The Free Particle Again (Problem 5.4)

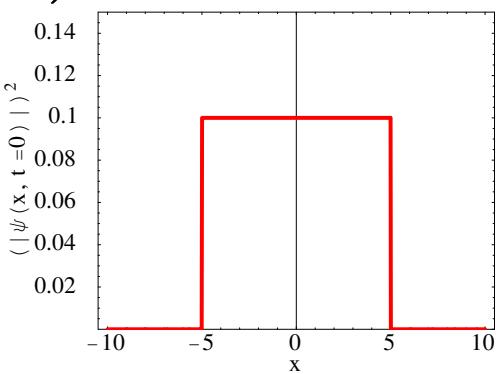
Consider a free particle ($V = 0$) with initial wave function

$$|\Psi(x,0)\rangle = \frac{1}{\sqrt{a}} e^{ik_0 x} \qquad |x| \leq \frac{a}{2}$$

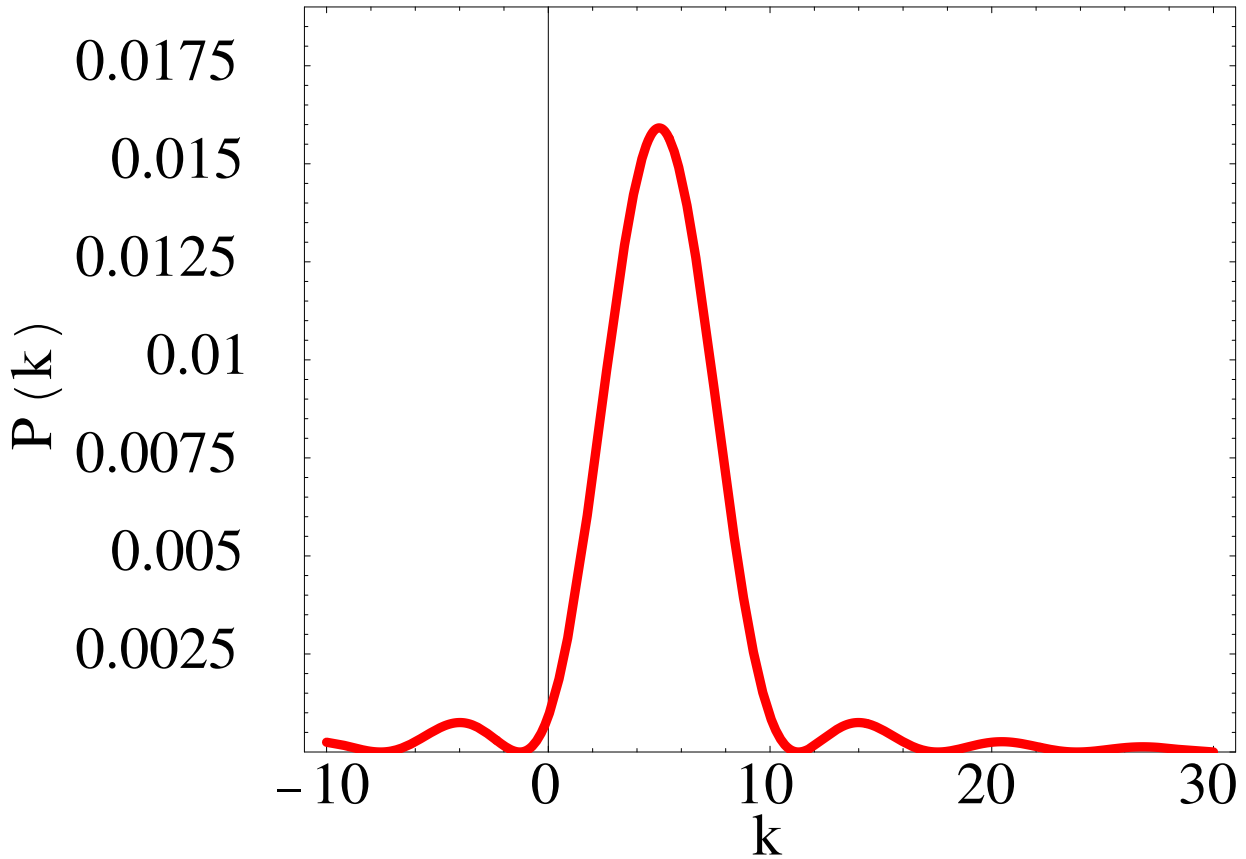
and zero elsewhere where $a = 1.0\ m$. The spectral distribution is then

$$|b(k)|^2 = \left(\sqrt{\frac{a}{2\pi}} \frac{\sin \Delta k a/2}{\Delta k a/2} \right)^2 \qquad \Delta k = k_0 - k$$

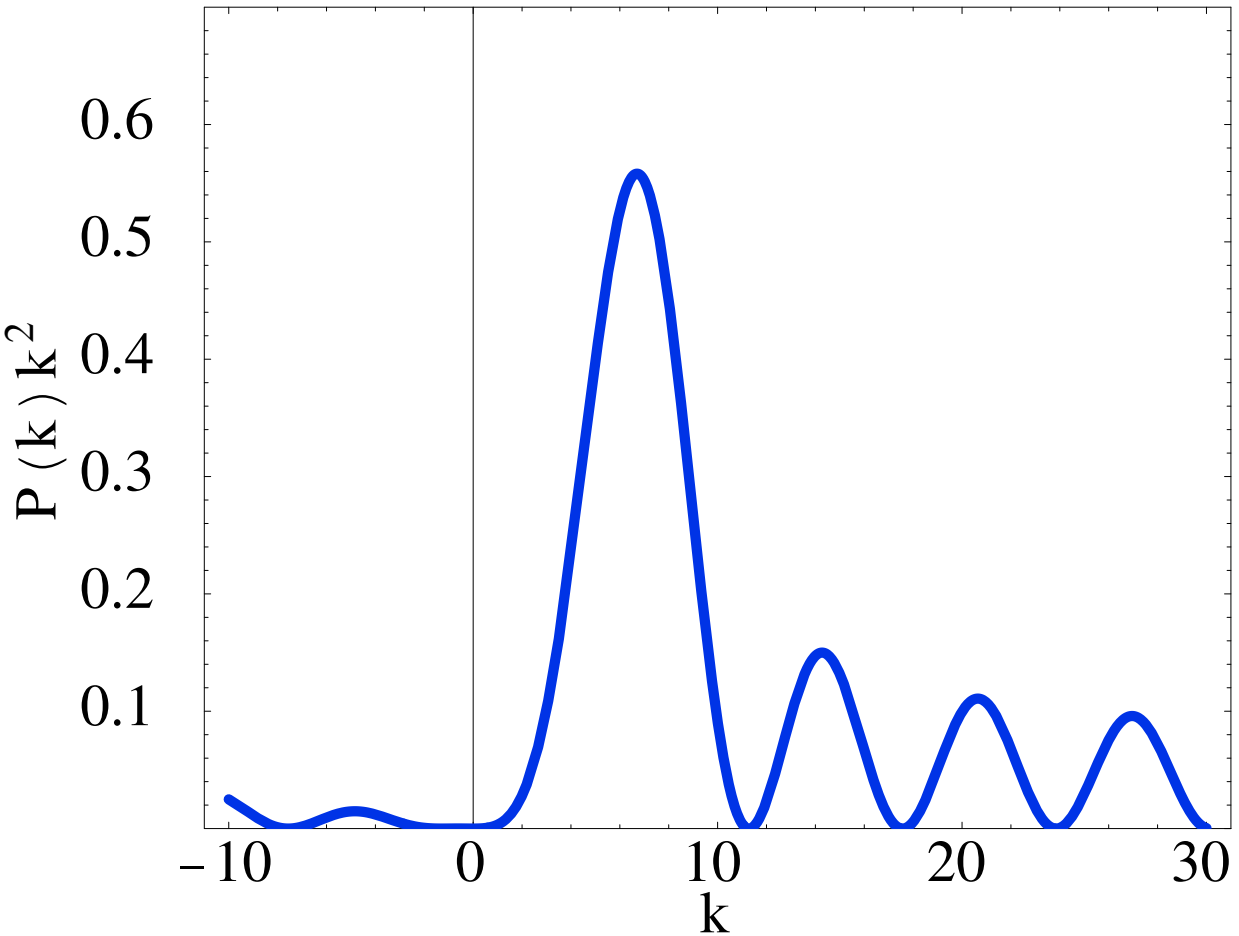
What is $\Delta x \Delta p$ for this state?



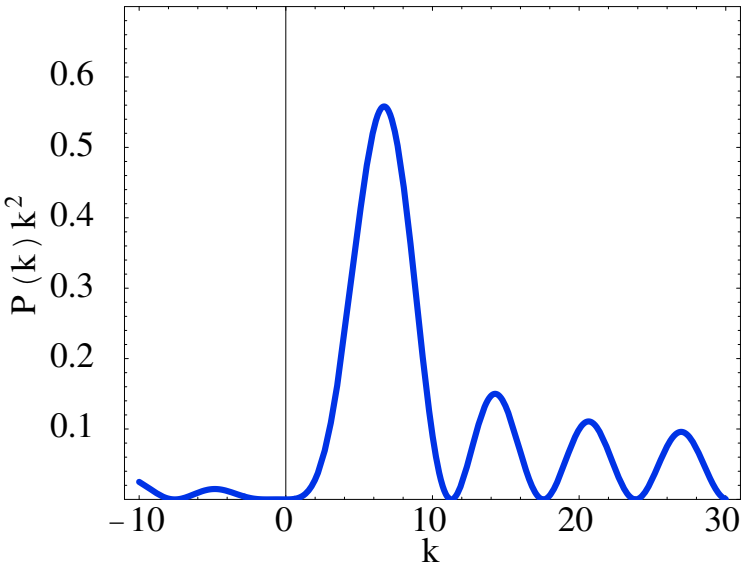
The Spectral Function

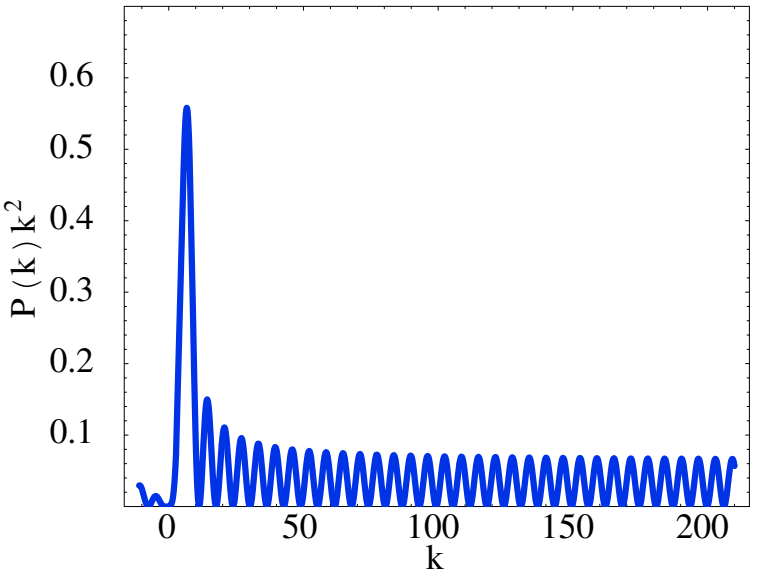


Before the Storm



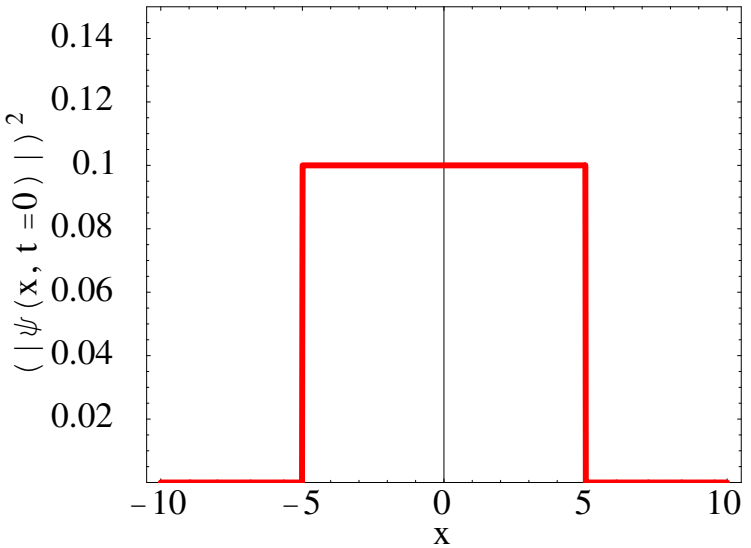
Disaster Strikes





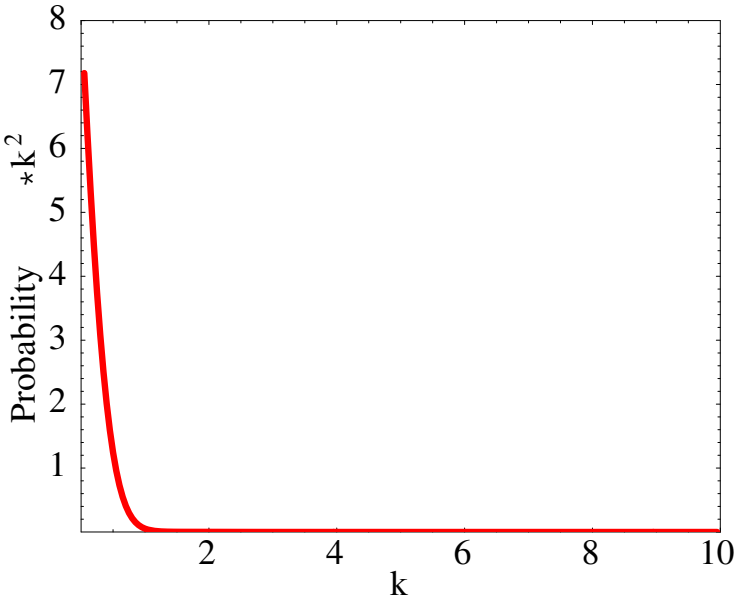
The integrand does not go to zero so the integral will not converge!

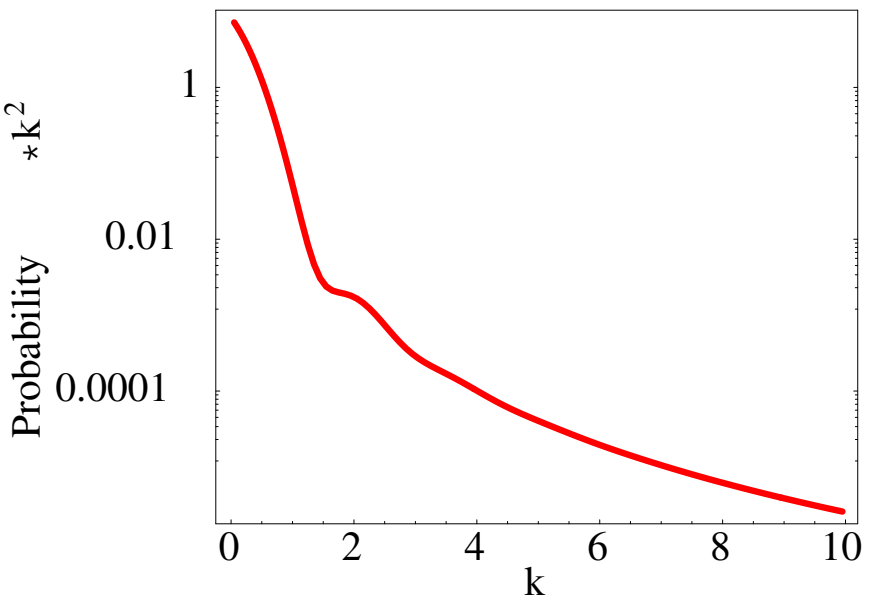
A slightly different $\psi(x, t = 0)$



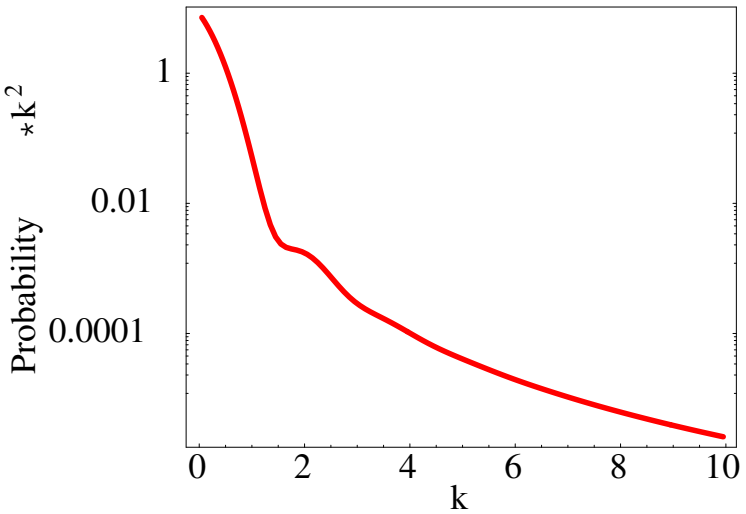
$$\psi(x, t = 0) = \frac{1}{\sqrt{a_1}} e^{ik_0 x} \frac{1}{e^{(\frac{x - a_1/2}{\delta})} + 1}$$

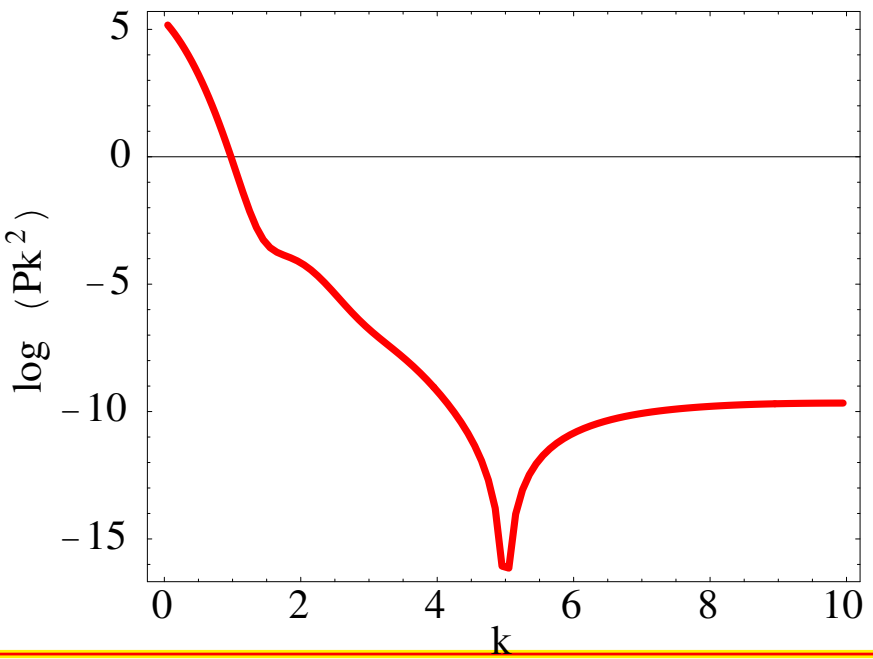
The results


Probability *k^2
k
1
2
3
4
5
6
7
8
2
4
6
8
10


Probability *k^2
k
1
0.01
0.0001
0
2
4
6
8
10

The results


Probability *k^2
1
0.01
0.0001
0
2
4
6
8
10
k


log (Pk^2)
5
0
-5
-10
-15
0
2
4
6
8
10
k