Alpha Decay

1. Introduction

One of the earliest and most convincing verifications of the validity of quantum mechanics was the resolution of the paradox of alpha decay observed in heavy nuclei. Gamow, Condon, and Gurney treated the problem as a barrier penetration problem and were able to provide a reasonable explanation of the decay rates for a broad range of alpha-emitting nuclei. In this exercise you will compile some of the existing data on alpha decay and make a comparison with a theoretical calculation based on the transfer matrix method.

2. Constants, Commands, and Functions

In this Section the ingredients necessary for the calculation of the transmission coefficient for the alpha potential barrier are generated. The commands in each Section should be executed since they will be used in the calculation of the transmission coefficient and they demonstrate the plotting of data with *Mathematica*.

2.1 Constants:

The mass and charge of the emitted particle are defined by "Aout" and "Zout" in units of atomic mass units electron charges respectively. The mass and charge of the parent nucleus is defined by "Anuc" and "Znuc". The other quantities below are necessary constants. To define the constants place the pointer in the cell, click, and then hit "Enter". The emitted particle is an alpha particle.

Note: *Mathematica* will warn you about "possible" spelling errors in some of the definitions below because of the similarity of some of the names used. You can usually, safely ignore this warning.

List of constants.

Aout - emitted particle mass
Zout - emitted particle charge
Anuc - atomic number of parent nucleus
Mout - mass of parent
Znuc - charge of parent
R0 - radius of parent
Rmax - classical turning point of emitted particle
Ndiv - Number of segments
step - width of an individual barrier
Ve - array of (radius, potential energy)

```
Aout = 4.0;
Zout = 2.0;
Anuc = 212;
Znuc = 84;
Ndiv = 10;
hbarc = 197.0;     (* h_bar times c *)
e2 = hbarc/137;    (* electron charge squared *)
```

■ 2.2 Necessary commands:

Some of the commands needed to calculate the transmission coefficient for a given emitted particle energy are demonstrated here. The technique for generating an array of numbers is shown first. You must determine what values R0, Rmax, and step should have in terms of the constants defined above. The values given below are for demonstration purposes only.

```
R0   = 8.0;
Rmax = 30.0;
step = 1.0;
Ve    = Table[{r, Zout*Znuc*e2/r},
              {r, R0, Rmax-step,step}];
```

Put zeros at each end of the array since we are treating the potential as if it is zero in those regions.

```
PrependTo[Ve, {R0-step, 0}];
AppendTo[Ve, {Rmax, 0}];
```

Initialize the transfer matrix to the unit matrix.

```
test = { {1,0},
         {0,1} };
```

Use the **Do** command to loop over the segments and calculate the transfer matrix by multiplying successive matrices together. The commands for extracting elements from the array Ve and multiplying matrices are also demonstrated here within the loop.

```
Do[
  Vtest = Ve[[i,2]];
  mat1  = {{ Vtest,     0},
           {     0, Vtest} };
  mat2  = {{-Vtest,     0},
           {     0, Vtest} };
  test  = test.mat1.mat2;,
  {i,1,Ndiv+1}
 ];
```

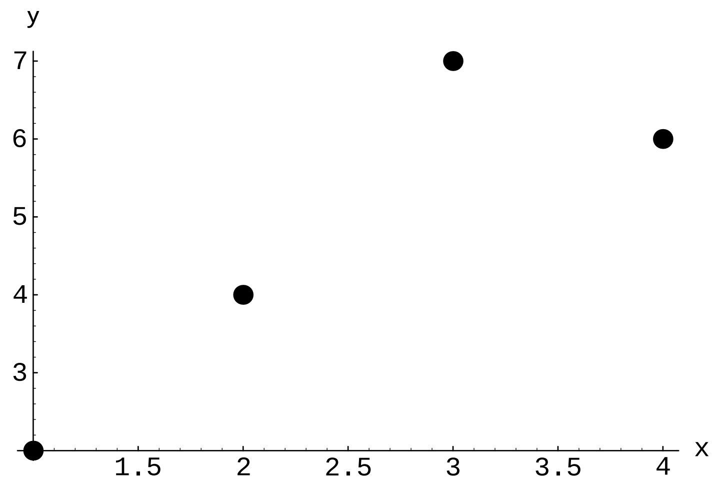
■ 2.3 Plotting commands

□ 2.3.1 Plotting data.

Mathematica can be used plots of data with the following commands that define an array of x and y ordered pairs ("Stuff") that is then plotted with the command "ListPlot" as demonstrated below.

```
Stuff= { {1,2},
         {2,4},
         {3,7},
         {4,6} }
ListPlot[Stuff,
         PlotStyle->PointSize[0.03],
         AxesLabel->{"x", "y"}]
```

```
{{1, 2}, {2, 4}, {3, 7}, {4, 6}}
```




```
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```

□ 2.3.2 Data entry for alpha decay.

Because of the broad range covered by the lifetime calculations it is best to plot the common log of the lifetime as a function of alpha energy. The *Mathematica* table below will hold the results from the theoretical calculations. The column on the left contains the alpha particle energy and the column on the right contains the lifetime calculation. Simply substitute your energies and lifetimes for those displayed here. Do NOT use the numbers displayed below. They are an example only.

Energy Theoretical Lifetime

```
theory = { { 8.78 , Log[10,5.9 10^-5] },
           { 9.00 , Log[10,1.2 10^-12]},
           { 12.0 , Log[10,2.0 10^-7 ] } }
```

```
{{8.78, -4.22915}, {9., -11.9208}, {12., -6.69897}}
```

The measured lifetimes taken from the table of isotopes are stored here in a manner similar to the table above. Substitute your results for the ones listed here.

Energy Measured Lifetime

```
data   = { { 8.78 , Log[10,3.0 10^-7] },
           { 9.00 , Log[10,2.0 10^-12]},
           { 12.0 , Log[10,5.0 10^-8 ] } }
```

```
{{8.78, -6.52288}, {9., -11.699}, {12., -7.30103}}
```

□ 2.3.3 Plotting data and theory together.

The code in this cell will take the numbers you entered above and plot them together. It defines two graphical objects, "dataplot" and "theoryplot" and then combines them. Place the pointer anywhere in the cell, click, and hit "Enter".

```
dataplot = ListPlot[
             data,
             Frame->True,
             FrameLabel->
                {"E(MeV)",
                 "Log(t)",
                 "Points->Data,  Line->Theory",
                 " "},
             Prolog->AbsolutePointSize[5],
             RotateLabel->False,
             DisplayFunction->Identity
            ]
theoryplot = ListPlot[
               Sort[theory],
               Frame->True,
               FrameLabel->
                {"E(MeV)",
                 "Log(t)",
                 "Points->Data,  Line->Theory",
                 " "},
               PlotJoined->True,
               RotateLabel->False,
               PlotStyle->{RGBColor[1,0,0]},
               DisplayFunction->Identity
              ]
Show[{dataplot,theoryplot},
     DisplayFunction->$DisplayFunction]
```

```
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```

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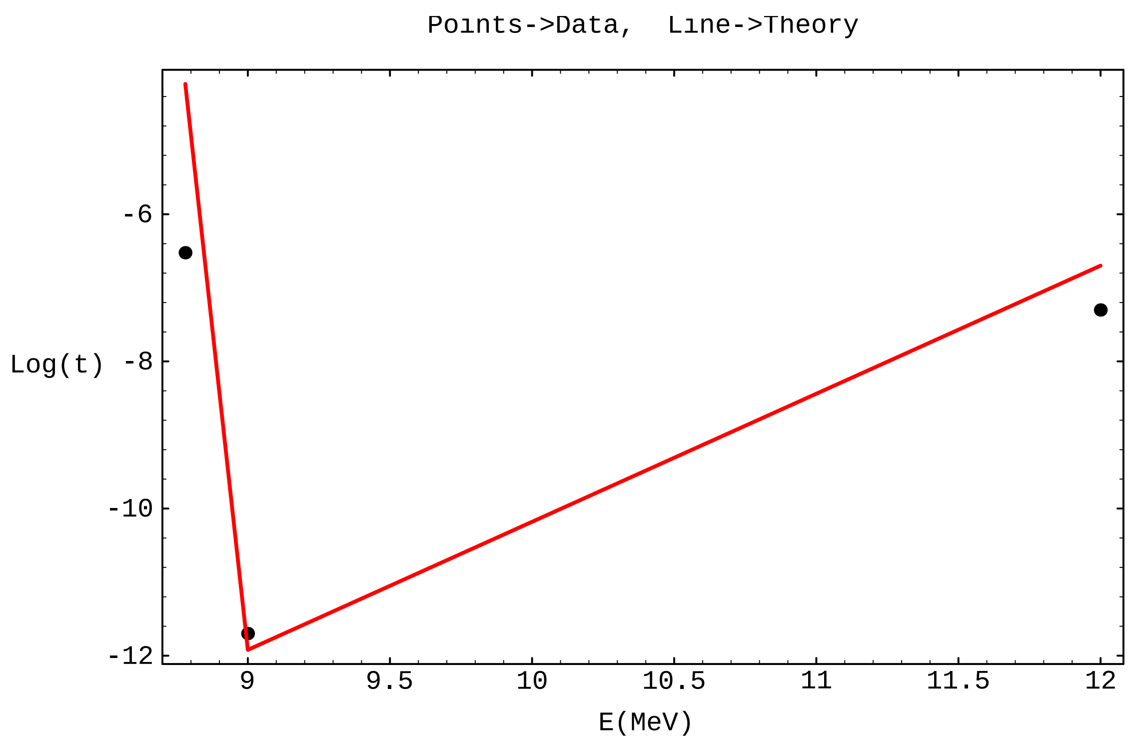
```



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3. Procedure

1. Clearly state the paradox associated with alpha decay.

2. Using the "Table of Isotopes" find seven examples of alpha emitters and make a table of nucleus, half-life, and alpha energy. See the accompanying sheet for examples of the data contained in the "Table of Isotopes". The nuclei that alpha decay are in the mass range A>200. It is best to find nuclei that emit alpha particles with primarily one kinetic energy.

3. The commands demonstrated in Section 2.2 can be used to calculate the transmission coefficient using the transfer matrix method and by breaking the potential barrier into a series of adjacent rectangular barriers. The accuracy of the calculation depends on the number of barriers that the potential energy curve is divided into. It is important to "calibrate" the calculation by exploring the dependence of the result on the parameters of the calculation. For one of the entries in your table calculate the transmission coefficient using many values of the number of barriers. Does your result converge to a limiting value as the number of barriers increases? It is best to show your results with a plot. Section 2.3 demonstrates the procedure for making plots of data points with *Mathematica*.

4. Compare the calculation of the transmission coefficient that you calibrated in Section 3.3 with the expectation of the WKB approximation. Do the two results agree?

5. Use the algorithm you programmed in part 3 to calculate the transmission coefficient for each of the entries in your table. Since the transmission coefficient represents the probability that a particle will penetrate the barrier during a collision, the inverse of it is the number of collisions

that will occur before, on average, transmission will occur. You will use this fact in the next Section.

4. Analysis and Discussion

1. If we know the number of collisions that must occur before a particle is transmitted (1/T), then the product of this number with the time between collisions will produce an estimate of the lifetime. Assume the kinetic energy of the alpha particle inside the nucleus is the same as when it is emitted. Show the time between collisions, Δt, is

$$\Delta t = 2\ R/v$$

where $R = 1.4A^{1/3}$ is the nuclear radius, v is the alpha particle speed inside the nucleus, and A is the mass number of the parent nucleus.

Calculate the theoretical lifetimes for each of the entries in your table. Be sure to include one sample calculation in your report.

2. Make a plot comparing your theoretical and experimental results using the plotting commands defined in Section 2.3. How well does the theory describe the data? Does it reproduce the systematic trends of the data set? What attributes of the data does the theory fail to account for? Make at least one criticism of the procedure you followed to calculate the lifetime and why it might be wrong. Your report should include Purpose, Calculations, Analysis, and a Discussion.