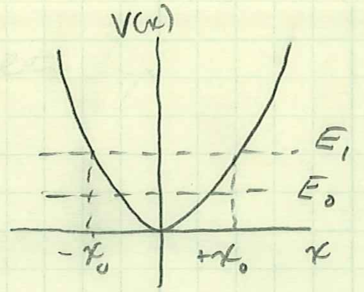
Supplement #2

$$E_1 - E_0 = 0.10\,eV \qquad m = m_p$$



Classically, the proton should oscillate between $\pm x_0$

answer = probability of finding proton outside the well = 1 − probability of finding proton in the well.

$$\psi_1 = \left(\frac{\pi}{4}\right)^{-\frac{1}{4}} \xi\, e^{-\xi^2/2}$$

Classically the turning points ($\pm x_0$) occur when:

$$E = \frac{p^2}{2m} + \frac{Cx^2}{2} = \frac{Cx_0^2}{2}$$

at the endpoints the classical speed vanishes.

$$\frac{2E}{C} = x_0^2$$

$$x_0 = \sqrt{\frac{2E}{C}} \qquad \text{classical turning point}$$

Get E

$$E_n = (n+\tfrac{1}{2})\,h\nu$$

$$n = 1$$

$$E_1 = \frac{3}{2}h\nu$$

to get $h\nu$

$$E_1 - E_0 = \frac{3}{2}h\nu - \frac{1}{2}h\nu$$

$$= h\nu$$

$$= 0.10\text{ eV}$$

$$\therefore h\nu = 0.10\text{ eV}$$

$$\therefore \underline{E_1 = 0.15\text{ eV}}$$

Get C

$$2\pi\nu = \sqrt{\frac{C}{m_p}} \qquad \text{from Phys. 201}$$

$$C = 4\pi^2\nu^2 m_p$$

$$\nu = \frac{E}{h}$$

$$= 4\pi^2 \frac{E_1^2}{h^2} m_p \frac{c^2}{c^2}$$

$$= \frac{E_1^2}{(\hbar c)^2} m_p c^2$$

$$= \frac{(0.15\text{ eV})^2 (938\times10^6\text{ eV})}{(197\times10^6\text{ eV-fm})^2}$$

$$= 5.44\times10^{-10}\text{ eV/fm}^2$$

$$\therefore x_0 = \sqrt{\frac{2E}{C}}$$

$$= \left[\frac{2\,(0.15\text{ eV})}{5.44\times10^{-10}\text{ eV/fm}^2}\right]^{1/2}$$

$x_0 = 2.35 \times 10^4 \text{ fm}$

$$\boxed{x_0 = 0.235 \text{ Å}}$$

$$\text{answer} = 1 - \int_{-\xi_0}^{\xi_0} \psi_1^* \psi_1 \, d\xi$$

$\xi_0 = \beta x_0 \qquad \beta^2 = \frac{m\omega_0}{\hbar}\frac{\hbar}{\hbar}$

$= \frac{m}{\hbar^2} \hbar\omega_0 \frac{c^2}{c^2} \qquad \hbar\omega_0 = 0.1 \text{ eV}$

$= \frac{mc^2}{(\hbar c)^2} \hbar\omega_0$

$= \frac{\left(938 \times 10^6 \frac{\cancel{eV}}{\cancel{c^2}}\right)\left(\cancel{c^2}\right)\left(0.1 \cancel{eV}\right)}{\left(197 \times 10^6 \cancel{eV}\text{-fm}\right)^2}$

$= 2.4 \times 10^{-9} \text{ fm}^{-2}$

$\beta = 4.9 \times 10^{-5} \text{ fm}^{-1}$

$\therefore \quad \xi_0 = \beta x_0$

$= \left(4.9 \times 10^{-5} \text{ fm}^{-1}\right)\left(0.235 \text{ Å}\right) 10^5 \frac{\text{fm}}{\text{Å}}$

$= 1.16$

$$\text{answer} = 1 - \int_{-\xi_0}^{\xi_0} \left(\frac{\pi}{4}\right)^{-\frac{1}{4}} \xi e^{-\xi^2/2} \left(\frac{\pi}{4}\right)^{-\frac{1}{4}} \xi e^{-\xi^2/2} \, d\xi$$

$$= 1 - \int_{-\xi_0}^{\xi_0} \left(\frac{4}{\pi}\right)^{\frac{1}{2}} \xi^2 e^{-\xi^2} \, d\xi$$

$$= 1 - 2 \frac{2}{\sqrt{\pi}} \int_0^{\xi_0} \xi^2 e^{-\xi^2} \, d\xi$$

$$= 1 - \frac{4}{\sqrt{\pi}} \cdot \underbrace{\frac{1}{4}\left(-2e^{-\xi_0^2} + \sqrt{\pi}\, \text{Erf}(\xi_0)\right)}_{= 0.26821}$$

$$= 1 - \frac{4}{\sqrt{\pi}} \cdot 0.26821$$

$$= 0.39$$