8)

$$F = ma = -Cx$$

$$m \frac{d^2x}{dt^2} = -Cx$$

$$\frac{d^2x}{dt^2} = -\frac{C}{m}x$$

$$= -\omega^2 x \qquad \omega = \sqrt{\frac{C}{m}}$$

let $x(t) = \sum_{n=0}^{\infty} a_n t^n = a_0 + a_1 t + a_2 t^2 + a_3 t^3 + \ldots$

then $\frac{dx}{dt} = a_1 + 2a_2 t + 3a_3 t^2 + 4a_4 t^3 + \ldots$

$$\frac{d^2x}{dt^2} = 2a_2 + 2 \cdot 3 a_3 t^1 + 3 \cdot 4 a_4 t^2 + 4 \cdot 5 a_5 t^3 + \ldots$$

$$2a_2 + 2 \cdot 3a_3 t + 3 \cdot 4 a_4 t^2 + 4 \cdot 5 a_5 t^3 + \ldots =$$

$$-\omega^2 \left[a_0 + a_1 t + a_2 t^2 + a_3 t^3 + \ldots \right]$$

$$\left[2a_2 + \omega^2 a_0 \right] t^0 +$$
$$\left[2 \cdot 3 a_3 + \omega^2 a_1 \right] t +$$
$$\left[3 \cdot 4 a_4 + \omega^2 a_2 \right] t^2 +$$
$$\left[4 \cdot 5 a_5 + \omega^2 a_3 \right] t^3 + \ldots = 0$$

the n^{th} term will be

$$(n+1)(n+2)a_{n+2} + \omega^2 a_n = 0$$

$$\frac{a_{n+2}}{a_n} = -\frac{\omega^2}{(n+1)(n+2)}$$

Choice #1 let $a_0 = 0$ $a_1 = \omega$

$a_n = 0$ $n = 0, 2, 4, \ldots$

$$a_{n+2} = -\frac{\omega^2}{(n+1)(n+2)} a_n \qquad n = 1, 3, 5, \ldots$$

n	a_n	a_{n+2}
1	ω	$-\omega^3/2\cdot3$
3	$-\omega^3/3!$	$\omega^5/2\cdot3\cdot4\cdot5$
5	$\omega^5/5!$	$-\omega^7/2\cdot3\cdot4\cdot5\cdot6\cdot7$
7	$-\omega^7/7!$	$\omega^9/2\cdot3\cdot4\cdot5\cdot6\cdot7\cdot8\cdot9$
9	$\omega^9/9!$	$-\omega^{11}/11!$
11	$-\omega^{11}/11!$	$\omega^{13}/13!$

$$x(t) = \omega t - \frac{\omega^3 t^3}{3!} + \frac{\omega^5 t^5}{5!} - \frac{\omega^7 t^7}{7!} + \frac{\omega^9 t^9}{9!} - \frac{\omega^{11} t^{11}}{11!} + \ldots$$

$$\boxed{x(t) = \sin \omega t}$$

Choice # 2 let $a_1 = 0$ $a_0 = 1$

$\therefore a_n = 0 \quad n = 1, 3, 5, 7, \ldots$

$$a_{n+2} = -\frac{\omega^2}{(n+1)(n+2)} a_n$$

n	a_n	a_{n+2}
0	1	$-\omega^2/1.2$
2	$-\omega^2/2!$	$+\omega^4/1.2.3.4$
4	$\omega^4/4!$	$-\omega^6/1.2.3.4.5.6$
6	$-\omega^6/6!$	
8	$\omega^8/8!$	
10	$\omega^{10}/10!$	

$$\therefore \; x(t) = 1 - \frac{\omega^2 t^2}{2!} + \frac{\omega^4 t^4}{4!} - \frac{\omega^4 t^6}{6!} + \frac{\omega^8 t^8}{8!} - \frac{\omega^{10} t^{10}}{10!} + \cdots$$

$$\boxed{x(t) = \cos \omega t}$$

$$x(t) = \sum_{n=0}^{\infty} a_n t^n$$

$$= a_0 + a_1 t - \frac{\omega^2}{1\cdot 2} a_0 t^2 - \frac{\omega^2}{2\cdot 3} a_1 t^3 - \frac{\omega^2}{3\cdot 4} a_2 t^4$$

$$- \frac{\omega^2}{4\cdot 5} a_3 t^5 + \ldots$$

$$= a_0 + a_1 t - \frac{\omega^2}{1\cdot 2} t^2 - \frac{\omega^2}{2\cdot 3} a_1 t^3 - \frac{\omega^2}{3\cdot 4}\left(-\frac{\omega^2}{1\cdot 2}\right) a_0 t^4$$

$$- \frac{\omega^2}{4\cdot 5}\left(-\frac{\omega^2}{2\cdot 3} a_1\right) t^5 + \ldots$$

$$= \left\{ a_0 - \frac{\omega^2}{1\cdot 2} a_0 t^2 + \frac{\omega^4}{1\cdot 2\cdot 3\cdot 4} a_0 t^4 + \ldots \right\} +$$

$$\left\{ a_1 t - \frac{\omega^2}{2\cdot 3} a_1 t^3 + \frac{\omega^4}{2\cdot 3\cdot 4\cdot 5} a_1 t^5 + \ldots \right\}$$

$$= a_0 \left\{ 1 - \frac{\omega^2}{1\cdot 2} t^2 + \frac{\omega^4}{1\cdot 2\cdot 3\cdot 4} t^4 - \ldots \right\}$$

$$+$$

$$a_1 \left\{ t - \frac{\omega^2}{2\cdot 3} t^3 + \frac{\omega^4 t^5}{2\cdot 3\cdot 4\cdot 5} + \ldots \right\}$$

$$= \sin \omega t \quad + \quad \cos \omega t \quad \text{for } a_0 = 1, \; a_1 = \omega$$