Physics 309 Test 1

I pledge that I have given nor received unauthorized assistance during the completion of this work.
Signature: ____

Questions (8 pts. apiece) Answer questions in complete, well-written sentences WITHIN the spaces provided.

1. Cite at least two observations or experimental results that motivated the development of quantum mechanics and why, *i.e.*, what problem did they fix?

2. What is the quantum program? Be succinct.

3. The time development of a wave packet is written as $\Psi(x,t) = (\sum b_n \phi_n) e^{-i\omega t}$. True or False? Explain.

4. Consider the function.

$$f(x) = \sqrt{\frac{2b^2}{\pi}}\; e^{-2x^2b^2}$$

   What is the width of this distribution in terms of parameters in $f(x)$ or any other constants? Explain.

DO NOT WRITE BELOW THIS LINE.

5. In quantum mechanics, one may picture a wave function in either momentum space (the spectral distribution) or in configuration space (the spatial distribution or $\psi^*\psi$). If the correct wave function in configuration space is $\psi(x) = N/(x^2 + \alpha^2)$, then what is the wave function in momentum space (aside from a multiplicative constant) from the choices below. Explain your reasoning.

a.	$e^{-\alpha^2 k^2/\hbar^2}$	d.	$e^{-\alpha x}$
b.	$\cos(px/\hbar)$	e.	e^{ikx}
c.	$\sin(kx/\hbar$		

Problems. Clearly show all work on a separate piece of paper for full credit.

Note 1: If you use *Mathematica* you have to reference it to get credit for the work.

Note 2: If you encounter integrals that you are unable to solve label them I_1, I_2, ... and so on and carry that notation through to the solution.

1. (15 pts.) The work function of aluminum is $W = 4.1\ eV$. What is the energy of the most energetic photoelectron emitted by ultraviolet light of wavelength $\lambda = 2500$ Å?

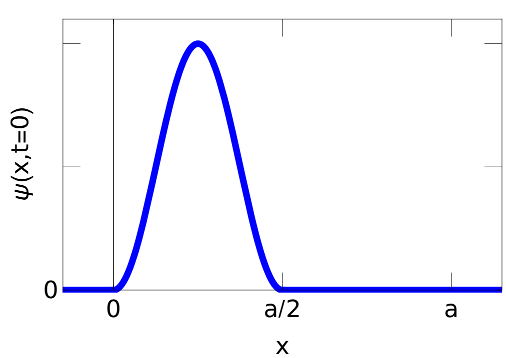
2. (20 pts.) The eigenfunctions and eigenvalues of the particle in a box with walls at $x = 0$ and $x = a$ are the following

$$\begin{aligned} \phi(x) &= \sqrt{\tfrac{2}{a}} \sin\left[\tfrac{n\pi x}{a}\right] \quad 0 < x < a \\ &= 0 \quad \text{otherwise} \end{aligned} \qquad E_n = \frac{n^2\hbar^2\pi^2}{2m_p a^2}$$

where m_p is the particle mass. At $t = 0$, the state of the particle is

$$\begin{aligned} \phi(x) &= A_0 \sin^2\left(\tfrac{\pi x}{a}\right) \quad 0 < x < a/2 \\ &= 0 \quad \text{otherwise} \end{aligned}$$

as shown in the figure and where A_0 is a known normalization factor. What is the probability the particle will have energy E_5 at $t = 0$? Get your answer in terms of the parameters above, *i.e.* get the formula. Don't plug in numbers. Clearly show all work.



3. (25 pts.) A wave packet consists of one thousand electrons moving in a region with $V = 0$. At $t = 0$, each electron is in the state

$$\psi(x, t = 0) = \alpha x^2 e^{-\beta x^2}$$

where α and β are constants and ψ is properly normalized.

1. What is the probability of finding an electron at the origin?
2. What is the spectral distribution?
3. What is the momentum distribution?

Get your answer in terms of the parameters above. Clearly show all work.

Physics 309 Equations

$$R_T(\nu) = \frac{Energy}{time \times area} \quad E = h\nu = \hbar\omega \quad v_{wave} = \lambda\nu \quad I \propto |\vec{E}|^2 \quad K_{max} = h\nu - W \quad K = \frac{p^2}{2m} = \frac{\hbar^2 k^2}{2m}$$

$$\lambda = \frac{h}{p} \quad p = \hbar k \quad -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\Psi(x,t) + V(x)\Psi(x,t) = i\hbar\frac{\partial}{\partial t}\Psi(x,t) \quad \hat{p}_{\ x} = -i\hbar\frac{\partial}{\partial x}$$

$$\hat{A}\ |\phi\rangle = a|\phi\rangle \quad \langle \hat{A}\ \rangle = \int_{-\infty}^{\infty} \psi^* \hat{A}\ \psi dx \quad |\phi\rangle = \frac{e^{\pm ikx}}{\sqrt{2\pi}} \quad E = \frac{\hbar^2 k^2}{2m} \quad |\phi\rangle = \sqrt{\frac{2}{a}}\sin\left(\frac{n\pi x}{a}\right) \quad E_n = \frac{n^2\hbar^2\pi^2}{2ma^2}$$

$$\langle\phi_{n'}|\phi_n\rangle = \int_{-\infty}^{\infty} \phi_{n'}^* \phi_n\ dx = \delta_{n',n} \quad \langle\phi(k')|\phi(k)\rangle = \int_{-\infty}^{\infty} \phi_{k'}^* \phi_k\ dx = \delta(k - k')$$

$$|\psi\rangle = \sum b_n|\phi_n\rangle \to b_n = \langle\phi_n|\psi\rangle \quad |\psi\rangle = \int b(k)|\phi(k)\rangle dk \to b(k) = \langle\phi(k)|\psi\rangle \quad |\psi(t)\rangle = \sum b_n|\phi_n\rangle e^{-i\omega_n t}$$

$$\Delta p \Delta x \propto \hbar \quad (\Delta x)^2 = \langle x^2\rangle - \langle x\rangle^2 \quad \langle f(x)\rangle = \int_{-\infty}^{\infty} f(x)P(x)dx \quad \langle f_n\rangle = \sum_{n=0}^{\infty} f_n P_n$$

$$\text{If } f(x) = \sqrt{\frac{1}{2\pi\sigma^2}}\ e^{-x^2/2\sigma^2}, \text{ then } \Delta x = \sigma$$

$$e^{ix} = \cos x + i\sin x \quad \sin x = \frac{e^{ix} - e^{-ix}}{2i} \quad \cos x = \frac{e^{ix} + e^{-ix}}{2} \quad \lim_{x\to c}\frac{f(x)}{g(x)} = \lim_{x\to c}\frac{f'(x)}{g'(x)}$$

The wave function, $\psi(\vec{r},t)$, contains all we know of a system and $|\psi|^2$ is the probability of finding it in the region $\vec{r}$ to $\vec{r}+d\vec{r}$. The wave function and its derivative are (1) *finite*, (2) *continuous*, and (3) *single-valued*.

$$\ln(ab) = \ln a + \ln b \quad \ln\left(\frac{a}{b}\right) = \ln a - \ln b \quad \ln x^n = n \ln x \quad \ln(e^a) = a \quad e^{\ln a} = a$$

$$\frac{df}{du} = \frac{df}{dx}\frac{du}{dx} \quad \frac{d}{dx}(x^n) = nx^{n-1} \quad \frac{d}{dx}(\sin x) = \cos x \quad \frac{d}{dx}(\cos x) = -\sin x \quad \frac{d}{dx}(e^{ax}) = ae^{ax}$$

$$\frac{d}{dx}(\ln x) = \frac{1}{x} \quad \frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx} \quad \int x^n dx = \frac{x^{n+1}}{n+1} \quad \int e^{ax}dx = \frac{e^{ax}}{a} \quad \int \frac{1}{x} = \ln x$$

$$\int \frac{1}{\sqrt{x^2+a^2}}dx = \ln\left[x + \sqrt{x^2+a^2}\right] \quad \int \frac{x}{\sqrt{x^2+a^2}}dx = \sqrt{x^2+a^2}$$

$$\int \frac{x^2}{\sqrt{x^2+a^2}}dx = \frac{1}{2}x\sqrt{x^2+a^2} - \frac{1}{2}a^2 \ln\left[x + \sqrt{x^2+a^2}\right] \quad \int \frac{x^3}{\sqrt{x^2+a^2}}dx = \frac{1}{3}(-2a^2+x^2)\sqrt{x^2+a^2}$$

$$\int \cos(Ax)dx = \frac{\sin(Ax)}{A} \quad \int \cos(Ax)x dx = \frac{\cos(Ax)}{A^2} + \frac{x\sin(Ax)}{A}$$

$$\int \sin(Ax)dx = -\frac{\cos(Ax)}{A} \quad \int \sin(Ax)x dx = \frac{\sin(Ax)}{A^2} - \frac{x\cos(Ax)}{A}$$

$$\int \sin(Ax)x^2 dx = \frac{2x\sin(Ax)}{A^2} - \frac{(A^2x^2-2)\cos(Ax)}{A^3} \quad \int \cos(Ax)x^2 dx = \frac{2x\cos(Ax)}{A^2} + \frac{(A^2x^2-2)\sin(Ax)}{A^3}$$

$$\int_a^b \sin(Ax)dx = \frac{\cos(aA) - \cos(Ab)}{A} \quad \int_a^b \sin(Ax)x dx = \frac{-\sin(aA) + aA\cos(aA) + \sin(Ab) - Ab\cos(Ab)}{A^2}$$

$$\int_a^b \cos(Ax)dx = \frac{\sin(Ab) - \sin(aA)}{A} \quad \int_a^b \cos(Ax)x dx = \frac{-aA\sin(aA) - \cos(aA) + Ab\sin(Ab) + \cos(Ab)}{A^2}$$

$$\int_{-\infty}^{\infty} x^2 \exp(-ax - bx^2)dx = \frac{\sqrt{\pi}}{4b^{5/2}}(a^2+2b)\exp\left(\frac{a^2}{4b}\right)$$

$$\int \sin(Ax)\sin(Bx)dx = \frac{\sin(x(A-B))}{2(A-B)} - \frac{\sin(x(A+B))}{2(A+B)}$$

$$\int \cos(Ax)\cos(Bx)dx = \frac{\sin(x(A-B))}{2(A-B)} + \frac{\sin(x(A+B))}{2(A+B)}$$

$$\int \sin(Ax)\cos(Bx)dx = -\frac{\cos(x(A-B))}{2(A-B)} - \frac{\cos(x(A+B))}{2(A+B)}$$

$$\int \sin Ax \sin^2 Bx dx = \frac{1}{4}\left(-\frac{2\cos Ax}{A} + \frac{cos(A-2B)x}{A-2B} + \frac{\cos(A+2B)x}{A+2B}\right)$$

Physics 309 Conversions, and Constants

Speed of light (c)	$2.9979 \times 10^8\ m/s$	fermi (fm)	$10^{-15}\ m$
Boltzmann constant (k_B)	$1.381 \times 10^{-23}\ J/K$	angstrom (Å)	$10^{-10}\ m$
	$8.62 \times 10^{-5}\ eV/K$	electron-volt (eV)	$1.6 \times 10^{-19}\ J$
Planck constant (h)	$6.621 \times 10^{-34}\ J-s$	MeV	$10^6\ eV$
	$4.1357 \times 10^{-15}\ eV-s$	GeV	$10^9\ eV$
Planck constant ($\hbar$)	$1.0546 \times 10^{-34}\ J-s$	Electron charge (e)	$1.6 \times 10^{-19}\ C$
	$6.5821 \times 10^{-16}\ eV-s$	e^2	$\hbar c/137$
Planck constant ($\hbar c$)	$197\ MeV-fm$	Electron mass (m_e)	$9.11 \times 10^{-31}\ kg$
	$1970\ eV-$ Å		$0.511\ MeV/c^2$
Proton mass (m_p)	$1.67 \times 10^{-27} kg$	atomic mass unit (u)	$1.66 \times 10^{-27}\ kg$
	$938\ MeV/c^2$		$931.5\ MeV/c^2$
Neutron mass (m_n)	$1.68 \times 10^{-27}\ kg$		
	$939\ MeV/c^2$		