have also observed phenomena where a changing electric field induced a magnetic field. In other words, a changing electric field induces a magnetic field and vice versa; a changing magnetic field induces an electric field. Notice these statements don't require the presence of electrons or other material. The fields can be induced in a vacuum. We are now going to explore via a simulation, what happens when charges are wiggled (*i.e.* oscillate) up and down.

1. To begin to explore our wiggling charges, consider some questions. Suppose the motion of the charges can be described as an oscillating dipole so the dipole moment as a function of time looks like Figure 1. Assume the dipole is aligned with the z-axis. What do you expect the electric field to look like as a function of time at some arbitrary distance r away from the source in the $x-y$ plane? What is the direction of the $\vec{E}$ field? Make a sketch of your answer on the plot and label your curve.

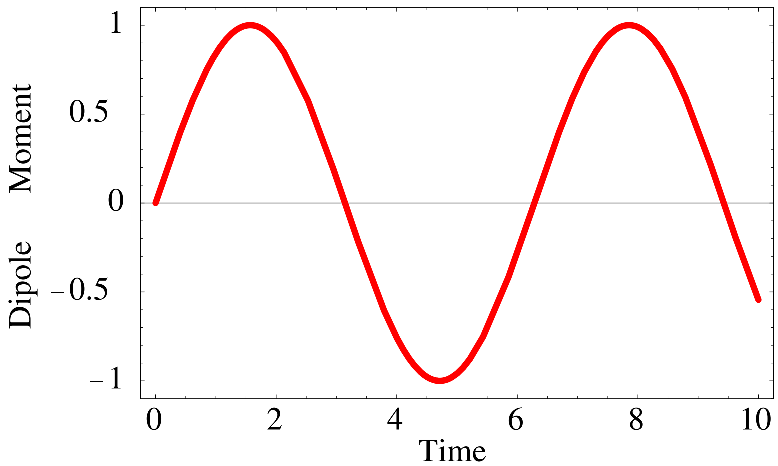


Figure 4: Time dependence of a dipole source oriented along the z-axis.

2. What would the magnetic field strength look like as a function of time at the same distance r from the source? In what direction does the $\vec{B}$ field point? How are the directions of the $\vec{E}$ and $\vec{B}$ field related? Draw a dashed line on Figure 1 to represent the magnetic field strength.

Activity 3: Simulating Electromagnetic Waves

We are now going to use a computer simulation to investigate the behavior of our oscillating dipole. The situation we are exploring is very similar to the generation of radio waves with an antenna. Charges (usually electrons) are driven up and down in the antenna and emit electromagnetic waves.

1. Open *Internet Explorer* (*IE*) and go to the website `http://www.falstad.com/emwave2`. A Java applet will pop up showing brightly colored waves like the ones in Figure 2 propagating outward from our oscillating electric dipole. If you don't see this window, consult your instructor.

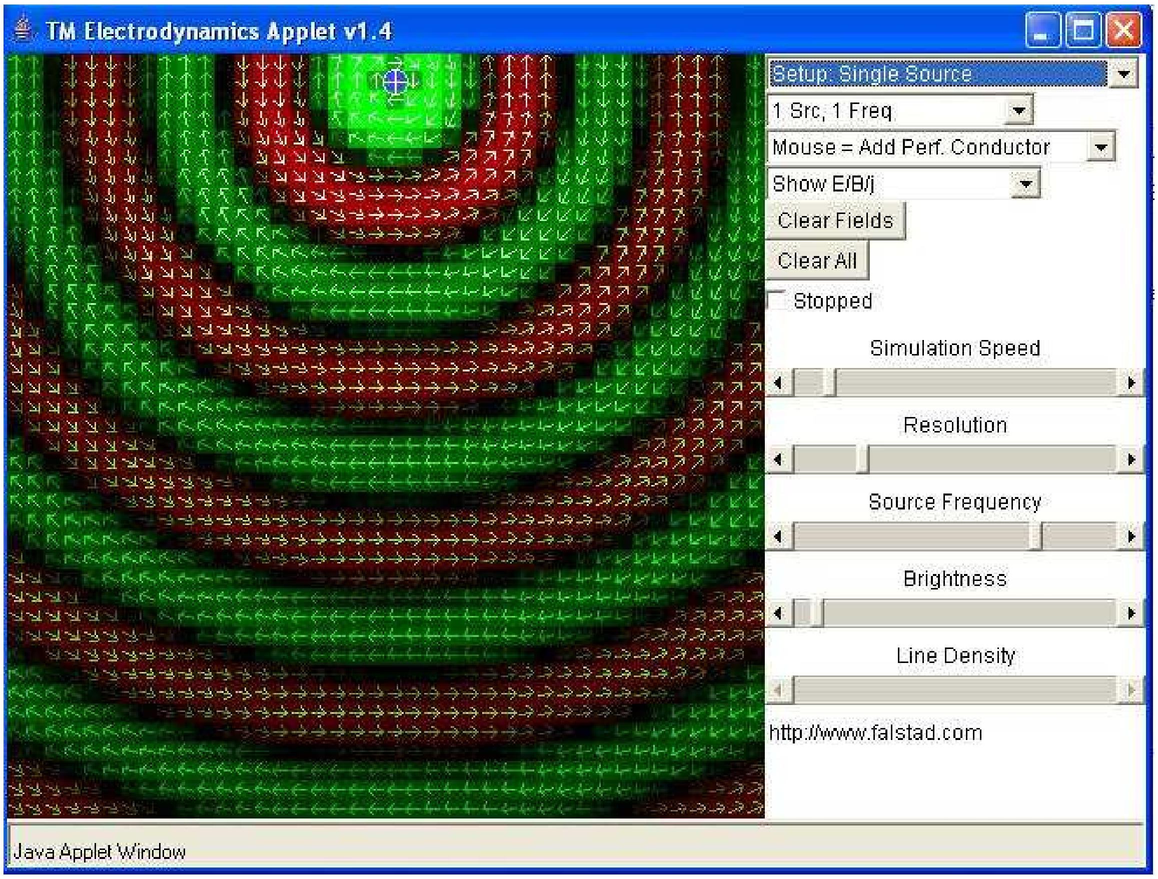


Figure 5: Applet showing electromagnetic waves.

2. It is useful to slow down the simulation speed to observe the waves more clearly. Do this using the slide labeled `Simulation Speed` on the right-hand side of the applet window. The alternating yellow and blue circle at the top of the applet is the source (the dipole) as viewed from above. The color indicates the electric field; green areas are positive ($\vec{E}$ toward you) and the red areas are negative ($\vec{E}$ away from you).

The electric field is always perpendicular to the plane of the screen. In addition to the red and green color, you will see arrows which indicate the direction of the magnetic field (which is always in the plane of the screen). Also, sources and conductors may show a blue or yellow color, indicating current. Yellow means that current is flowing toward you and blue means it is flowing away from you. Describe what you see in your own words.

3. Are these waves spherical or plane waves? Why?

4. What is the orientation of the $\vec{E}$ field? What is the orientation of the $\vec{B}$ field? How are the two related?

5. Go back to your predictions in Activity 2 about the directions of the $\vec{E}$ and $\vec{B}$ fields. Compare your predictions with these observations. Correct any disagreements.

6. Reduce the frequency of the oscillation of the dipole using the slide labeled `Source Frequency` on the right-hand side of the applet. You may have to increase the brightness of the applet using the slide labeled `Brightness`. What happens to the distance between equal positions on successive waves (*i.e.*, successive peaks or crests of the waves defined by the red regions in the applet)? This distance is called the wavelength and it is characteristic of a particular wave. Light, for example, is an electromagnetic wave and different wavelengths correspond to different colors.

7. One can calculate an quantity known as the Poynting vector which points in the direction of the flow of energy in an individual wave. Make the applet draw the Poynting vector by clicking on the arrow in the box with `Show E/B/j` entered in it. Scroll up or down until you find `Show Poynting vector` and highlight it. In what direction does the energy flow?

8. Go back to `Show E/B/j` mode you were in before. Just for fun, we want to introduce one of the central phenomena associated with waves known as interference. Go to the second menu from the top of the right-hand side of the applet. Click and highlight `2 Src, 1 Freq`. This will place an additional oscillating dipole at the bottom of the applet. Describe what happens. When waves add together they can cancel

one another (this is called destructive interference). Do you observe destructive interference? What do you think happens for constructive interference?

9. Now use your mouse to grab one of the sources and drag it to a position near the other one. How does the interference pattern change? Try different positions for the source (move it closer or further away). Describe what you see. If these were light waves, where would you see bright light? Where would you see dark regions?

Activity 4: Plane Waves

You just completed a study of spherical waves emitted by a dipole source. We now want to consider another type of wave that we will study when we explore light.

1. Use *IE* to go to the site `http://www.amanogawa.com/archive/PlaneWave/PlaneWave-2.html` and a Java applet will appear like the one in Figure 3. If you don't see this window, consult your instructor. The top panel of the applet shows a plane, electromagnetic wave. The red lines represent the direction and magnitude of the $\vec{E}$ field and the blue ones do the same for the $\vec{B}$ field. NOTE: The magnetic field in this applet is labeled $\vec{H}$. In this case, this is exactly the same as our previous $\vec{B}$. To eliminate some unnecessary lines, click the box beside `Phasors` so the check in the box disappears.

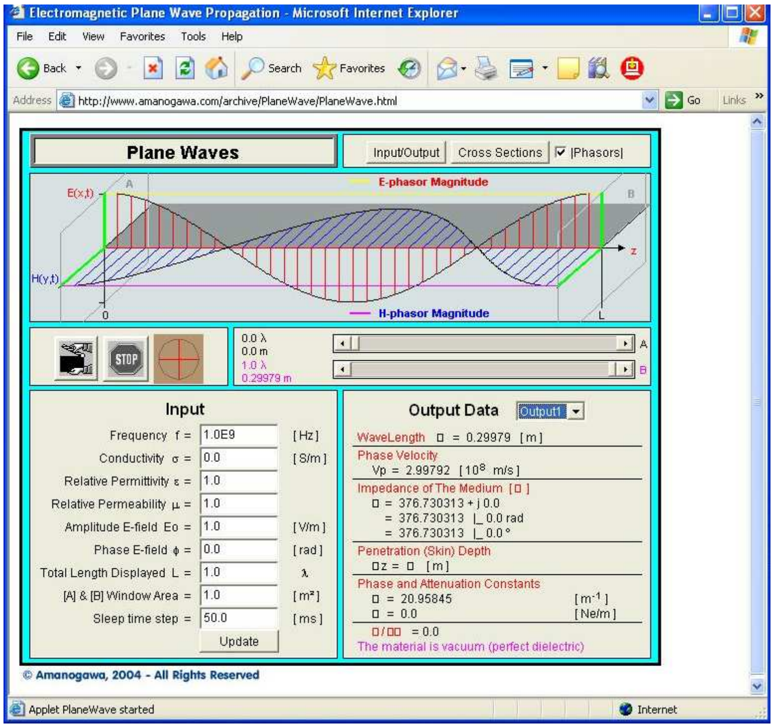


Figure 6: Applet showing plane electromagnetic waves.

2. Why do you think it's called a plane wave?

3. What is the orientation of the $\vec{E}$ field? What is the orientation of the $\vec{B}$ field? Compare your answer here with your predictions in Activity 2 and your observations in part 3.6. Correct any disagreements.

4. Click the start button to watch the wave move or propagate. The button is the one to the left of the STOP button on the left-hand side of the applet. Describe what happens to the electric and magnetic fields and how they are related (*i.e.* When the $\vec{E}$ is large, what is the $\vec{B}$ field doing?).

5. Click on the `Cross Sections` button at the top of the applet. You will see new panels that show the $\vec{E}$ (red) and $\vec{B}$ (blue) vectors in cross section at the planes A and B labeled in the top panel. Use these vectors to confirm your observations in parts 4.3-4.4.

6. Use the B slide in the middle of the right-hand-side of the applet to move the B cross section position. Set it one-half wavelength from the A cross section. How are the $\vec{E}$ and $\vec{B}$ vectors in the A cross section related to their partners in the B cross section. What will be the total electric and magnetic fields if two waves are added that are out of line by one-half wavelength?

24 The Interference of Light

Name ______________________ Section __________ Date __________

Objective

- To investigate the interference of light waves as they pass through a set of slits.

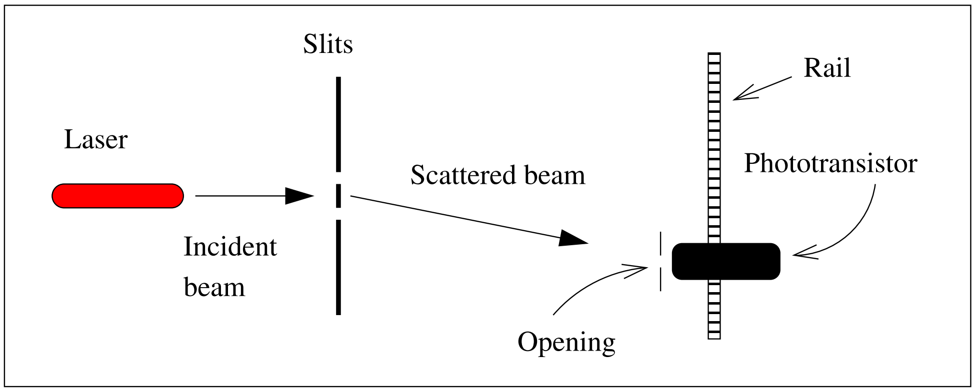
Apparatus

- Laser.
- Phototransistor for measuring light intensity (mounted on rotary motion sensor).
- Set of narrow slits.
- *DataStudio* 750 Interface.
- Plumb line.

Introduction

In this laboratory you will investigate the interference of light produced by a laser beam passing through a set of narrow, adjacent slits. When light passes the slits each opening acts as an independent source of waves that can overlap one another to produce a distinctive pattern of bright and dark spots on a screen. The position of the bright spots depends on the separation of the adjacent slits and the wavelength of the incident light.

You can measure this interference pattern with the setup shown below. A phototransistor is seated behind the narrow opening on top of the large, metal mount sitting on a rail. The phototransistor can translate the intensity of the light falling on it into a voltage signal that can be read by the computer. In addition, the phototransistor can be moved back and forth on a rotary motion sensor that measures the position of the mount. These two signals can be combined to make a graph of the intensity as a function of position.



In this unit you will pass light from the laser through slits of known separation and use the interference pattern to determine the wavelength of the light.

Activity 1: An Alternative View

Isaac Newton believed that light was made up of small, unseen particles that obeyed (surprisingly enough!) Newton's Laws. This view is known as the corpuscular theory. We want to consider how this model of light predicts different behavior from the wave theory.

(a) Consider a laser beam shining on a circular hole. If a beam of light consisted of small, unseen particles that behaved as tiny billiard balls what would you see on a screen that is downstream from the circular hole? A sketch might be useful here.

(b) Now consider the same laser beam shining on a pair of narrow slits. What would you see on a screen downstream from the slits if light were made of corpuscles?

For the questions above you should have predicted that the laser would form a single bright spot (for part a) or two parallel lines (for part b). The experiment you are about to perform provided compelling evidence that Newton's corpuscular theory was wrong.

Activity 2: The Interference of Light

(a) You are now ready to turn on the laser. DO NOT LOOK DIRECTLY INTO THE BEAM OR POINT THE LASER CARELESSLY ABOUT THE ROOM. Turn on the laser and you should see the bright red spot of the beam striking the wall. You should have a glass plate with a green border and several different slit arrangements on it. Place the opening in the center of the plate in the path of the laser beam. The adjacent slits in the center hole are 0.03295 mm apart. What do you see?

(b) Position the glass plate about 30-40 cm from the phototransistor mount with the central maximum (the brightest spot) striking the center hole. Measure and record this distance. You may find it useful to use the plumb line to measure this distance. The phototransistor sits about 25.4 mm behind the opening.

(c) Position the phototransistor mount so the interference pattern is at the same height as the opening in the center of the phototransistor mount. The phototransistor is mounted behind this hole. To make accurate measurements it is important to carefully determine the geometry of your setup. Check to see if the slits and the phototransistor mount are perpendicular to the incident laser beam. You want to make sure the phototransistor can "see" as many bright spots as possible. Carefully slide the phototransistor mount back and forth to make sure the it stays centered on the interference pattern. Start the "Interference" activity in the **132 Workshop** folder. When you are ready, click **Start** and slowly move the phototransistor from one side of the slide to the other by turning the wheel on the rotary motion sensor. Move carefully and take about 4-5 seconds to complete the motion. Click **Stop**. When data acquisition is complete you will see a graph representing the intensity reading versus the position reading. You should see several distinct peaks. This graph is the interference pattern. If you do not see this pattern consult your instructor. Make a hardcopy of this graph and attach it to this unit.

(d) Is the spacing between the intensity peaks constant? Is the intensity of each peak the same? Does it appear that any peaks are missing? The more peaks you see the more (and hopefully better) data you can collect. There is a button on top of the phototransistor labeled "Gain" which changes the size of the intensity signal. Trying changing this setting to see if you can get more peaks in your spectrum.

(e) When you are satisfied with the quality of your spectrum record the position of each peak in the table below. Use the **Smart Tool** to accurately read off the peak positions by clicking on the appropriate button along the top of the graph. A set of cross-hairs will appear on the plot. Grab the cross-hairs by clicking on them and dragging them to the point you want to measure. The coordinates will be printed by the cross-hairs. Turn off the laser when you are finished.

Position Reading (m)	Change in Reading (m)

Activity 3: Determining the Wavelength of the Laser

(a) For the data you recorded in the previous activity calculate the difference between each pair of adjacent readings and record it in your data table.

(b) Calculate the average and standard deviation of the separation between adjacent peaks.

(c) The position of the interference maxima can be described by

$$y_m = \frac{\lambda D}{d} m$$

where y_m is the distance of a bright spot from the central maximum (the distance along the slide in this experiment) and D is the distance from the slits to the phototransistor. The quantity d is the slit separation, λ is the wavelength of the light, and m is the order of the bright spot. Generate an expression for the distance between adjacent bright spots.

(d) Use the expression you calculated above and the average separation between bright spots to calculate the wavelength of the laser light. Compare your result with the expected value of 6328 angstroms. Are the peaks of the interference pattern the same intensity? Describe the pattern you observe.

(e) Collect the results for the wavelength from the other teams in class and calculate the average and standard deviation. Record the results here. Are you results consistent with the class results? Why or why not?

(f) Recall the earlier discussion of Newton's corpuscular theory of light. Does your data support Newton's theory or the wave theory? Why?

25 Diffraction of Light

Name ______________________ Section ____________ Date ____________

Objective

To investigate how the interference and diffraction of light waves combine to form a distinctive pattern and how that pattern can be used to measure the size of an object.

Introduction

In this laboratory you will investigate the interference and diffraction of light produced by a laser beam passing through a set of narrow, adjacent slits. When light passes through a set of slits, each opening acts as an independent source of waves that can overlap one another to produce a distinctive pattern of bright and dark spots on a screen. The position of the bright spots depends on the separation of the adjacent slits and the wavelength of the incident light. In addition, diffraction produced by the individual slits modifies the intensity of each spot. To perform the following activities you will need:

- Laser.
- Phototransistor for measuring light intensity (mounted on rotary motion sensor).
- Set of narrow slits.
- *DataStudio* 750 Interface.
- Plumb line.

You can measure this interference/diffraction pattern with the setup shown below. A phototransistor is seated behind the narrow opening on top of the metal mount. The phototransistor can translate the intensity of the light falling on it into a voltage signal that can be read by the computer. In addition, the phototransistor can be moved back and forth on a rotary motion sensor that measures the position of the mount. These two signals can be combined to make a graph of the intensity as a function of position.

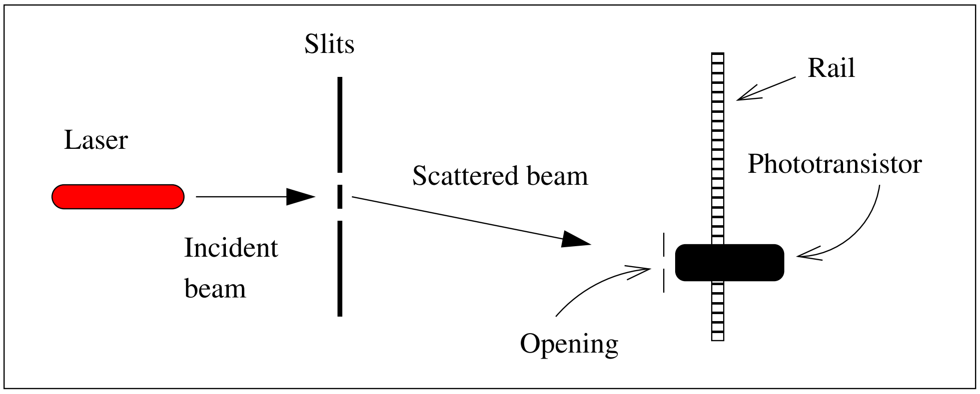


Fig. 1. View of diffraction apparatus from above.

In this unit you will pass light of known wavelength through slits and use the diffraction pattern to determine the size of the individual slit through which the light passed.

Intensity of Interference

For light that passes through two very narrow slits one can calculate a theoretical expression for the interference pattern that would be produced in such a situation. The expression is

$$I_{int} = 4I_0 cos^2(\frac{\pi d}{\lambda}\sin\theta)$$

where I_{int} is the intensity of the light at the phototransistor, I_0 is the maximum intensity of the incident light, d is the slit separation, θ is the angular position of the scattered light relative to the incident beam, and λ is the wavelength of the light. This expression has a characteristic shape shown below. We will compare this prediction of "pure" interference (without diffraction effects) with the measured pattern in the next activity.

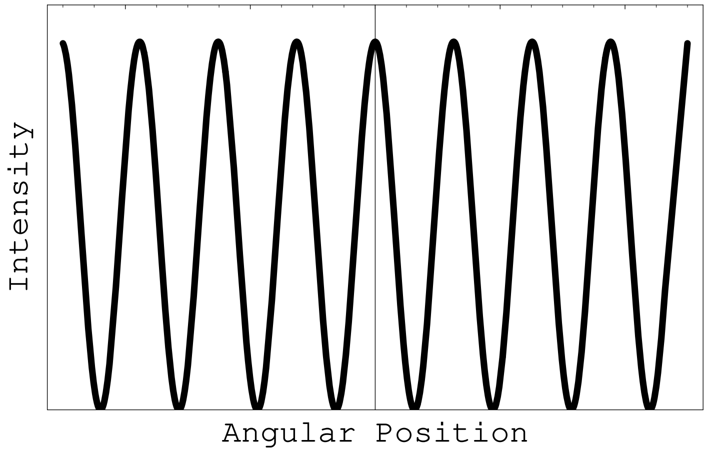


Fig. 2. Intensity distribution of pure interference.

Activity 1: The Interference of Light

(a) You are now ready to turn on the laser. DO NOT LOOK DIRECTLY INTO THE BEAM OR POINT THE LASER CARELESSLY ABOUT THE ROOM. Turn on the laser and you should see the bright red spot of the beam striking the wall. You should have a glass plate with a green border and several different slit arrangements on it. Place the opening in the center of the plate in the path of the laser beam. The adjacent slits in the center hole are 0.03295 mm apart.What do you see?

(b) Position the glass plate about 30-40 cm from the phototransistor apparatus with the beam striking the center opening. Place the apparatus so the intensity pattern is at the same height as the opening in the center of the device mounted on a movable slide. The phototransistor is mounted behind this opening. To make accurate measurements it is important to carefully determine the geometry of your setup. Check to see if the slits and the phototransistor mount are perpendicular to the incident laser beam. You also want the phototransistor to "see" as many bright spots as possible. Gently slide the mount back and forth to make sure the phototransistor stays centered on the interference pattern. You need to know the perpendicular distance from the center hole to the position of the phototransistor. Place the phototransistor mount at the position of the central maximum (the brightest spot) and measure the distance from the center hole of the slits to the front face of the phototransistor mount. You may find it useful to use the plumb line to measure this distance. Record your result. The phototransistor itself sits 25.4 mm behind the opening on the front face of the phototransistor mount.

(c) Start the "Interference" activity in the **132 Workshop** folder. When you are ready, click **Start** and slowly move the phototransistor from one side of the slide to the other by turning the wheel on the rotary motion sensor. Move carefully and take about 4-5 seconds to complete the motion. When data acquisition is complete, click **Stop** and you will see a graph representing the intensity reading versus the position reading. You should see several distinct peaks. This graph is the interference pattern. If you do not see this pattern, consult your instructor. Make a hardcopy of this graph and attach it to this unit.

(d) Is the spacing between the intensity peaks constant? Is the intensity of each peak the same? Does it appear that any peaks are missing? How does the measured intensity spectrum compare with the one predicted for "pure" interference discussed above? What is different? What is the same?

(e) Measure and record the Position Reading and Intensity Reading of each peak in the table below. Use the **Smart Tool** to accurately read off the peak positions by clicking on the appropriate button along the top of the graph. A set of cross-hairs will appear on the plot. Grab the cross-hairs by clicking on them and dragging them to the point you want to measure. The coordinates will be printed by the cross-hairs. Turn off the laser when you are finished.

Position Reading (m)	Intensity Reading (%)

Activity 2: Intensity of Diffraction

Applying the same theoretical techniques that were applied to interference (see equation above) one can derive a prediction for the intensity pattern due to diffraction of light passing through a single slit. The result is

$$I_{diff} = I_m\left(\frac{\sin(\frac{\pi a}{\lambda}\sin\theta)}{\frac{\pi a}{\lambda}\sin\theta}\right)^2$$

where a is the size of the single slit, θ is the angular position of the phototransistor relative to the incident beam, I_m is the maximum intensity at the center of the diffraction pattern, and λ is the wavelength of the light. The shape of this intensity distribution is shown in Figure 3.

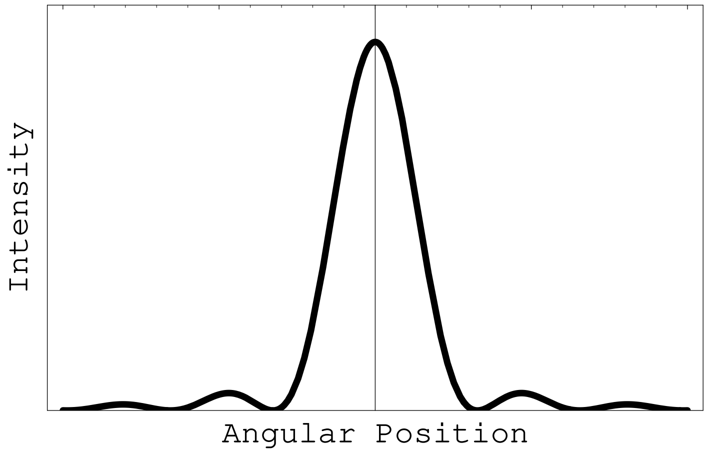


Fig. 3. Intensity distribution of diffraction from a single slit.

(a) The figure above shows the diffraction pattern has a central maximum with a series of points where the intensity goes to zero at positive and negative angles. When is the expression for the intensity in the equation above equal to zero?

(b) Using the result of part (a), what is the angular position of the minima on either side of the central maximum?

(c) Finally, generate an expression for the angular width of the central maximum in terms of a, λ, and any other constants you need.

Combining Interference and Diffraction

By now you should have realized that your measured intensity distribution does not completely agree with the distribution predicted by "pure" interference as represented by the first equation and Figure 2. When light passes through a pair of slits diffraction occurs at each individual slit and casts the characteristic pattern described by the second equation. At the same time there is interference between the light from different slits that creates an interference pattern described by the first equation. The net effect is a multiplication of these two equations to yield

$$I_{total} = I_m \cos^2(\frac{\pi d}{\lambda}\sin\theta)(\frac{\sin(\frac{\pi a}{\lambda}\sin\theta)}{\frac{\pi a}{\lambda}\sin\theta})^2$$

where I_m is the intensity of the central maximum, θ is the position of the phototransistor, d is the separation of the slits, a is the size of an individual slit, and λ is the wavelength of the light. The shape of this distribution is shown by the solid curve in Figure 4.

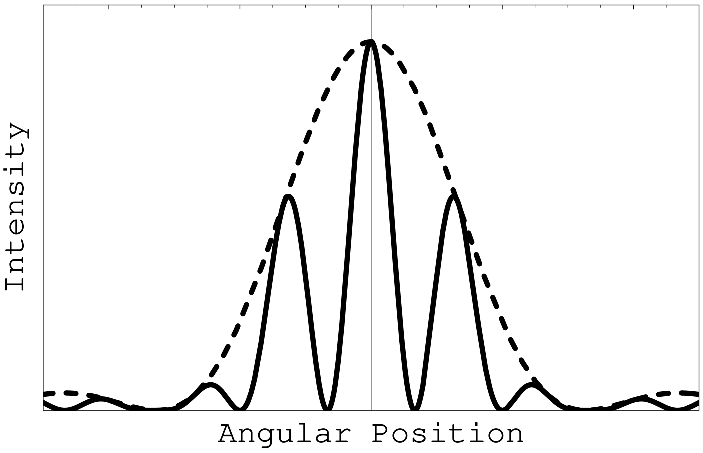


Fig. 4. Intensity distribution of light passing through a pair of slits.

The intensity of the interference peaks is no longer constant, but is modulated by the diffraction envelope represented by the dashed curve. This dashed curve is a plot of the second equation normalized to the maximum intensity at zero degrees. The intensity of each peak in the distribution represents the intensity due to the diffraction effects. If more slits are added, then the widths of the individual peaks in Figure 4 become narrower, but their intensity remains the same. In the next Activity you will use your data to determine the diffraction pattern and the angular width of the central diffraction envelope. This width can be used to measure the size of the individual slits that produced the distribution.

Activity 3: Measuring the Size of the Slit with the Diffraction Pattern

(a) Does the intensity distribution you measured with the phototransistor resemble the pattern shown in Figure 4? If not, consult your instructor.

(b) In Activity 1 you recorded the position and intensity of the interference peaks you measured with the phototransistor. How would you calculate the angular position of each peak relative to the central maximum? A sketch might be helpful here.

(c) Use your expression to calculate the angular position of each interference peak that you recorded and enter your results in the table below. Plot intensity versus angular position. Does your plot resemble the shape of the diffraction pattern shown in Figure 3? If not, consult your instructor. Print the plot and attach it to this unit.

Angular Position (radians)	Intensity Reading (V)

(d) What is the angular width of the central maximum in your data? Use the expression from part 2(c) to calculate the size of the slit. The expected result is 0.015 mm. What is your percent difference? Use the wavelength for the laser light that you found in the unit on interference of light.

(e) Collect the results for the slit width from the other teams in class and calculate the average and standard deviation. Record the result here. Are your results consistent with the class results? Why or why not?

(f) Can you think of any other methods for measuring small separations like this accurately?

26 The Optical Spectrum of Hydrogen

Name ______________________ Section ____________ Date ____________

Objective

- To determine the wavelengths of the visible lines in the hydrogen spectrum using a spectrometer and a diffraction grating.
- To determine the value of Rydberg's constant.
- To compare the predicted energy levels with the measured ones.

Introduction

The spectral lines of the hydrogen spectrum that fall in the visible region are designated as the H_α, H_β, H_γ, and H_δ lines. All (there happen to be four of them) belong to the Balmer series. In general, the spectrum of hydrogen can be represented by Rydberg's formula:

$$\frac{1}{\lambda} = R_H \left(\frac{1}{n_f^2} - \frac{1}{n_i^2} \right) \tag{3}$$

where n_f can be any positive integer and n_i takes on the values of $n_f + 1$, $n_f + 2$, $nf + 3$, and so on and R_H is the Rydberg constant for hydrogen and equals $1.097 \times 10^7 m^{-1}$.

If one writes equation 1 twice–once, say for the H_α wavelength λ_α, and once for the H_β wavelength, λ_β, then one can eliminate R_H:

$$\frac{1}{n_\beta^2} = \frac{1}{n_f^2} - \left(\frac{\lambda_\alpha}{\lambda_\beta} \right) \left(\frac{1}{n_f^2} - \frac{1}{n_\alpha^2} \right) \tag{4}$$

Thus, once one finds λ_α and λ_β so through trial and error one can determine the value of the three n's in equation 2 (recall they all must be integers and ($n_f < n_\alpha < n_\beta$).

Activity 1: Measuring Spectral Lines

Use the spectrometer to measure the angle (once on each side) for each line and get an average angle for each line. Calculate the wavelength of each line using the relation:

$$\lambda = d \sin\theta \tag{5}$$

where d is the diffraction grating spacing.

Diffraction grating spacing d = Å

Line	θ_{left} (degrees, minutes)	θ_{right} (degrees, minutes)	$\theta_{average}$ (decimal degrees)	Wavelength (Å)	n
H_α					
H_β					
H_γ					

Activity 2: Calculating the Rydberg Constant

Using pairs of measured wavelengths and guesses for n_f and one of the n_i's, calculate the other n_i in equation 2. When this calculated number is close to an integer you may have the correct value for the other n's. Once you have determined the proper n's, calculate a value of R_H for each line and compare the average of these with the accepted value. Use your results to predict the value of the next line in the series H_δ. It's measured value is 4101.2Å. How does your prediction compare?

$R_\alpha\ =$

$R_\beta\ =$

$R_\gamma\ =$

$R_{average}\ =$ % difference =

$H_\delta\ =$ % difference =

Collect values of $R_{average}$ from the other groups in the class and calculate an average and standard deviation. Record it below. How does this result compare with the accepted value? How does it compare with your individual measurement? Be quantitative in your answer.

Activity 3: The Hydrogen Level Diagram

Make an energy level diagram showing the transitions you believe you have measured. How do your measured transition energies compare with the predicted ones?

27 Conservation of Angular Momentum

Name ____________________ Section __________ Date __________

Objectives

To test the Law of Conservation of Angular Momentum and to explore the applicability of angular momentum conservation among objects that experience no external torques.

Apparatus

- A Rotating Disk System
- A mass of 1 kg
- A meter stick and a ruler
- A video analysis system (*VideoPoint*)

Overview

As a consequence of Newton's laws, angular momentum like linear momentum is believed to be conserved in isolated systems. This means that, no matter how many internal interactions occur, the total angular momentum of any system should remain constant. When one of the objects gains some angular momentum another part of the system must lose the same amount. If angular momentum isn't conserved, then we believe that there is some outside torque acting on the system. By expanding the boundary of the system to include the source of that torque we can always preserve the Law of Angular Momentum Conservation.

In this unit you will test the notion of the conservation of angular momentum. As in the test of the conservation of linear momentum, we will investigate what happens when two bodies undergo a "rotational" collision. You will drop a large weight onto a rotating disk and determine the moment of inertia, the angular speed, and finally, the angular momentum of the rotator-disk-weight system before and after this perfectly inelastic collision.

Activity 1: The Moment of Inertia Before and After the Collision

(a) Calculate the theoretical value of the rotational inertia of the metal disk using basic measurements of its radius and mass. Be sure to state units and show the expression you used!

$r_d =$ $M_d =$

$I_d =$

(b)The rotating fixture that holds the disk has a complex shape. We have determined its moment of inertia without the disk and recorded the result. Record that value here. Be sure to state units.

$I_f =$

(c) After dropping the weight on the rotating disk, the system will have a new moment of inertia. Derive a formula for the moment of inertia of a disk-shaped weight of mass m_w and radius r_w revolving about the origin at a distance, r_r. Measure the mass of the weight and use a vernier caliper to measure its diameter.

$I_w =$

$m_w =$ $r_w =$

(d) Come up with a formula for the rotational inertia, I, of the whole system before and after the collision and calculate the moment of inertia before the collision. Don't forget to include the units.

$I_{\text{before}} =$

$I_{\text{after}} =$

Activity 2: Measurement of Angular Acceleration

(a) Place the video camera about 1 m above the rotator, align the camera with the center of the rotator using the pendulum, and center the rotator in the field of view of the camera by viewing it with the *VideoPoint Capture* software. Place a ruler of known length in the field of view of the camera and parallel to one side of the frame. Check that the rotator is flat with the small water-bubble level.

(b) Give the rotator a push and begin recording its motion with the video camera. See **Appendix D: Video Analysis** for details. While the rotator is moving hold the 1 kg weight near the rim of the metal disk and close to, but not quite touching, the surface of the moving metal disk. After at least one revolution of the metal disk drop the 1-kg mass onto the disk and record the motion of the disk for at least one revolution afterward.

(c) Measure the distance of the center of the weight you dropped from the center of the rotator r_r. To do this, measure the distance from the center of the rotator to the edge of the weight and use the result from Activity 1 part (c) for the diameter of the weight. Calculate the distance from the origin to the center of the weight. Use these results and those from Activity 1 part (d) to calculate the final moment of inertia.

$r_{\text{edge}} =$ $\qquad$ $r_w =$ $\qquad$ $r_r =$

$I_{\text{after}} =$

(d) Determine the angular speed before and after the collision. To do this task see the instructions in **Appendix D: Video Analysis** for creating and analyzing a movie file.

1. Find the last frame before you dropped the weight on the rotator and click on the position of the white marker on the metal disk. Under the **Edit** menu highlight **Leave/Hide Trails**. Now go backward through the film until the rotator has gone through one full rotation. Estimate to the nearest fraction of a frame how many frames there are in one revolution. You also need to know the time between frames Δt_{frame}, which you can get from the data table in *VideoPoint*.

   $N_{\text{before}} =$ $\qquad$ $\Delta t_{\text{frame}} =$

   Calculate the time for one revolution before the collision and the angular speed.

   $t_{\text{before}} =$ $\qquad$ $\omega_{\text{before}} =$

2. We now follow a similar procedure to determine the angular speed after the collision. Under the **Edit** Menu highlight **Clear All...** to get rid of your previous results. Find the first frame after you dropped the weight on the rotator and click on the position of the white marker. Now click forward and estimate to the nearest fraction of a frame the number of frames in one full revolution.

   $N_{\text{after}} =$ $\qquad$ $\Delta t_{\text{frame}} =$

   Calculate the time for one revolution and the angular speed after the collision.

   $t_{\text{after}} =$ $\qquad$ $\omega_{\text{after}} =$

(e) Calculate the angular momentum before and after the collision. Calculate the percent difference between the two results. Is momentum conserved within the experimental uncertainty? What is the value of the experimental uncertainty? What is your evidence?

$L_{\text{before}} =$

$L_{\text{after}} =$

(f) Would the procedure you followed above change if the weight was moving horizontally at a constant velocity when you dropped it? If it changed, what would be different?

28 A Theory for the Hydrogen Atom

Name ______________________ Section ____________ Date ____________

Objective

To use a numerical technique to solve the Schroedinger equation for the hydrogen atom, compare calculations with spectroscopic measurements from hydrogen, and explore the implications of the quantum theory especially the quantization of energy in bound systems.

Apparatus

- *Schrödinger Shooter* software
- Results from previous measurements of hydrogen energy transitions.

Overview

Quantum mechanics is the theoretical structure used to make sense of the atomic and sub-atomic world. It is based on a small set of assumptions and a lot of associated mathematics.

The Postulates of Quantum Mechanics

1. The quantum state of a particle is characterized by a wave function $\Psi(\vec{r},t)$, which contains all the information about the system an observer can possibly obtain. The square of the magnitude of the wave function $|\Psi(\vec{r},t)|^2$ is interpreted as a probability or probability density for the particle's presence.
2. The things we measure (*e.g.* energy, momentum) are called observables. Each observable has a corresponding mathematical object called an operator that does 'something' to the wave function $\Psi(\vec{r},t)$. For example, the momentum $\vec{p}$ is associated with the operator
$$\frac{\hbar}{i}\left(\frac{\partial}{\partial x}\hat{i}+\frac{\partial}{\partial y}\hat{j}+\frac{\partial}{\partial z}\hat{k}\right)\Psi(\vec{r},t) \tag{1}$$
where $\hbar$ is Planck's constant. Making a measurement applies these operators to the wave function.
3. The spatial dependence of the wave function $\Psi(\vec{r},t)$ is governed by the time-independent Schrödinger equation.
$$-\frac{\hbar^2}{2m}\left(\frac{\partial^2}{\partial r^2}+\frac{L^2}{2mr^2}\right)\Psi(\vec{r},t)+V\Psi(\vec{r},t)=E\Psi(\vec{r},t) \tag{2}$$
4. The temporal or time dependence of the wave function $\Psi(\vec{r},t)$ is governed by a more general version of the Schrödinger equation which is the following.
$$-\frac{\hbar^2}{2m}\left(\frac{\partial^2}{\partial r^2}+\frac{L^2}{2mr^2}\right)\Psi(\vec{r},t)+V\Psi(\vec{r},t)=i\hbar\frac{\partial}{\partial t}\Psi(\vec{r},t) \tag{3}$$

We will focus on the first three postulates in this laboratory and in particular on understanding and solving Equation 2, the Schrödinger equation.

Activity 1: Energy in Three Dimensions

Consider a particular form of the total mechanical energy of a particle in three dimensions

$$E=\frac{1}{2}m\vec{v}\cdot\vec{v}+V(\vec{r})=\frac{1}{2}m(v_x^2+v_y^2+v_z^2)+V(\vec{r})=\frac{1}{2}mv^2+V(\vec{r})=\frac{p^2}{2m}+V(\vec{r}) \tag{4}$$

where m is the particle's mass, $\vec{v}$ is its velocity, and $V(\vec{r})$ is the potential energy of the particle and depends only on the particle's position. We have used the definition of the momentum $\vec{p} = m\vec{v}$. The potential energy between the electron and proton in a hydrogen atom is the Coulomb potential so

$$V(\vec{r}) = K\frac{q_1 q_2}{r} = -K\frac{e^2}{r} \tag{5}$$

where we have used the known proton and electron charges.

If we have an electron in the vicinity of a proton (*i.e.* a hydrogen atom), then it is convenient to rewrite the kinetic energy part of Equation 4 in polar coordinates. The proton, electron and the electron's velocity vector form a plane where we can describe the velocity in terms of a radial component and an angular one

$$\vec{v} = v_r\hat{r} + v_\theta\hat{\theta} \tag{6}$$

where $\hat{r}$ points radially along a line from the origin to the electron's position and $\hat{\theta}$ is perpendicular to $\hat{r}$ and points so counter-clockwise rotations are positive.

(a) Take the dot product of this form of $\vec{v}$ with itself and substitute your result in Equation 4.

(b) Recalling that for circular motion $v_\theta = r\omega$ where r is the distance of the electron from the origin and ω is the magnitude of the angular velocity rewrite your previous result in terms of ω and r.

(c) We can now define the momentum associated with the radial motion as $\vec{p}_r = m\vec{v}_r$ and the momentum associated with the angular motion $\vec{L} = mr^2\vec{\omega}$. The angular momentum L is a constant of the motion so it is unchanging. Rewrite your energy equation using in terms of p_r, L, r, and any other constants. Include the explicit version of the potential in Equation 5.

(d) You should have found in part 1.c that the three-dimensional energy of a particle in a Coulomb field can be written as

$$E = \frac{p_r^2}{2m} + \frac{L^2}{2mr^2} - K\frac{e^2}{r} \tag{7}$$

where L is a constant so the energy only depends on r and p_r. Note the similarities with the forms of the Schrödinger equation in Equations 2-3. Make a sketch of the potential energy as a function of r. What are the limiting values of the potential?

(e) The angular momentum L is a constant of the motion so the last two terms in Equation 7 can be treated as a single 'effective' potential that governs the radial motion of particles. Make a sketch of those last two terms as a function of r for constant L. How is your figure different from the one in 1.d?

(f) Since the energy is constant you can draw it on your previous sketch as a flat, straight line. We are considering bound states (the hydrogen atom) so $E < 0$. Draw an energy line for a bound state. Does your energy 'curve' intersect the effective potential curve anywhere? For a classical particle like a ball rolling on a hill or a satellite orbiting the Earth, what happens at this intersection? Describe the motion of a classical particle in this potential. What restrictions are there on the energy E of a classical particle?

Activity 2: Quantum Predictions

(a) What do you expect the wave function of the electron in a hydrogen atom to look like? Copy your drawing from part 1.b and add a sketch of your expectation on the same graph. Consider what happens as the effective potential changes with r.

(b) What happens to your wave function as the energy increases? Make another copy here of the graph of the effective potential energy that you made in 2.a. Draw an energy curve for a higher energy state than your previous one. Now sketch the new, higher-energy wave function. How is it different from the curve you made in 2.a?

Activity 3: Solving the Schrödinger Equation

(a) We are now ready to start solving the Schrödinger Equation. Go first to the `All Programs` menu, then to `Physics Applications` and click on `Schrödinger Shooter`. You will see the `Schrödinger Shooter` window like the one in the next figure. If you don't see this window, consult your instructor.

(b) The first step is to select a potential energy function. Click on `File` and go to `New Potential Energy Function`. Select the potential energy function most appropriate for the hydrogen atom. Next select the charge of the atomic nucleus by entering that value in the box labeled `Z:` in units of the electronic charge e. Set the

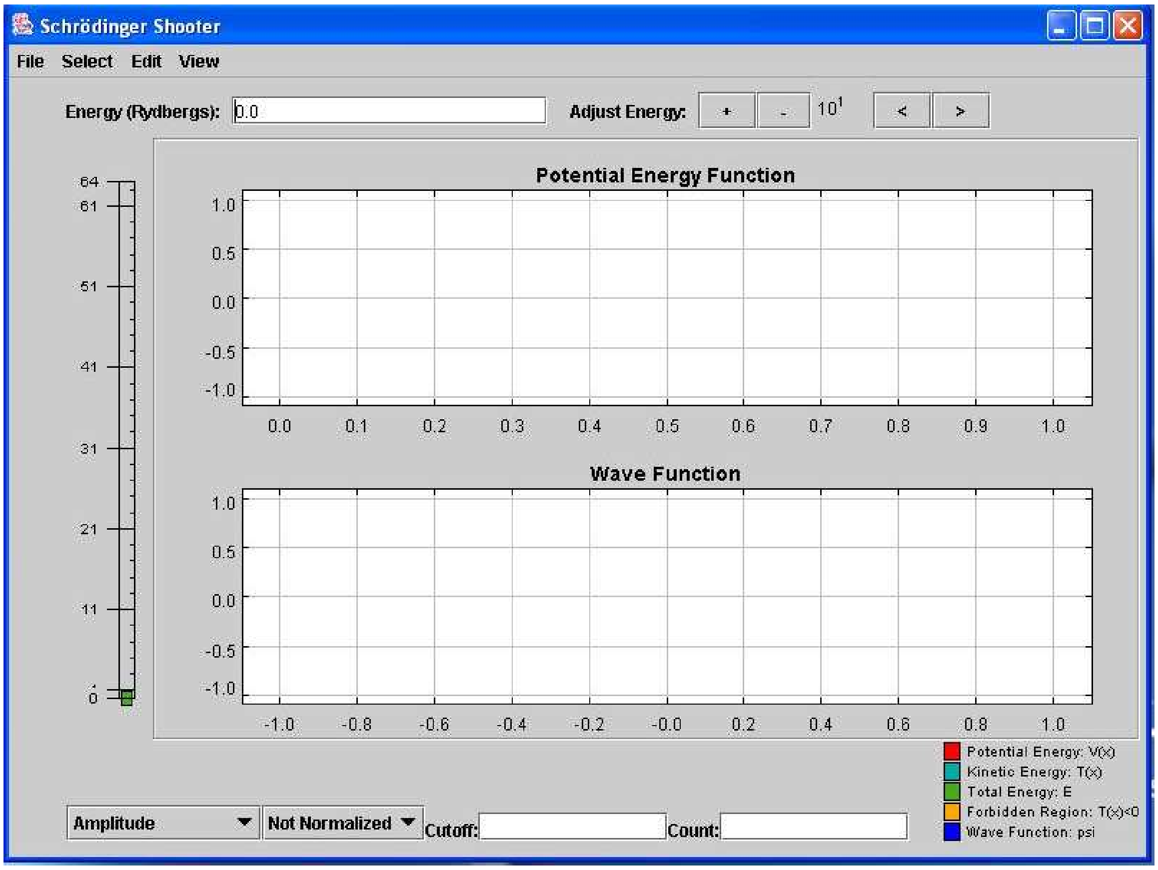


Figure 1: Initial panel for *Schrödinger Shooter*

quantum number of the angular momentum to zero in the box labeled `L:`. Use the second menu button from the left-hand side at the bottom of the `Schrödinger Shooter` window to choose `Normalized` wave functions (it should be labeled `Unnormalized` when you start the program). This last choice makes it easier to compare wave functions for different quantum numbers. Last, set the initial value of the energy to -1.5 in the box labeled `Energy (Rydbergs):` and hit return. The conversion between eV and rydbergs is 13.6056923(12) eV/rydberg. The number in parentheses is the uncertainty on the last two digits in the experimental value. At this point you should see curves in both of the panels. If you don't, consult your instructor.

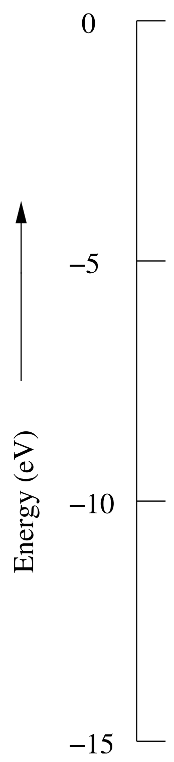
(c) You should see several curves in the upper panel. What quantities do the blue, red, yellow, and green curves represent? Use the legend in the lower right portion of the `Schrödinger Shooter` window.

(d) You can now adjust the energy of the electron in the field of the proton to find the energy levels of hydrogen. There are three ways to change this energy. There is a slide on the left-hand side of the `Schrödinger Shooter` window. Click and drag the slide to change the energy and read the value in the box labeled `Energy(Rydbergs)` located near the top of the `Schrödinger Shooter` window. You can also enter the energy in the same box labeled `Energy(Rydbergs):` as you did in part 3.b. Last, to the right of this last box there are buttons labeled `Adjust Energy:` that will increase or decrease the energy and change the step size.

(e) Tune the energy to find lowest five energy levels in your hydrogen atom by obtaining physically acceptable wave functions. You may need to increase the horizontal range to view the full wave function. You can do this by increasing the value in the box labeled `Cutoff:` at the bottom of the `Schrödinger Shooter` window. If you increase the horizontal scale, then increase the next box labeled `Count:` (next to the `Cutoff` box) by the same ratio. In other words, if you double the cutoff, then double the count. This last parameter controls the stepsize used in the numerical integration of the Schrödinger equation. What does the wave function look like when you have found the correct energy level? What postulate did you exploit to find the correct solutions? How does the radial wave function change as the energy increases? Record you values for each of the energy levels below.

Activity 4: Analysis and Comparison with Previous Data

(a) Enter the values you found for the energies in an *Excel* spreadsheet and convert the values to eV. Use the scale below to make an energy level diagram for hydrogen.



(b) In the lab entitled *Optical Spectrum of Hydrogen* you measured and identified transitions in hydrogen between different principle quantum levels. Using the results in part 4.a calculate the same transitions and record them below.

(c) Compare your measured results from the *Optical Spectrum of Hydrogen* with the predictions of your model in part 4.b. Calculate the percentage difference for each transition. Does your model agree with the data? Be quantitative in your answer.

Activity 5: The Meaning of Quantum Theory

(a) For one of your hydrogen energy states, change the angular momentum of the quantum state by setting $L = 1$ in the box labeled `L:`. How does the radial wave function change? How much does the energy change? Do the same thing with the next higher energy state. Can you jump to any conclusions about the effect of angular momentum on the energy of a quantum state in hydrogen?

(b) Return to the questions in Activity 2 and examine your predictions. How did you do? Correct any statements that you now find are wrong.

(c) What requirement or postulate forces us to choose particular energy states (*i.e.* what causes energy quantization)?

(d) Reconsider a variation of one of the questions in part 1.c. In that question you were asked to describe the motion of a classical particle like a satellite orbiting the Earth. Describe the motion of a quantum particle in the hydrogen atom potential.

A Treatment of Experimental Data

Recording Data

When performing an experiment, record all required original observations as soon as they are made. By "original observations" is meant what you actually see, not quantities found by calculation. For example, suppose you want to know the stretch of a coiled spring as caused by an added weight. You must read a scale both before and after the weight is added and then subtract one reading from the other to get the desired result. The proper scientific procedure is to record both readings as seen. Errors in calculations can be checked only if the original readings are on record.

All data should be recorded with units. If several measurements are made of the same physical quantity, the data should be recorded in a table with the units reported in the column heading.

Significant Figures

A laboratory worker must learn to determine how many figures in any measurement or calculation are reliable, or "significant" (that is, have physical meaning), and should avoid making long calculations using figures which he/she could not possibly claim to know. *All sure figures plus one estimated figure are considered significant.*

The measured diameter of a circle, for example, might be recorded to four significant figures, the fourth figure being in doubt, since it is an estimated fraction of the smallest division on the measuring apparatus. How this doubtful fourth figure affects the accuracy of the computed area can be seen from the following example.

Assume for example that the diameter of the circle has been measured as $.526\underline{4}$ cm, with the last digit being in doubt as indicated by the line under it. When this number is squared the result will contain eight digits, of which the last five are doubtful. Only one of the five doubtful digits should be retained, yielding a four-digit number as the final result.

In the sample calculation shown below, each doubtful figure has a short line under it. Of course, each figure obtained from the use of a doubtful figure will itself be doubtful. The result of this calculation should be recorded as 0.2771 cm^2, including the doubtful fourth figure. (The zero to the left of the decimal point is often used to emphasize that no significant figures precede the decimal point. This zero is not itself a significant figure.)

$$(.526\underline{4}\ \mathrm{cm})^2 = .277\underline{09696}\ \mathrm{cm}^2 = 0.277\underline{1}\ \mathrm{cm}^2$$

In multiplication and division, the rule is that a calculated result should contain the same number of significant figures as the least that were used in the calculation.

In addition and subtraction, do not carry a result beyond the first column that contains a doubtful figure.

Statistical Analysis

Any measurement is an intelligent estimation of the true value of the quantity being measured. To arrive at a "best value" we usually make several measurements of the same quantity and then analyze these measurements statistically. The results of such an analysis can be represented in several ways. Those in which we are most interested in this course are the following:

<u>Mean</u> - The mean is the sum of a number of measurements of a quantity divided by the number of such measurements, which is just the arithmetic mean or the so-called average. It generally represents the best estimate of true value of the measured quantity.

<u>Standard Deviation</u> - The standard deviation (σ) is a measure of the range on either side of the mean within which approximately two-thirds of the measured values fall. For example, if the mean is 9.75 m/s^2 and the standard deviation is 0.10 m/s^2, then approximately two-thirds of the measured values lie within the range 9.65 m/s^2 to 9.85 m/s^2. A customary way of expressing an experimentally determined value is: Mean$\pm\sigma$, or (9.75$\pm$ 0.10) m/s^2. Thus, the standard deviation is an indicator of the spread in the individual measurements, and a small σ implies high precision. Also, it means that the probability of any future measurement falling in this range is approximately two to one. The equation for calculating the standard deviation is

$$\sigma = \sqrt{\frac{\Sigma\,(x_i - \langle x \rangle)^2}{N-1}}$$

where x_i are the individual measurements, $\langle x \rangle$ is the mean, and N is the total number of measurements.

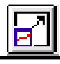
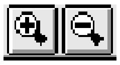
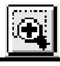
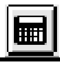
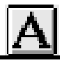
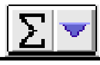
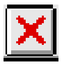
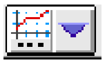
<u>% Difference</u> - Often one wishes to compare the value of a quantity determined in the laboratory with the best known or "accepted value" of the quantity obtained through repeated determinations by a number of investigators. *The % difference is calculated by subtracting the accepted value from your value, dividing by the accepted value, and multiplying by 100.* If your value is greater than the accepted value, the % difference will be positive. If your value is less than the accepted value, the % difference will be negative. The % difference between two values in a case where neither is an accepted value can be calculated by choosing one as the accepted value.

B Introduction to DataStudio

Quick Reference Guide

Shown below is the quick reference guide for DataStudio.

What You Want To Do	How You Do It	Button
Start recording data	Click the 'Start' button or select 'Start Data' on the Experiment menu (or on the keyboard press CTRL - R (Windows) or Command - R (Mac))	
Stop recording (or monitoring) data	Click the 'Stop' button or select 'Stop Data' on the Experiment menu (or on the keyboard press CTRL - . (period) (Win) or Command - . (Mac))	
Start monitoring data	Select 'Monitor Data'on the Experiment menu (or on the keyboard press CTRL - M (Win) or Command - M (Mac))	none

On the Graph Display	In the Graph Toolbar	Button
Re-scale the data so it fills the Graph display window	Click the 'Scale to Fit' button.	
Pinpoint the x- and y-coordinate values on the Graph display	Click the 'Smart Tool' button. The coordinates appear next to the 'Smart Tool'.	
'Zoom In' or 'Zoom Out'	Click the 'Zoom In' or 'Zoom Out' buttons.	
Magnify a selected portion of the plotted data	Click the 'Zoom Select' button and drag across the data section be to magnified.	
Create a Calculation	Click the 'Calculate' button	
Add a text note to the Graph	Click the 'Note' button.	
Select from the Statistics menu	Click the Statistics menu button	
Add or remove a data run	Click the 'Add/Remove Data' menu button	
Delete something	Click the 'Delete' button	
Select Graph settings	Click the 'Settings' menu button	

Selecting a Section of Data

1. To select a data section, hold the mouse button down and move the cursor to draw a rectangle around the data of interest. The data in the region of interest will be highlighted.
2. To unselect the data, click anywhere in the graph window.

Fitting a Section of Data

1. Select the section of data to be fitted.

2. Click on the **Fit** button on the Graph Toolbar and select a mathematical model. The results of the fit will be displayed on the graph.

3. To remove the fit, click the **Fit** button and select the checked function type.

Finding the Area Under a Curve

1. Use the **Zoom Select** button on the Graph Toolbar to zoom in around the region of interest in the graph. See the quick reference guide above for instructions.

2. Select the section of data that you want to integrate under.

3. Click the **Statistics** button on the Graph Toolbar and select **Area**. The results of the integration will be displayed on the graph.

4. To undo the integration, click on the **Statistics** button and select **Area**.

C Video Analysis

Making a Movie

To make a movie, perform the following steps:

1. Start up *Videopoint Capture* by going to *Start* → *Programs* → *Physics Applications* → *VideoPoint* → *VP Capture*. The program will first ask you to choose a file name and location for the video you are going to make. You should choose to put the file on the Desktop.
2. Before you start making a movie, click on the *Capture rate* box and set the capture rate to 30 frames per second. Also, go to the *Size & Colors* under the *Capture Options* menu, and choose the largest available size for the video.
3. Go to *Preferences* under the *Edit* menu. Check the box that says *Convert Captured File Before Editing.* A dialog box will pop up; select the *Intel Indeo Video 4.5* option and click OK. Click OK again to close the Preferences box. (**If you forget to do this step, you won't be able to analyze the video.**)
4. You should always have the camera about 1 m or more from the object you will be viewing. This constraint is required to reduce the effect of perspective for objects viewed near the edge of the field of view. Point the camera so that its field of view is centered on the expected region where you will perform the experiment.
5. Make sure the camera is focused by rotating the barrel on the outside of the lens until you have a clear picture.
6. During the analysis of the movie (see next section) we need to determine the size and position of objects in the movie. Place a ruler or some object of known size in the field of view where it won't interfere with the experiment. The object should be the same distance away from the camera as the motion so the horizontal and vertical scales will be accurately determined. It should also be parallel to one of the sides of the movie frame.
7. One member of your group should perform the computer tasks while the others do the experiment.
8. To start recording your video, click *Record.* When you're done, click *Stop.*
9. The next step is to decide how much of the movie to save. Use the slider to step through the movie frame by frame. When you find the first frame you want to save, click *First.* When you find the last frame you want to save, click *Last.* (You may want to save the entire movie, in which case *First* and *Last* really will be the first and last frames. Often, though, there will be "dead" time either at the beginning or the end of the movie, which you might as well cut out before saving.)
10. After you've selected the range of frames you want to save, the button at the lower right should say *Keep.* Make sure that the box next to this button says *All.* (If it says *Double*, change it to *All.*) Then click *Keep* and *Save.* You will see a quick replay of the movie as Videopoint converts and saves it.
11. Click *Open in VideoPoint.*

Analyzing the Movie

To determine the position of an object at different times during the motion, perform the following steps:

1. *VideoPoint* will request the number of objects you want to track in the movie. Carefully read the instructions for the unit you are working on to find this number. Enter it in the space provided. You will now see several windows. (Note: You may have to move the movie window out of the way to see the other windows.) One contains the movie and has control buttons and a slider along the bottom of the frame to control the motion of the film. Experiment with these controls to learn their function. Another window below the movie frame (labeled Table) contains position and time data and a third window to the right of the frame (labeled Coordinate Systems) describes the coordinate system in use.

2. This is a good time to calibrate the scale. Go to a frame where an object of known size is clearly visible (see item 6 in the previous section). Under the **Movie** menu highlight **Scale Movie**. A dialog box will appear. Enter the length of the object and set **Scale Type** to **Fixed**. Click **Continue**. Move the cursor over the frame and click on the ends of scaling object.

3. You are now ready to record the position and time data. Go to the first frame of interest. Move the cursor over the frame and it will change into a small circle with an attached label. Place the circle over the object of interest in the frame and click. The x and y positions will be stored and the film advanced one frame. Move the circle over the position of the object in the frame and repeat. Continue this process until you have mapped out the motion of the object. If you entered more than one object to keep track of when you opened the movie, then you will click on all those objects in each frame before the film advances.

4. When you have entered all the points you want, go to the **File** menu and select **Export data**. This will allow you to save your data table as an Excel file. Save this file on the desktop (by clicking on the "Open" button, which actually doesn't open anything), and double-click on the saved file to start up Excel. You will now be able to continue your data analysis in Excel.

5. Once you have looked at your data in Excel and made sure everything looks OK, you can quit Videopoint Analysis. If you are sure you have exported your data correctly to Excel, there is no need to save in Videopoint.

Changing the Origin

To change the position of the origin take the following steps.

1. Click on the arrow icon near the top of the menu bar to the left. The cursor will be shaped like an arrow when you place it on the movie frame.

2. Click at the origin (where the axes cross) and drag the origin to the desired location.

3. Click on the circle at the top of the menu bar to the left to return to the standard cursor for marking points on the film.

Using a Moving Coordinate System

To record the position of an object and to change the coordinate system from frame to frame take the following steps.

1. Open the movie as usual and enter one object to record. First we have to select the existing origin and change it from a fixed one to a moving one. Click on the arrow near the top of the menu bar to the left. The cursor will have the shape of an arrow when you place it on the movie frame. Click on the existing origin (where the axes cross) and it will be highlighted.

2. Under the **Edit** menu drag down and highlight **Edit Selected Series**. A dialog box will appear. Click on the box labeled **Data Type** and highlight the selection **Frame-by-Frame**. Click OK.

3. Click on the circle at the top of the menu bar to the left to change the cursor back to the usual one for marking points. Go to the first frame of interest. When the cursor is placed in the movie frame it will be labeled with "Point S1." Click on the object of interest. The film will NOT advance and the label on the cursor will change to "Origin 1." Click on the desired location of the origin in that frame. The film will advance as usual. Repeat the procedure to accumulate the x- and y-positions relative to the origin you've defined in each frame.

D Instrumentation

Introduction

Being both quantitative and experimental, physics is basically a science of measurement. A great deal of effort has been expended over the centuries improving the accuracy with which the fundamental quantities of length, mass, time, and charge can be measured.

It is important that the appropriate instrument be used when measuring. Ordinarily, a rough comparison with a numerical scale, taken at a glance and given in round numbers, is adequate. Increasing precision, though, requires a more accurate scale read to a fraction of its smallest division. The "least count" of an instrument is the smallest division that is marked on the scale. This is the smallest quantity that can be read directly without estimating fractions of a division.

Even at the limit of an instrument's precision, however, accidental errors— which cannot be eliminated—still occur. These errors result in a distribution of results when a series of seemingly identical measurements are made. The best value, known as the most probable value, is the arithmetic mean or average of the measurements.

Other errors, characteristic of all instruments, are known as systematic errors. These can be minimized by improving the equipment and by taking precautions when using it.

Length Measurement

Three instruments will be available in this class for length measurements: a ruler (one- or two-meter sticks, for example), the vernier caliper, and the micrometer caliper.

The Meter Stick

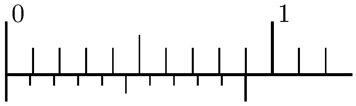
A meter stick, by definition, is 1 meter (m) long. Its scaled is divided, and numbered, into 100 centimeters (cm). Each centimeter, in turn, is divided into 10 millimeters. Thus 1 cm $= 10^{-2}$ m, and 1 mm $= 10^{-1}$ cm $= 10^{-3}$ m.

When measuring a length with a meter stick, different regions along the scale should be used for the series of measurements resulting in an average value. This way, non-uniformities resulting from the meter stick manufacturing process will tend to cancel out and so reduce systematic errors. The ends of the stick, too, should be avoided, because these may be worn down and not give a true reading. Another error which arises in the reading of the scale is introduced by the positioning of the eyes, an effect known as parallax. Uncertainty due to this effect can be reduced by arranging the scale on the stick as close to the object being measured as possible.

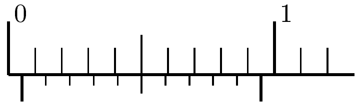
The Vernier Caliper

A vernier is a small auxiliary scale that slides along the main scale. It allows more accurate estimates of fractional parts of the smallest division on the main scale.

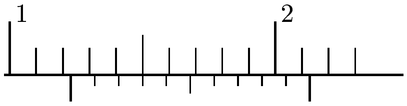
On a vernier caliper, the main scale, divided into centimeters and millimeters, is engraved on the fixed part of the instrument. The vernier scale, engraved on the movable jaw, has ten divisions that cover the same spatial interval as nine divisions on the main scale: each vernier division is $\frac{9}{10}$ the length of a main scale division. In the case of a vernier caliper, the vernier division length is 0.9 mm. [See figures below.]



0.00



0.05



1.23

Examples of vernier caliper readings

To measure length with a vernier caliper, close the jaws on the object and read the main scale at the position indicated by the zero-line of the vernier. The fractional part of a main-scale division is obtained from the first vernier division to coincide with a main scale line. [See examples above.]

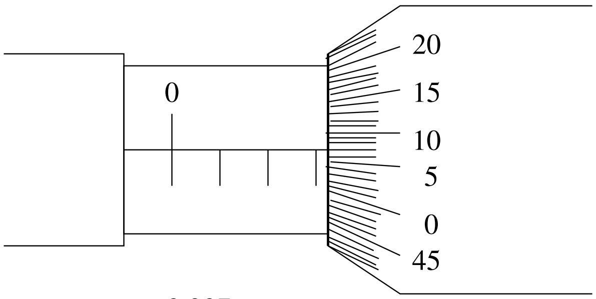
If the zero-lines of the main and vernier scales do not coincide when the jaws are closed, all measurements will be systematically shifted. The magnitude of this shift, called the zero reading or zero correction, should be noted and recorded, so that length measurements made with the vernier caliper can be corrected, thereby removing the systematic error.

The Micrometer Caliper

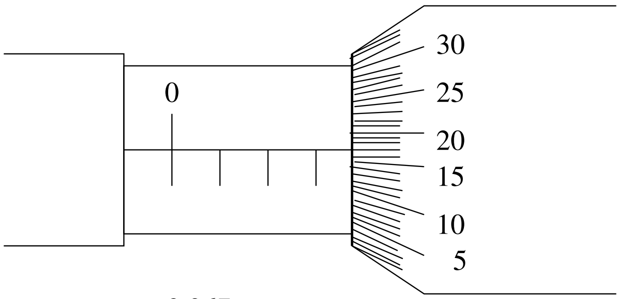
A micrometer caliper is an instrument that allows direct readings to one hundredth of a millimeter and estimations to one thousandth of a millimeter or one millionth of a meter (and, hence, its name). It is essentially a carefully machined screw housed in a strong frame. To measure objects, place them between the end of the screw and the projecting end of the frame (the anvil). The screw is advanced or retracting by rotating a thimble on which is engraved a circular scale. The thimble thus moves along the barrel of the frame which contains the screw and on which is engraved a longitudinal scale divided in millimeters. The pitch of the screw is 0.5 mm, so that a complete revolution of the thimble moves the screw 0.5 mm. The scale on the thimble has 50 divisions, so that a turn of one division is $\frac{1}{50}$ of 0.5 mm, or 0.01 mm.

Advance the screw until the object is gripped gently. Do not force the screw. A micrometer caliper is a delicate instrument.

To read a micrometer caliper, note the position of the edge of the thimble along the longitudinal scale and the position of the axial line on the circular scale. The first scale gives the measurement to the nearest whole division; the second scale gives the fractional part. It takes two revolutions to advance one full millimeter, so note carefully whether you are on the first or second half of a millimeter. The result is the sum of the two scales. (See examples below).



0.307 cm



0.367 cm

As with the vernier caliper, the zero reading may not be exactly zero. A zero error should be checked for and recorded, and measurements should be appropriately corrected.

Mass Measurement

Three kinds of instruments will be available to determine mass: a digital scale and two types of balances. The operation of the first instrument is trivial, and so will not be explained here.

Please understand that with each of these instruments we are really comparing weights, not masses, but the proportionality of weight and mass allows the instruments to be calibrated for mass.

The Equal-Arm Balance

The equal-arm balance has two trays on opposite sides of a pivot. The total mass placed on one tray required to balance the object on the other gives the mass of the object. Most equal-arm balances have a slider, as well, that can move along a scale and allow for greater precision than the smallest calibrated mass available. Typically, this scale has 0.5 g divisions.

The Triple-Beam Balance

The triple-beam balance, so-called because of its three slider scales, can be read to 0.1 g and estimated to half that. With an object on the tray, the masses of the different scales are slid to notches until balanced. Get close with the larger masses first and then fine-adjust with the smallest slider.

Time Measurement

Time measurements in this course will be made either with a computer or with a stop watch. This first is out of your control.

The Stop Watch

The stop watches you will use in class have a time range of from hours to hundredths of a second. There are two buttons at the top: a stop/start button and a reset button. The operation of these should be evident, although once the watch is reset, the reset button also starts the watch (but doesn't stop it). Please be aware of this feature.

Charge Measurements

The magnitude of charge is among the most difficult measurements to make. Instead a number of indirect measurements are undertaken to understand electric phenomena. These measurements are most often carried out with a digital multimeter

The Digital Multimeter

The digital multimeters available for laboratory exercises have pushbutton control to select five ac and dc voltage ranges, five ac and dc current ranges, and six resistance ranges. The ranges of accuracy are 100 microvolts to 1200 volts ac and dc, 100 nanoamperes to 1.999 amperes ac and dc, and 100 milliohms to 19.99 megaohms.

To perform a DC voltage measurement, select the DCV function and choose a range maximum from one of 200 millivolts or 2, 20, 200, or 1200 volts. Be sure the input connections used are V-Ω and COMMON. The same is true for AC voltage, regarding range and inputs, but the ACV function button should be selected.

For DC current choose DC MA (for DC milliamperes), while for AC current choose AC MA. Your choices for largest current are 200 microamperes or 2, 20, 200, or 2000 milliamperes. Check that the input are connected to MA and COMMON.

There are two choices for resistance measurement: Kilohms (KΩ) and Megohms (20MΩ). The input connectors are the same as when measuring voltage, namely V-Ω and COMMON. The range switches do not function with the Megohm function, but one of the range buttons must be set. The maximum settings for Kilohm readings are 200Ω or 2, 20, 200, or 2000kΩ.

E Introduction to Excel

Microsoft Excel is the spreadsheet program we will use for much of our data analysis and graphing. It is a powerful and easy-to-use application for graphing, fitting, and manipulating data. In this appendix, we will briefly describe how to use Excel to do some useful tasks.

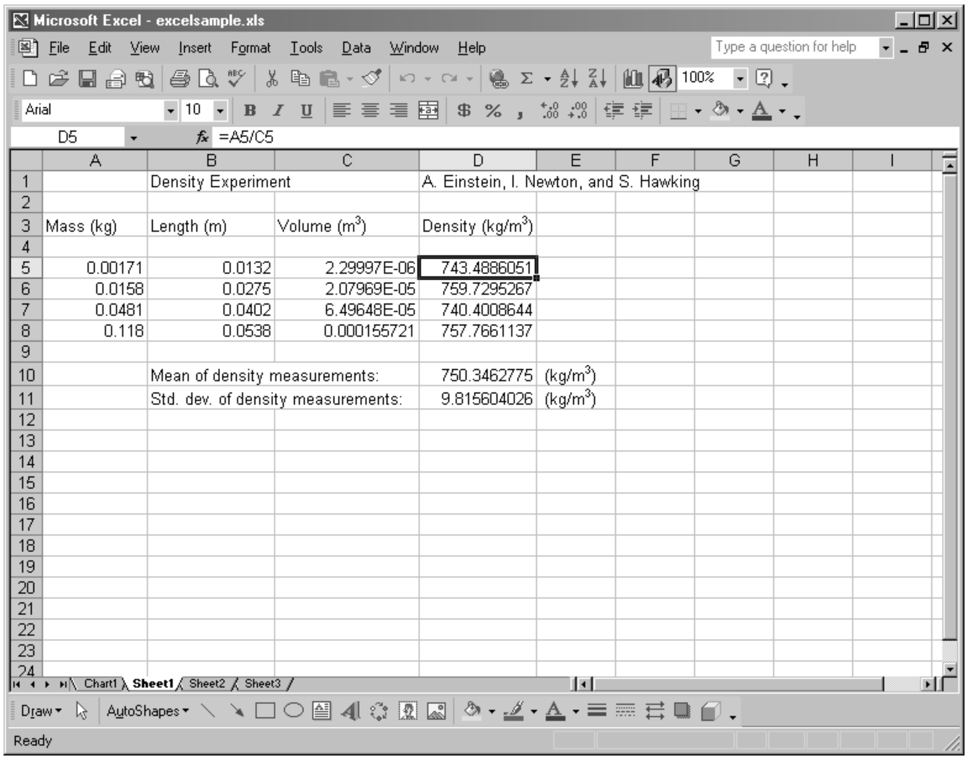
E.1 Data and formulae

The figure below shows a sample Excel spreadsheet containing data from a made-up experiment. The experimenter was trying to measure the density of a certain material by taking a set of cubes made of the material and measuring their masses and the lengths of the sides of the cubes. The first two columns contain her measured results. **Note that the top of each column contains both a description of the quantity contained in that column and its units.** You should make sure that all of the columns of your data tables do as well. You should also make sure that the whole spreadsheet has a descriptive title and your names at the top.

In the third column, the experimenter has figured out the volume of each of the cubes, by taking the cube of the length of a side. To avoid repetitious calculations, she had Excel do this automatically. She entered the formula "=B3∧3" (without the quotes) into cell C3. Note the equals sign, which indicates to Excel that a formula is coming. The ∧ sign stands for raising to a power. After entering a formula into a cell, you can grab the square in the lower right corner of the cell with the mouse and drag it down the column. This will copy the cell, making the appropriate changes, into the rest of the column. For instance, in this case, cell C4 contains the formula "=B4∧3," and so forth.

Column D was similarly produced with a formula that divides the mass in column A by the volume in column C.

At the bottom of the spreadsheet we find the mean and standard deviation of the calculated densities (that is, of the numbers in cells D3 through D6). Those are computed using the formulae "=average(D3:D6)" and "=stdev(D3:D6)".



E.2 Graphs

Making graphs in Excel is relatively easy. First, use the mouse to select the columns of numbers you want to graph. (If the two columns aren't next to each other, select the first one, then hold down the control key while selecting the second one.) Then click on the "chart wizard" button (which looks like this ).

There are a number of different styles of graphs that the chart wizard can generate. In nearly every case, you will want to select an "XY (scatter)" graph. Click "Next" to proceed to customize your graph. The most useful customization options come in step 3 of the process. Under "Title," you can put appropriate labels on the x and y axes of your graphs and give the overall graph a descriptive title. **All graphs must have correctly labeled axes (including units).** If the graph contains only one set of data points, you may wish to uncheck the box that says "Show legend": the information in the legend is probably already contained in the title and axis labels, so the legend just takes up space.

Sometimes, you may want to make a graph in Excel where the x column is to the right of the y column in your worksheet. In these cases, Excel will make the graph with the x and y axes reversed. There are at least two ways to fix this problem. The simplest way is to make a copy of the y column in the worksheet and paste it so that it's to the right of the x column. If you don't want to do that, here's another way. In step 2 of the chart wizard, look under the "Series" tab. Click on this icon  next to the place where it says "X values." You can now select which column of data you want to go on the x axis. Do the same thing to select the correct y column.

E.3 Fitting lines and curves

After you've made a graph, you can have Excel draw a straight line or curve that is a best fit to the data. Under the "chart" menu, select "Add trendline." The most common sort of trendline you will add is a linear fit, but you can also have Excel draw other sorts of best-fit curves. Under the "options" tab, you can check a box that causes Excel to display the equation for the line or curve it has drawn. **Excel will not put the correct units on the numbers in this equation, but you should.**

F Nuclear Safety

All of the radioactive sources we will use in class are very low-level isotopes referred to as "license-free" sources. The following guidelines should be followed for handling radioactive materials in the classroom.

1. Eating, drinking, and application of cosmetics in the laboratory are not permitted.
2. Pipetting by mouth is never permitted. Use suction devices such as pipette filters.
3. Gloves and lab coats should be worn when working with all liquid isotopes.
4. Before leaving the lab, wash your hands thoroughly and check for possible contamination with a survey instrument.
5. All radioactive liquid wastes are to be poured into the liquid waste container, NEVER into a sink.
6. Report all spills, wounds, or other emergencies to your instructor.
7. Maintain good housekeeping at all times in the lab.
8. Store radioactive material only in the designated storage area. Do not remove sources from the lab.