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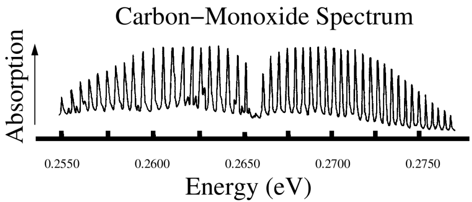
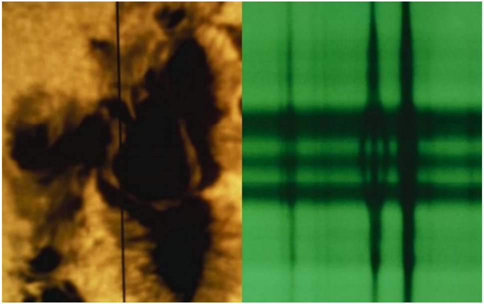


Carbon monoxide rigid rotator



Zeeman effect in the Sun

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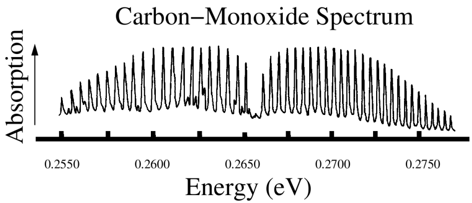

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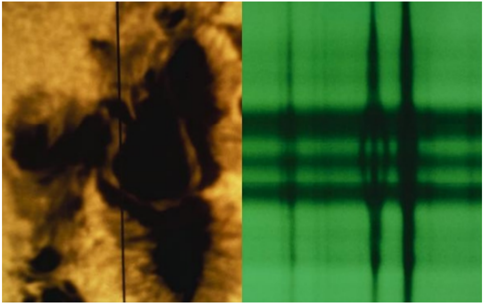

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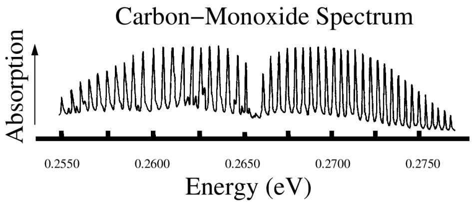

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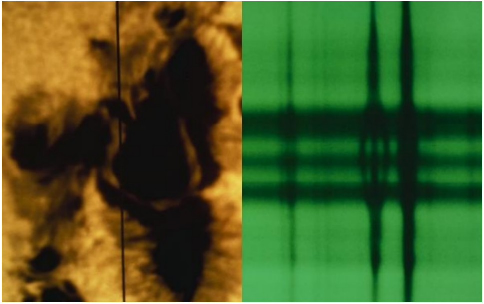

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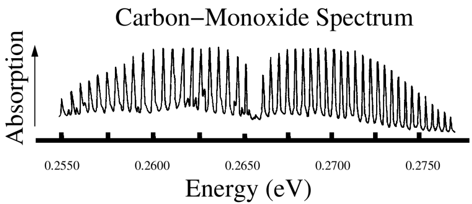
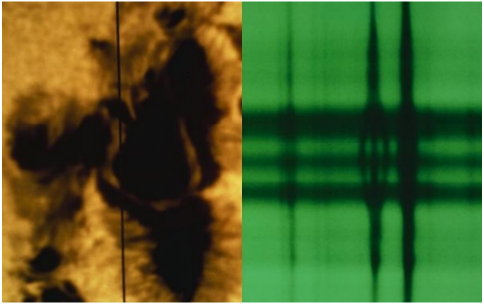
WHY? WHY? WHY?

The Question





To explain the carbon monoxide spectrum and the Zeeman effect we invoked angular momentum selection rules: $\Delta l = \pm 1$, $\Delta m = 0, \pm 1$ to understand light emission from the transitions between atomic energy states.

Where do these selections rules come from?

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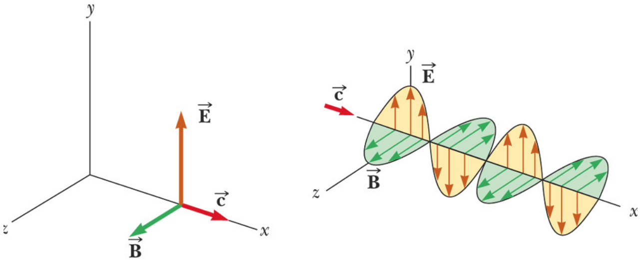
4. If the charge oscillates sinusoidally, then you get 'typical' electromagnetic (EM) waves.



(a) (b)

More on Oscillating Charges

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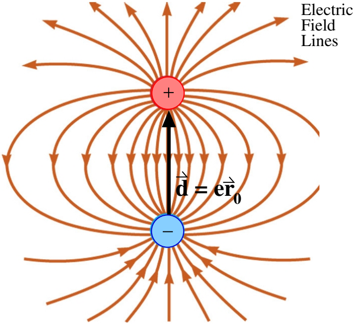

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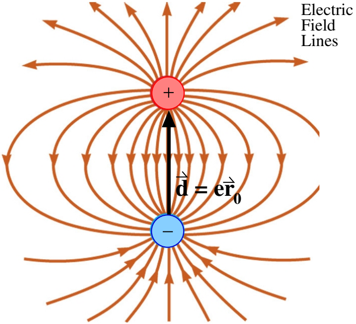

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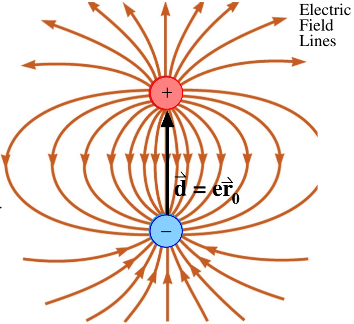
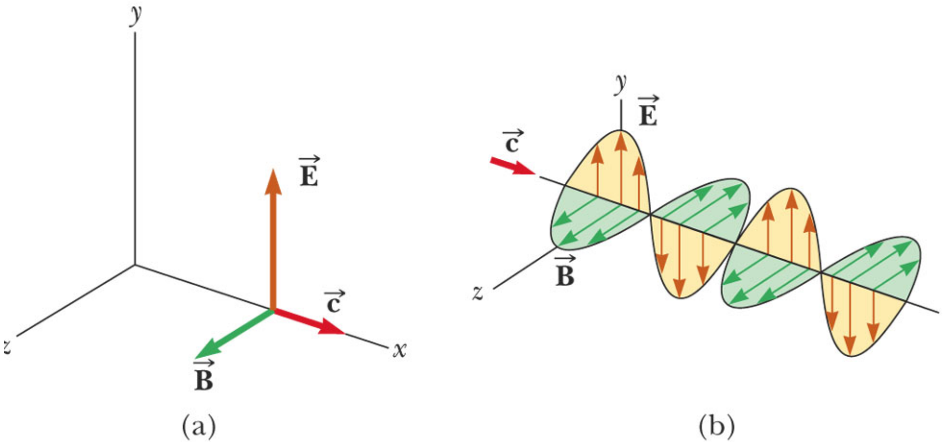
   $$\vec{E} \propto \vec{r}_0 \cos\omega t \rightarrow e\vec{r}_0 \cos\omega t = \vec{d} \cos\omega t$$

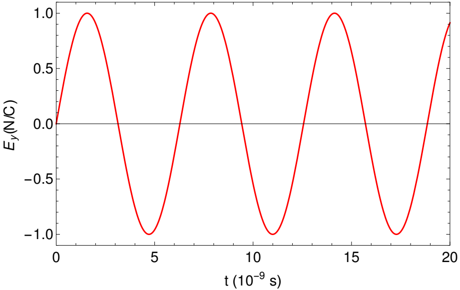


Energy Transfer in an Electromagnetic Wave



$$\vec{S} = \frac{1}{\mu_0}\vec{E} \times \vec{B}$$

Rapidly Oscillating Energy Transfer



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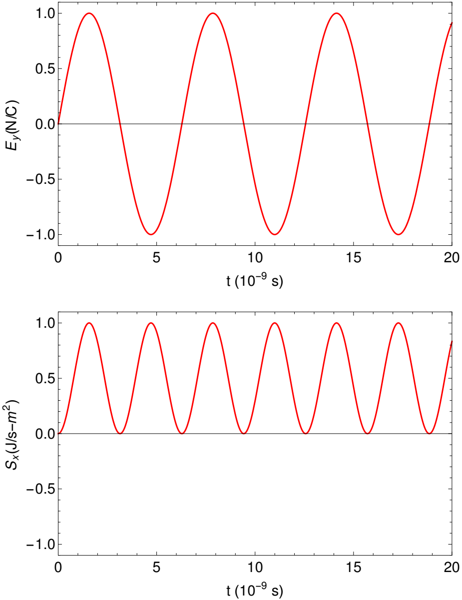

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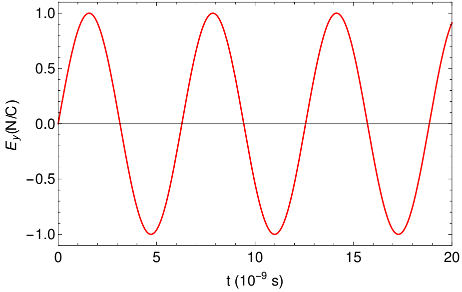

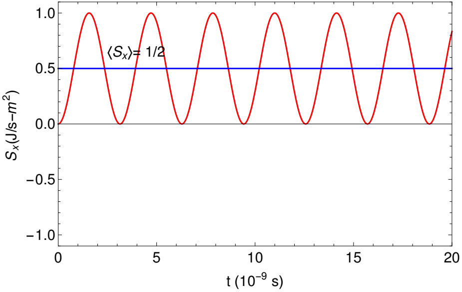

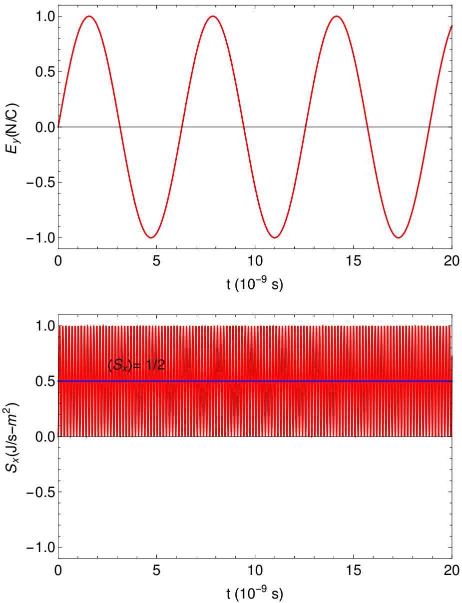
1.0
0.5
0.0
−0.5
−1.0
E_y(N/C)
0
5
10
15
20
t (10^{-9} s)
1.0
⟨S_x⟩= 1/2
0.5
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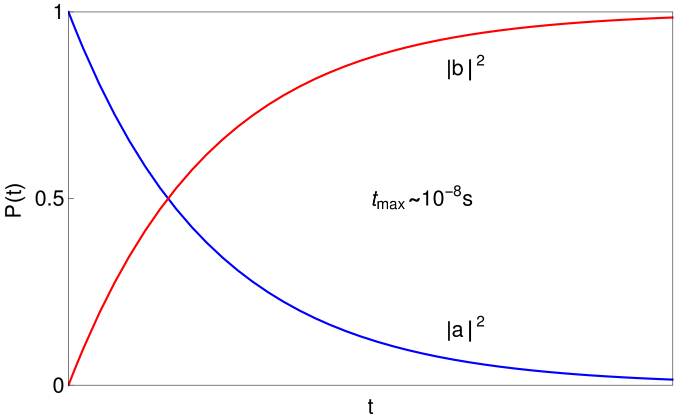
Time Dependence of Coefficients



$$P(t) = |\Psi(\vec{r}, t)|^2 = |\ ae^{iE_n t/\hbar}|nlm\rangle + be^{iE_{n'} t/\hbar}|n'l'm'\rangle\ |^2$$

Some Necessary Math Results

$$P_l^m(\mu) = (-1)^m (1-\mu^2)^{m/2} \frac{d^m}{d\mu^m} P_l(\mu); \qquad P_l^0 = P_l$$

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For these equations, m is taken as ≥ 0. In the formulas below, however, m may be < 0 also; $l = 0, 1, 2, \ldots, |m| \leq l$.

Differential Equation

$$(1-\mu^2)\frac{d^2 P_l^m(\mu)}{d\mu^2} - 2\mu \frac{d P_l^m(\mu)}{d\mu} + \left[l(l+1) - \frac{m^2}{1-\mu^2} \right] P_l^m(\mu) = 0$$

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$$Y_l^m(\theta,\phi) = \left[\frac{2l+1}{4\pi}\frac{(l-m)!}{(l+m)!}\right]^{1/2} P_l^m(\cos\theta)e^{im\phi}$$

$$\int_{-1}^{1} d\cos\theta \int_0^{2\pi} d\phi\, Y_l^m (Y_{l'}{}^{m'})^* = \delta_{mm'}\delta_{ll'}$$

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