24.6 MOMENTUM AND RADIATION PRESSURE

Electromagnetic waves transport linear momentum as well as energy. Hence, it follows that pressure is exerted on a surface when an electromagnetic wave impinges on it. In what follows, let us assume that the electromagnetic wave strikes a surface at normal incidence and transports a total energy U to a surface in a time interval Δt. If the surface absorbs all the incident energy U in this time, Maxwell showed that the total momentum $\vec{\mathbf{p}}$ delivered to this surface has a magnitude

■ Momentum delivered to a perfectly absorbing surface

$$p = \frac{U}{c} \qquad \text{(complete absorption)} \qquad [24.32]$$

The pressure exerted on the surface is defined as force per unit area F/A. Let us combine this definition with Newton's second law:

$$P = \frac{F}{A} = \frac{1}{A}\frac{dp}{dt}$$

If we now replace p, the momentum transported to the surface by radiation, from Equation 24.32, we have

$$P = \frac{1}{A}\frac{dp}{dt} = \frac{1}{A}\frac{d}{dt}\left(\frac{U}{c}\right) = \frac{1}{c}\frac{(dU/dt)}{A}$$

We recognize $(dU/dt)/A$ as the rate at which energy is arriving at the surface per unit area, which is the magnitude of the Poynting vector. Therefore, the radiation pressure P exerted on the perfectly absorbing surface is

■ Radiation pressure exerted on a perfect absorbing surface

$$P = \frac{S}{c} \qquad \text{(complete absorption)} \qquad [24.33]$$

An absorbing surface for which all the incident energy is absorbed (none is reflected) is called a **black body.** A more detailed discussion of a black body will be presented in Chapter 28.

If the surface is a perfect reflector, the momentum delivered in a time interval Δt for normal incidence is twice that given by Equation 24.32, or $p = 2U/c$. That is, a momentum U/c is delivered first by the incident wave and then again by the reflected wave, a situation analogous to a ball colliding elastically with a wall.[5] Finally, the radiation pressure exerted on a perfect reflecting surface for normal incidence of the wave is twice that given by Equation 24.33, or $P = 2S/c$.

Although radiation pressures are very small (about 5×10^{-6} N/m^2 for direct sunlight), they have been measured using torsion balances such as the one shown in Figure 24.11. Light is allowed to strike either a mirror or a black disk, both of which are suspended from a fine fiber. Light striking the black disk is completely absorbed, so all its momentum is transferred to the disk. Light striking the mirror (normal incidence) is totally reflected and hence the momentum transfer is twice as great as that transferred to the disk. The radiation pressure is determined by measuring the angle through which the horizontal connecting rod rotates. The apparatus must be placed in a high vacuum to eliminate the effects of air currents.

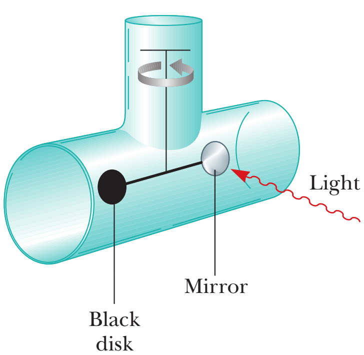

FIGURE 24.11 An apparatus for measuring the pressure exerted by light. In practice, the system is contained in a high vacuum.

QUICK QUIZ 24.4 In an apparatus such as that in Figure 24.11, suppose the black disk is replaced by one with half the radius. Which of the following are different after the disk is replaced? **(a)** radiation pressure on the disk **(b)** radiation force on the disk **(c)** radiation momentum delivered to the disk in a given time interval

[5]For *oblique* incidence, the momentum transferred is $2U\cos\theta/c$ and the pressure is given by $P = 2S\cos^2\theta/c$, where θ is the angle between the normal to the surface and the direction of propagation.

Thinking Physics 24.2

A large amount of dust occurs in the interplanetary space in the Solar System. Although this dust can theoretically have a variety of sizes, from molecular size upward, very little of it is smaller than about 0.2 μm in our Solar System. Why? (*Hint:* The Solar System originally contained dust particles of all sizes.)

Reasoning Dust particles in the Solar System are subject to two forces: the gravitational force toward the Sun and the force from radiation pressure due to sunlight, which is away from the Sun. The gravitational force is proportional to the cube of the radius of a spherical dust particle because it is proportional to the particle's mass. The radiation force is proportional to the square of the radius because it depends on the circular cross-section of the particle. For large particles, the gravitational force is larger than the force from radiation pressure. For small particles, less than about 0.2 μm, the larger force from radiation pressure sweeps these particles out of the Solar System. ■

EXAMPLE 24.3 Solar Energy

The Sun delivers about 1 000 W/m^2 of energy to the Earth's surface.

A Calculate the total power incident on a roof of dimensions 8.00 m × 20.0 m.

Solution The Poynting vector has an average magnitude $I = S_{avg} = 1\,000$ W/m^2, which represents the power per unit area. Assuming that the radiation is incident normal to the roof, we can find the power for the whole roof:

$$\mathcal{P} = IA = (1\,000 \text{ W/m}^2)(8.00 \times 20.0 \text{ m}^2) = 1.60 \times 10^5 \text{ W}$$

If solar energy could all be converted to electric energy, it would provide more than enough power for the average home. Solar energy is not easily harnessed, however, and the prospects for large-scale conversion are not as bright as they may appear from this simple calculation. For example, the conversion efficiency from solar to electric energy is typically 10% for photovoltaic cells. Solar energy has other practical problems that must also be considered, such as overcast days, geographic location, and energy storage.

B Determine the radiation pressure and radiation force on the roof, assuming that the roof covering is a perfect absorber.

Solution Using Equation 24.33 with $I = 1\,000$ W/m^2, we find that the average radiation pressure is

$$P = \frac{I}{c} = \frac{1\,000 \text{ W/m}^2}{3.00 \times 10^8 \text{ m/s}} = 3.33 \times 10^{-6} \text{ N/m}^2$$

Because pressure equals force per unit area, this value of P corresponds to a radiation force of

$$F = PA = (3.33 \times 10^{-6} \text{ N/m}^2)(160 \text{ m}^2) = 5.33 \times 10^{-4} \text{ N}$$

INTERACTIVE EXAMPLE 24.4 Pressure from a Laser Pointer

Many people giving presentations use a laser pointer to direct the attention of their audience. If a 3.0-mW pointer creates a spot that is 2.0 mm in diameter, determine the radiation pressure on a screen that reflects 70% of the light striking it. The power of 3.0 mW is a time-averaged power.

Solution In conceptualizing this problem, we certainly do not expect the pressure to be very large. We categorize this problem as one in which we calculate the radiation pressure by using something like Equation 24.33, but which is complicated by the 70% reflection. To analyze the problem, we begin by determining the Poynting vector of the beam. We divide the time-averaged power delivered via the electromagnetic wave by the cross-sectional area of the beam. Thus,

$$I = \frac{\mathcal{P}}{A} = \frac{\mathcal{P}}{\pi r^2} = \frac{3.0 \times 10^{-3} \text{ W}}{\pi\left(\dfrac{2.0 \times 10^{-3} \text{ m}}{2}\right)^2} = 9.6 \times 10^2 \text{ W/m}^2$$

Now we can determine the radiation pressure from the laser beam. A completely reflected beam would apply an average pressure of $P_{avg} = 2S_{avg}/c$. We can model the actual reflection as follows. Imagine that the surface absorbs the beam, resulting in pressure

$P_{avg} = S_{avg}/c$. Then the surface emits the beam, resulting in additional pressure $P_{avg} = S_{avg}/c$. If the surface emits only a fraction f of the beam (so that f is the amount of the incident beam reflected), the pressure due to the emitted beam is $P_{avg} = fS_{avg}/c$. Therefore, the total pressure on the surface due to absorption and re-emission (reflection) is

$$P_{avg} = \frac{S_{avg}}{c} + f\frac{S_{avg}}{c} = (1 + f)\frac{S_{avg}}{c}$$

For a beam that is 70% reflected, the pressure is

$$P = (1 + 0.70)\frac{9.6 \times 10^2 \text{ W/m}^2}{3.0 \times 10^8 \text{ m/s}} = 5.4 \times 10^{-6} \text{ N/m}^2$$

To finalize the problem, consider first the magnitude of the Poynting vector. It is about the same as the intensity of sunlight at the Earth's surface. (Therefore, it is not safe to shine the beam of a laser pointer into a person's eyes; that may be more dangerous than looking directly at the Sun.) To finalize further, note that the pressure has an extremely small value, as expected. (Recall from Section 15.1 that atmospheric pressure is approximately 10^5 N/m^2.)

PhysicsNow™ Log into PhysicsNow at **www.pop4e.com** and go to Interactive Example 24.4 to investigate the pressure on the screen for various laser and screen parameters.

Space Sailing

When imagining a trip to another planet, we normally think of traditional rocket engines that convert chemical energy in fuel carried on the spacecraft to kinetic energy of the spacecraft. An interesting alternative to this approach is that of **space sailing.** A space-sailing craft includes a very large sail that reflects light. The motion of the spacecraft depends on pressure from light, that is, the force exerted on the sail by the reflection of light from the Sun. Calculations performed (before U.S. government budget cutbacks shelved early space-sailing projects) showed that sailing craft could travel to and from the planets in times similar to those for traditional rockets, but for less cost.

Calculations show that the radiation force from the Sun on a practical sailcraft with large sails could be equal to or slightly larger than the gravitational force on the sailcraft. If these two forces are equal, the sailcraft can be modeled as a particle in equilibrium because the inward gravitational force of the Sun balances the outward force exerted by the light from the Sun. If the sailcraft has an initial velocity in some direction away from the Sun, it would move in a straight line under the action of these two forces, with no necessity for fuel. A traditional spacecraft with its rocket engines turned off, on the other hand, would slow down as a result of the gravitational force on it due to the Sun. Both the force on the sail and the gravitational force from the Sun fall off as the inverse square of the Sun–sailcraft separation. Therefore, in theory, the straight-line motion of the sailcraft would continue forever with no fuel requirement.

By using just the motion imparted to a sailcraft by the Sun, the craft could reach Alpha Centauri in about 10 000 years. This time interval can be reduced to 30 to 100 years using a *beamed power system*. In this concept, light from the Sun is gathered by a transformation device in orbit around the Earth and is converted to a laser beam or microwave beam aimed at the sailcraft. The force from this intense beam of radiation increases the acceleration of the craft, and the transit time is significantly reduced. Calculations indicate that the sailcraft could achieve design speeds of up to 20% of the speed of light using this technique.

24.7 THE SPECTRUM OF ELECTROMAGNETIC WAVES

Electromagnetic waves travel through vacuum with speed c, frequency f, and wavelength λ. The various types of electromagnetic waves, all produced by accelerating charges, are shown in Figure 24.12. Note the wide range of frequencies and wavelengths. Let us briefly describe the wave types shown in Figure 24.12.

Radio waves are the result of charges accelerating, for example, through conducting wires in a radio antenna. They are generated by such electronic devices as LC oscillators and are used in radio and television communication systems.