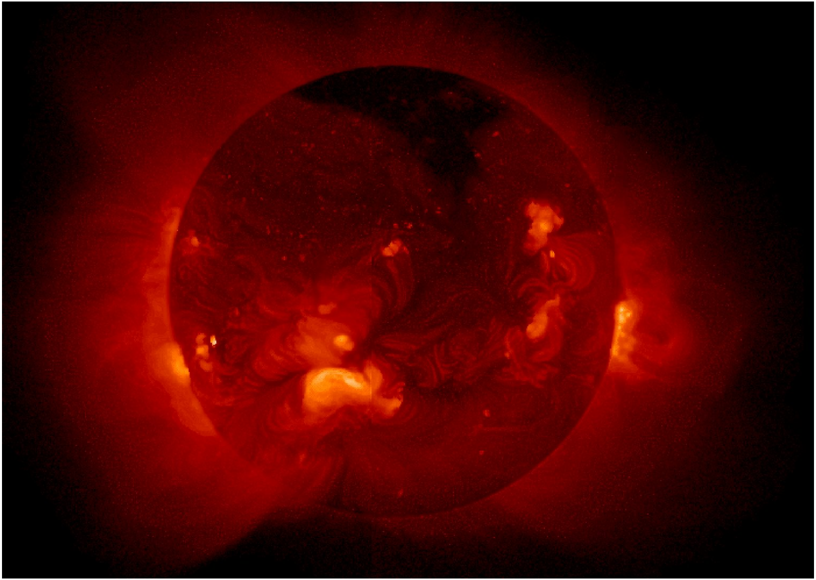
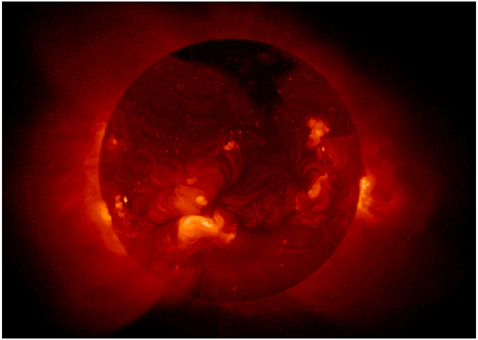
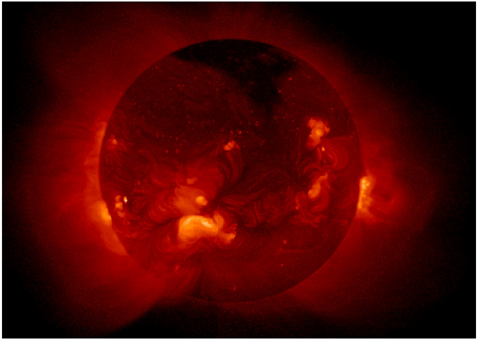


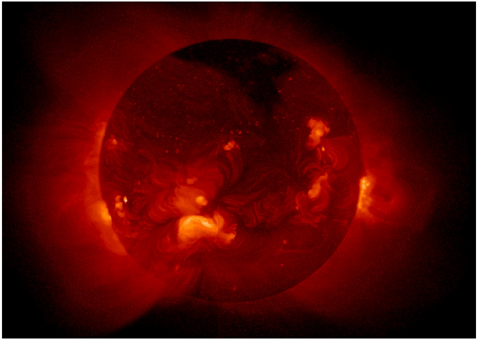
1. What is the energy production of the Sun ($G_{SC} = 1.36\ kW/m^2$)?
2. How is the energy generated? The Sun has lots of protons. Would $\mathrm{H} + \mathrm{H} \rightarrow \mathrm{H}_2$ work ($\Delta E = 4.48\ \mathrm{eV/rxn}$)?
3. How long would the Sun ($M_{Sun} = 2 \times 10^{30}\ kg$) last making $\mathrm{H_2O}$?

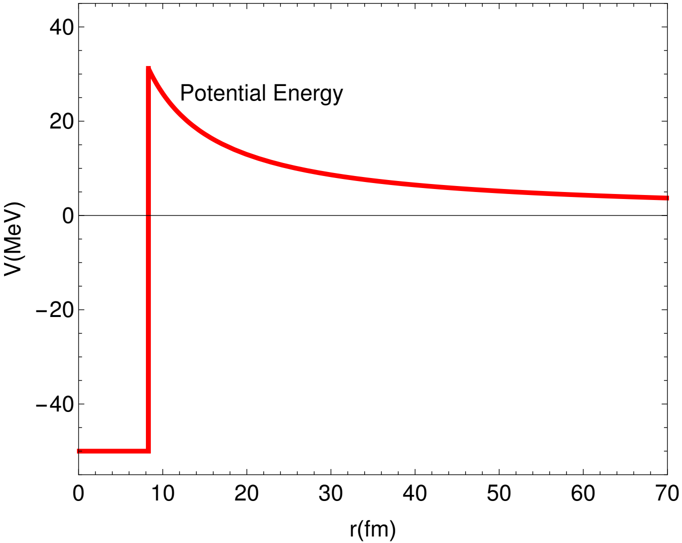


1. Would $pp \rightarrow d + \beta^+ + \nu_e$, $\Delta E = 1.442\ \mathrm{MeV}$ work?



1. Would $pp \to d + \beta^+ + \nu_e$, $\Delta E = 1.442\ \mathrm{MeV}$ work?
2. How close must protons approach each other for fusion to occur?




Potential Energy
40
20
0
−20
−40
V(MeV)
0
10
20
30
40
50
60
70
r(fm)

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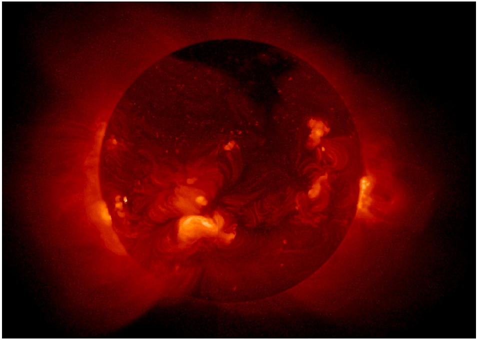

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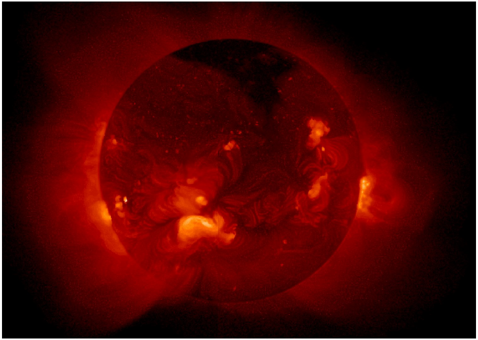
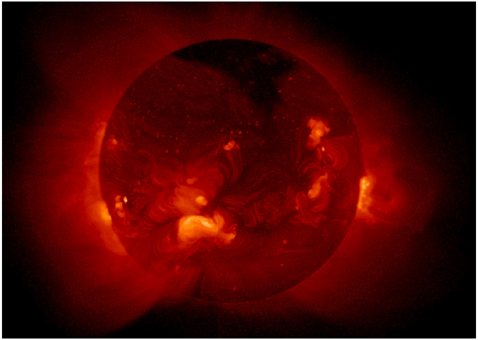
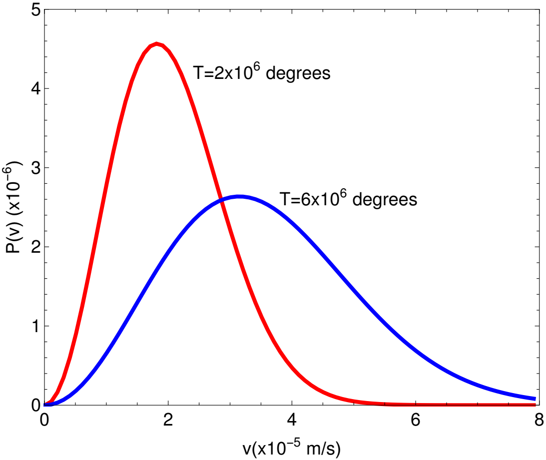


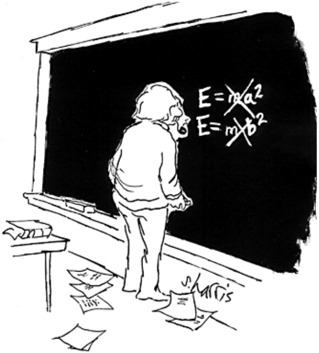
1. Would $pp \rightarrow d + \beta^+ + \nu_e$, $\Delta E = 1.442\ \mathrm{MeV}$ work?
2. How close must protons approach each other for fusion to occur?
3. Do the protons in the Sun have enough energy, on average, to overcome the Coulomb barrier? $(T_{Sun} = 2 \times 10^6\ K)$
4. Would protons in the high-velocity tail of the Maxwellian distribution have enough energy to overcome the barrier?



$$P(v)d\vec{v} = 4\pi \left(\frac{m}{2\pi k_B T}\right)^{3/2} v^2 e^{-mv^2/2k_B T} dv$$



1. Our Sun generates enormous power $P_{Sun} = 3.8 \times 10^{26}\ J/s$.
2. The power source was utterly unknown until Einstein discovered his famous result $E = mc^2$.
3. And Rutherford discovered nuclear physics.
4. Even then, statistical mechanics told us the chances of the reaction $p + p \rightarrow d + \beta^+ + \nu_e$ occurring were small because of the height of the Coulomb barrier.
5. How can the two protons overcome their mutual repulsion to fuse and release enough energy to power the Sun?



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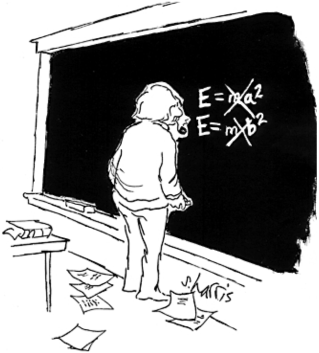
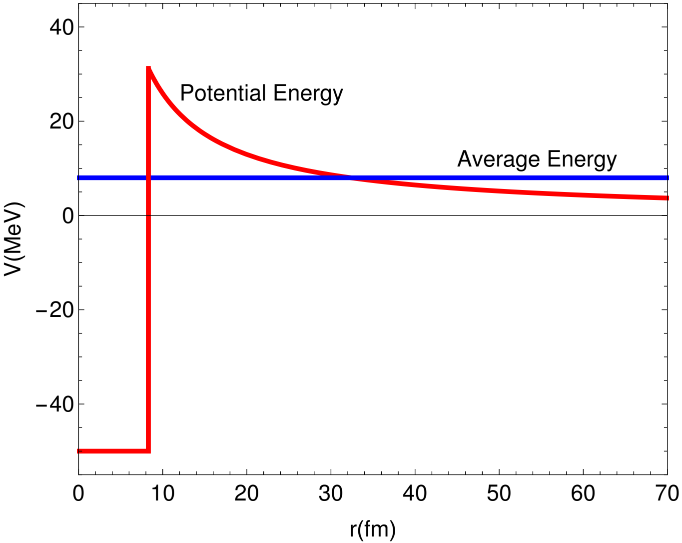
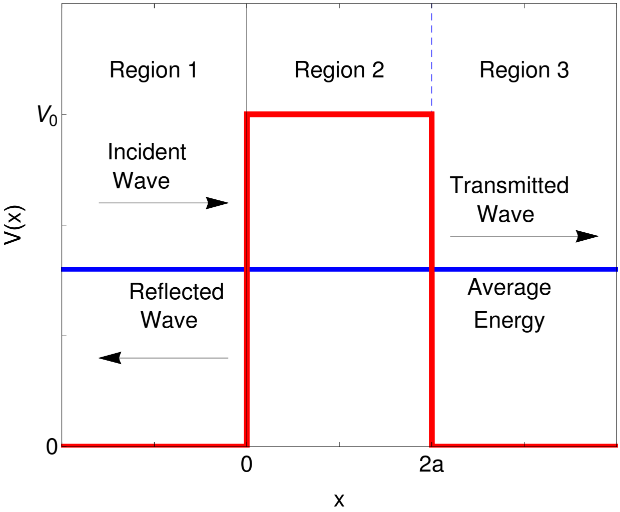


Quantum Tunneling!

The Coulomb Barrier 12



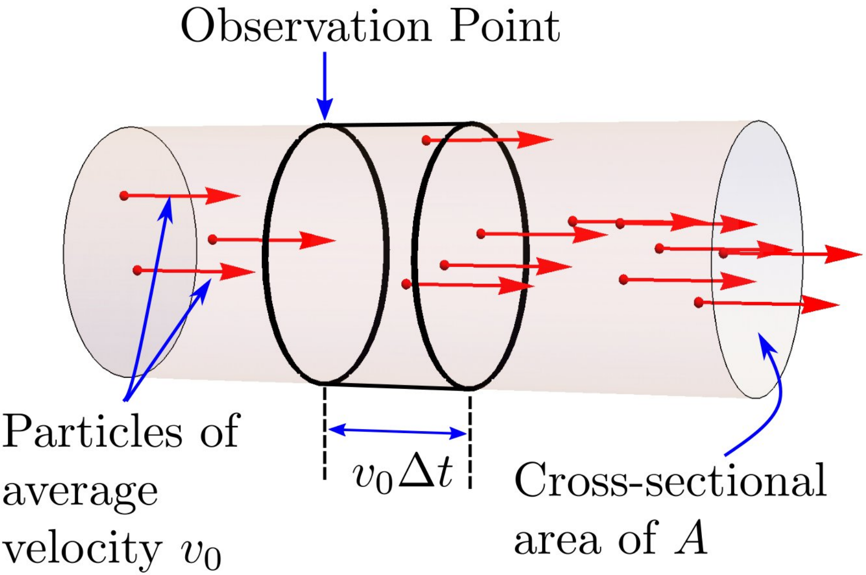

Region 1
Region 2
Region 3
V_0
Incident Wave
Transmitted Wave
V(x)
Reflected Wave
Average Energy
0
0
2a
x

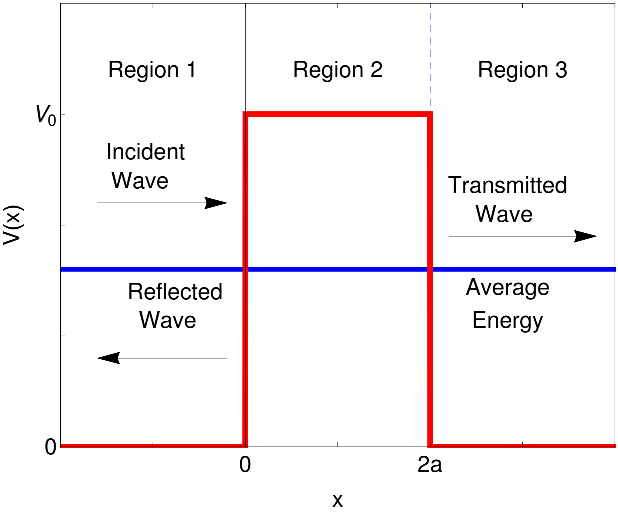
1. Approximate the Coulomb barrier with a rectangular barrier.
2. Develop the notion of particle flux or flow.
3. Solve the Schroedinger equation for the rectangular barrier potential.
4. Determine the flux penetrating the barrier.
5. Calculate the probability of the following reaction occurring.

$$p + p \rightarrow d + \beta^{+} + \nu_e \qquad \Delta E = 1.442\ MeV$$

6. Compare the results of the previous calculation with the prediction of classical physics using the Maxwellian velocity distribution.

$$P(v)d\vec{v} = 4\pi \left(\frac{m}{2\pi kT}\right)^{3/2} v^2 e^{-mv^2/2kT} dv$$

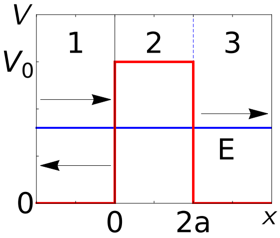

Observation Point
Particles of average velocity v_0
$v_0 \Delta t$
Cross-sectional area of A


Region 1
Region 2
Region 3
V_0
Incident Wave
Transmitted Wave
V(x)
Reflected Wave
Average Energy
0
0
2a
x

1. Each physical, measurable quantity, A, has a corresponding operator, $\hat{A}$, that satisfies the eigenvalue equation $\hat{A}\,\phi = a\phi$ and measuring that quantity yields the eigenvalues of $\hat{A}$.
2. Measurement of the observable A leaves the system in a state that is an eigenfunction of $\hat{A}$.
3. The state of a system is represented by a wave function Ψ which is continuous, differentiable and contains all the information about it.
   - The average value of any observable A is determined by $\langle A \rangle = \int_{all\ space} \Psi^* \hat{A}\, \Psi d\vec{r}$.
   - The 'intensity' is proportional to$|\Psi|^2$.
4. The time development of the wave function is determined by

$$i\hbar \frac{\partial \Psi(\vec{r},t)}{\partial t} = -\frac{\hbar^2}{2\mu}\nabla^2\Psi(\vec{r},t) + V(\vec{r})\Psi(\vec{r},t) \qquad \mu \equiv \text{reduced mass.}$$

The Potential



Eigenfunctions	$e^{\pm i k_1 x}$	$e^{\pm i k_2 x}$	$e^{\pm i k_3 x}$
Wave Numbers	$k_1 = \sqrt{\frac{2mE}{\hbar^2}}$	$k_2 = \sqrt{\frac{2m(E - V_0)}{\hbar^2}}$	$k_3 = k_1$
General Solution	$\phi_{1,2,3} = \mathrm{coeff}_1 \times e^{i k_{1,2} x} + \mathrm{coeff}_2 \times e^{i k_{1,2} x}$		
Coefficients	A, B	C, D	F, G
Boundary Conditions 1	$\phi_1(0) = \phi_2(0)$	$\phi_2(2a) = \phi_3(2a)$	
Boundary Conditions 2	$\frac{\partial \phi_1(0)}{\partial x} = \frac{\partial \phi_2(0)}{\partial x}$	$\frac{\partial \phi_2(2a)}{\partial x} = \frac{\partial \phi_3(2a)}{\partial x}$	

Transfer Matrix method

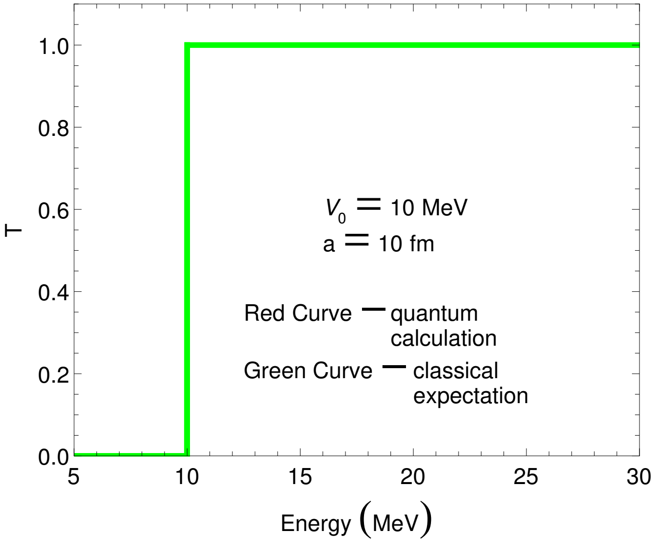
$$\begin{aligned}\tilde{\psi}_1 &= \mathbf{d}_{12}\mathbf{p}_2\mathbf{d}_{21}\mathbf{p}_1^{-1}\tilde{\psi}_3 \\ &= \mathbf{t}\tilde{\psi}_3\end{aligned}$$

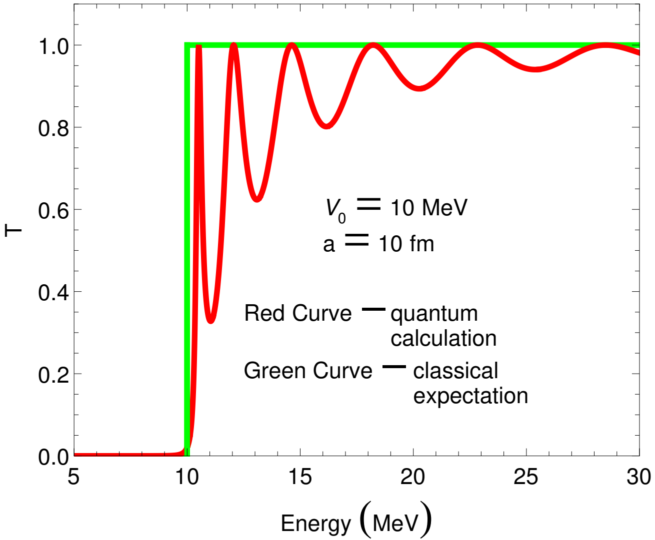
$$\mathbf{d_{12}} = \frac{1}{2}\begin{pmatrix} 1+\frac{k_2}{k_1} & 1-\frac{k_2}{k_1} \\ 1-\frac{k_2}{k_1} & 1+\frac{k_2}{k_1}\end{pmatrix} \qquad \mathbf{d_{21}} = \frac{1}{2}\begin{pmatrix} 1+\frac{k_1}{k_2} & 1-\frac{k_1}{k_2} \\ 1-\frac{k_1}{k_2} & 1+\frac{k_1}{k_2}\end{pmatrix}$$

$$\mathbf{p_2} = \begin{pmatrix} e^{-ik_2 2a} & 0 \\ 0 & e^{+ik_2 2a}\end{pmatrix} \qquad \mathbf{p_1^{-1}} = \begin{pmatrix} e^{+ik_1 2a} & 0 \\ 0 & e^{-ik_1 2a}\end{pmatrix}$$

Flux

$$\text{flux} = |\psi|^2 v_n \quad \text{where } n \text{ is the region}$$


1.0
0.8
0.6
0.4
0.2
0.0
T
5
10
15
20
25
30
Energy (MeV)
V_0 = 10 MeV
a = 10 fm
Red Curve – quantum calculation
Green Curve – classical expectation


1.0
0.8
0.6
0.4
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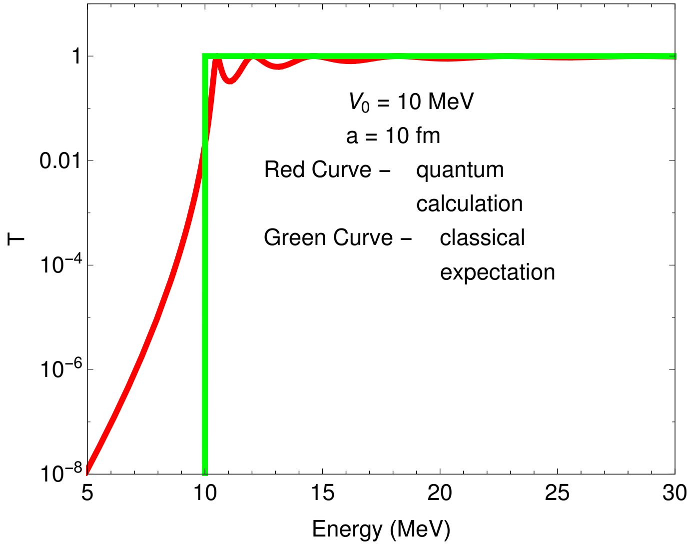
1
0.01
10^{-4}
10^{-6}
10^{-8}
T
5
10
15
20
25
30
Energy (MeV)
V_0 = 10 MeV
a = 10 fm
Red Curve – quantum calculation
Green Curve – classical expectation

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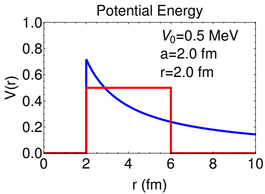
$$P(v)d\vec{v} = 4\pi \left(\frac{m}{2\pi kT}\right)^{3/2} v^2 e^{-mv^2/2kT} dv$$

1. Quantum Tunneling
   1. Pick reasonable values for the barrier.
   2. Get $\langle KE \rangle$ in the solar core:

$$\langle KE \rangle = \frac{3}{2} k_B T \quad \text{where}\, T \approx 10^7 \ k$$

   3. Calculate $T = 1/|t_{11}|^2$ with:



$$t_{11} = \frac{1}{4}\left[\left(1+\frac{k_2}{k_1}\right)e^{-ik_2 2a}\left(1+\frac{k_1}{k_2}\right)+\left(1-\frac{k_2}{k_1}\right)e^{ik_2 2a}\left(1-\frac{k_1}{k_2}\right)\right] \quad \text{and}$$

$$k_n = \sqrt{\frac{2m_p(\langle KE \rangle - V_n)}{\hbar^2}}$$

2. Maxwellian velocity
   1. Get the Coulomb barrier height and the proton velocity.

$$V_{top} = \frac{Z_1 Z_2 e^2}{r} \quad \text{so} \quad v_{top} = \sqrt{\frac{2V_{top}}{m_p}}$$

   2. Integrate the velocity distribution from v_{top}.

$$P = \int_{v_{top}}^{\infty} 4\pi \left(\frac{m}{2\pi kT}\right)^{3/2} v^2 e^{-mv^2/2kT} dv$$

3. Compare.