Physics 309 Test 2

I pledge that I have given nor received unauthorized assistance during the completion of this work.

Name ______________________ Signature ______________________

Questions (8 pts. apiece) Answer questions in complete, well-written sentences WITHIN the spaces provided. For multiple-choice questions circle the correct answer.

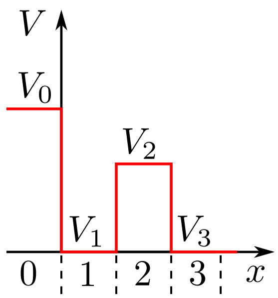
1. The Maxwellian velocity distribution in a gas is the following.
$$n(v) = 4\pi N \left(\frac{m}{2\pi kT}\right)^{3/2} v^2 \exp\left[\frac{-mv^2}{2kT}\right]$$
In the classical fusion lab we used this distribution to estimate the probability that fusion could occur without resorting to quantum tunneling. How would you do that? Don't do any calculations. Describe the method in words and/or equations.

2. In solving the CO rotator problem we changed coordinates from Cartesian ones to spherical ones and the relative momentum of the CO molecule became
$$\vec{p}_{rel} = p_r\hat{r} + \frac{L}{r}\hat{\theta}$$
where p_r is the momentum along the line connecting the two atoms, L is the angular momentum, and r is the distance between the atoms. The unit vectors point along the line connecting the atoms ($\hat{r}$) and perpendicular to that direction in the plane of the rotation of the molecule ($\hat{\theta}$). What, if any, advantage is there to writing the momentum this way? Explain.

3. Recall the lab on the time development of a free particle with an initial wave packet that was a Gaussian so $\psi(x) = (2\pi\sigma^2)^{-1/4}\exp[-x^2/(4\sigma^2)]$. How did the nature of the probability density change for $t > 0$? What was the average momentum of the wave packet and how did it change with time? Explain.

4. What is a solid angle?

5. Consider the potential barrier shown below. How would you use the transfer-matrix approach to connect the wave function $\tilde{\psi}_0$ in region 0 to the wave function $\tilde{\psi}_3$ in region 3? Give your answer in the appropriate notation used in class for the discontinuity and propagation matrices shown below. What is the form of the wave number k in each region in terms of E, V_i, and any other constants?



Problems. Write your solutions on a separate sheet and clearly show all work for full credit.

1. (15 pts.) A harmonic oscillator consists of a mass $m = 2\ g$ on a spring. Its frequency is $\nu = 1.0\ Hz$ and the mass passes through the equilibrium position with a velocity $v = 0.1\ m/s$. What is the order of magnitude of the quantum number associated with the energy of the system?

2. (20 pts.) A water molecule consists of an oxygen atom ($m_O = 16u$) with two hydrogen atoms each of mass $m_H = 1u$ bound to it. The angle between the two bonds is $\theta = 106°$. Put the origin at the position of the oxygen atom. If the bonds are 0.96 Å long, then where is the center of mass?

3. (25 pts.) Recall our old friends, Newton's Second Law, $\vec{F} = m\vec{a}$ and Hooke's Law, $|\vec{F}| = -Kx$. They can be combined with a friction force $\vec{F}_f = -b\vec{v}$ to form a differential equation
$$m\frac{d^2x}{dt^2} = -Kx - bv \qquad \text{or} \qquad \frac{d^2x}{dt^2} + \beta^2\frac{dx}{dt} + \omega_0^2 x = 0$$
where $\omega_0 = \sqrt{\frac{K}{m}}$ and $\beta = \sqrt{\frac{b}{m}}$. Solve this differential equation using the Method of Frobenius and generate a recursion relationship. Discuss all the information you need to form a complete solution.

Physics 309 Equations and Constants

$$E = h\nu = \hbar\omega \qquad v_{wave} = \lambda\nu \qquad I \propto |\vec{E}|^2 \qquad \lambda = \frac{h}{p} \qquad p = \hbar k \qquad K = \frac{1}{2}mv^2 = \frac{p^2}{2m} \quad E = \frac{\hbar^2 k^2}{2m}$$

$$-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\Psi(x,t)+V(x)\Psi(x,t) = i\hbar\frac{\partial}{\partial t}\Psi(x,t) \quad \hat{p}_{\ x} = -i\hbar\frac{\partial}{\partial x} \quad \hat{A}\ |\phi\rangle = a|\phi\rangle \quad \langle \hat{A}\ \rangle = \int_{-\infty}^{\infty}\psi^* \hat{A}\ \psi dx$$

$$\langle\phi_{n'}|\phi_n\rangle = \int_{-\infty}^{\infty}\phi_n^*\phi\, dx = \delta_{n',n} \quad \langle\phi(k')|\phi(k)\rangle = \int_{-\infty}^{\infty}\phi_{k'}^*\phi_k\, dx = \delta(k-k') \quad e^{i\phi} = \cos\phi + i\sin\phi$$

$$|\psi\rangle = \sum b_n|\phi_n\rangle \rightarrow b_n = \langle\phi_n|\psi\rangle \qquad |\phi\rangle = e^{\pm ikt} \qquad |\psi\rangle = \int b(k)|\phi(k)\rangle dk \rightarrow b(k) = \langle\phi(k)|\psi\rangle$$

$$|\psi(t)\rangle = \sum b_n|\phi_n\rangle e^{-i\omega t} \quad |\psi(t)\rangle = \int b(k)|\phi(k)\rangle e^{-i\omega(k)t}dk \quad \Delta p\Delta x \geq \frac{\hbar}{2} \qquad (\Delta x)^2 = \langle x^2\rangle - \langle x\rangle^2$$

The wave function, $\Psi(\vec{r},t)$, contains all we know of a system and its square is the probability of finding the system in the region $\vec{r}$ to $\vec{r}+d\vec{r}$. The wave function and its derivative are (1) *finite*, (2) *continuous*, and (3) *single-valued* ($\psi_1(a) = \psi_2(a)$ and $\psi_1'(a) = \psi_2'(a)$) .

$$V_{HO} = \frac{\kappa x^2}{2} \quad \omega = 2\pi\nu = \frac{2\pi}{T} = \sqrt{\frac{\kappa}{m}} \quad E_n = (n+\frac{1}{2})\hbar\omega_0 = \hbar\omega \quad |\phi_n\rangle = e^{-\xi^2/2}H_n(\xi) \quad \xi = \beta x \quad \beta^2 = \frac{m\omega_0}{\hbar}$$

$$\tilde{\psi}_1 = \mathbf{t}\psi_3 = \mathbf{d_{12}p_2d_{21}p_1^{-1}}\tilde{\psi}_3 \quad \mathbf{d_{nm}} = \frac{1}{2}\begin{pmatrix} 1+\frac{k_m}{k_n} & 1-\frac{k_m}{k_n} \\ 1-\frac{k_m}{k_n} & 1+\frac{k_m}{k_n}\end{pmatrix} \quad k = \sqrt{\frac{2m(E-V)}{\hbar^2}}$$

$$\mathbf{p_n} = \begin{pmatrix} e^{-ik_n2a} & 0 \\ 0 & e^{+ik_n2a}\end{pmatrix} \quad \mathbf{p_n^{-1}} = \begin{pmatrix} e^{ik_n2a} & 0 \\ 0 & e^{-ik_n2a}\end{pmatrix} \quad T = \frac{\text{transmitted flux}}{\text{incident flux}} = \frac{1}{|t_{11}|^2}$$

$$R = \frac{\text{reflected flux}}{\text{incident flux}} \quad R+T = 1 \quad \text{flux} = |\psi|^2 v \quad V(r) = \frac{Z_1Z_2e^2}{r} \quad E = \frac{1}{2}\mu v^2 + V(r)$$

$$\frac{dN_s}{dt} = \frac{d\sigma}{d\Omega}\frac{dN_{inc}}{dt}n_{tgt}d\Omega \quad n_{tgt} = \frac{\rho_{tgt}}{A_{tgt}}N_A\frac{V_{hit}}{a_{beam}} = \frac{\rho_{tgt}}{A_{tgt}}N_A L_{tgt} \quad d\Omega = \frac{dA}{r^2} \quad \frac{dN_{inc}}{dt} = \frac{I_{beam}}{Ze}$$

$$\mu = \frac{m_1 m_2}{m_1 + m_2} \quad \langle K \rangle = \frac{3}{2}kT \quad n(v) = 4\pi N \left(\frac{m}{2\pi k_B T}\right)^{3/2} v^2 e^{-mv^2/2k_B T} \quad \vec{L} = \vec{r} \times \vec{p} = \mathcal{I}\vec{\omega}$$

$$\vec{R}_{cm} = \frac{\sum m_i \vec{r}_i}{\sum m_i} \quad V_{coul} = \frac{Z_1 Z_2 e^2}{r} \quad ME = \frac{p_r^2}{2\mu} + \frac{L^2}{2\mu r^2} + V(r)$$

Speed of light (c)	$2.9979 \times 10^8\ m/s$	fermi (fm)	$10^{-15}\ m$
Boltzmann constant (k_B)	$1.381 \times 10^{-23}\ J/K$	angstrom (Å)	$10^{-10}\ m$
	$8.62 \times 10^{-5}\ eV/k$	electron-volt (eV)	$1.6 \times 10^{-19}\ J$
Planck constant (h)	$6.621 \times 10^{-34}\ J-s$	MeV	$10^6\ eV$
	$4.1357 \times 10^{-15}\ eV-s$	GeV	$10^9\ eV$
Planck constant ($\hbar$)	$1.0546 \times 10^{-34}\ J-s$	Electron charge (e)	$1.6 \times 10^{-19}\ C$
	$6.5821 \times 10^{-16}\ eV-s$	e^2	$\hbar c/137$
Planck constant ($\hbar c$)	$197\ MeV-fm$	Electron mass (m_e)	$9.11 \times 10^{-31}\ kg$
	$1970\ eV-$Å		$0.511\ MeV/c^2$
Proton mass (m_p)	$1.67 \times 10^{-27} kg$	atomic mass unit (u)	$1.66 \times 10^{-27}\ kg$
	$938\ MeV/c^2$		$931.5\ MeV/c^2$
Neutron mass (m_n)	$1.68 \times 10^{-27}\ kg$		
	$939\ MeV/c^2$		

$$\frac{df}{du} = \frac{df}{dx}\frac{du}{dx} \quad \frac{d}{dx}(x^n) = nx^{n-1} \quad \frac{d}{dx}(\sin x) = \cos x \quad \frac{d}{dx}(\cos x) = -\sin x \quad \frac{d}{dx}(e^{ax}) = ae^{ax}$$

$$\frac{d}{dx}(\ln ax) = \frac{1}{x} \quad \int x^n dx = \frac{x^{n+1}}{n+1} \quad \int e^{ax} dx = \frac{e^{ax}}{a} \quad \int \frac{1}{x} = \ln x \quad \int \frac{1}{\sqrt{x^2+a^2}} dx = \ln\left[x + \sqrt{x^2+a^2}\right]$$

$$\int \frac{x}{\sqrt{x^2+a^2}} dx = \sqrt{x^2+a^2} \quad \int \frac{x^2}{\sqrt{x^2+a^2}} dx = \frac{1}{2}x\sqrt{x^2+a^2} - \frac{1}{2}a^2 \ln\left[x + \sqrt{x^2+a^2}\right]$$

$$\sin x = \frac{e^{ix} - e^{-ix}}{2i} \quad \cos x = \frac{e^{ix} + e^{-ix}}{2} \quad \sinh x = \frac{e^x - e^{-x}}{2} \quad \cosh x = \frac{e^x + e^{-x}}{2}$$

Hermite polynomials ($H_n(\xi)$)

$$H_0(\xi) = \frac{1}{\sqrt{\sqrt{\pi}}}$$

$$H_1(\xi) = \frac{1}{\sqrt{2\sqrt{\pi}}}2\xi$$

$$H_2(\xi) = \frac{1}{\sqrt{8\sqrt{\pi}}}(4\xi^2 - 2)$$

$$H_3(\xi) = \frac{1}{\sqrt{48\sqrt{\pi}}}(8\xi^3 - 12\xi)$$

$$H_4(\xi) = \frac{1}{\sqrt{384\sqrt{\pi}}}(16\xi^4 - 48\xi^2 + 12)$$

$$H_5(\xi) = \frac{1}{\sqrt{3840\sqrt{\pi}}}(32\xi^5 - 160\xi^3 + 120\xi)$$

$$H_6(\xi) = \frac{1}{\sqrt{46080\sqrt{\pi}}}(64\xi^6 - 480\xi^4 + 720\xi^2 - 120)$$