9.19) $\hat{L}^2 Y_2^2 = -\hbar^2 \left[\frac{1}{\sin\theta} \frac{\partial}{\partial\theta}\left(\sin\theta \frac{\partial}{\partial\theta}\right) + \right.$

$\left. \frac{1}{\sin^2\theta} \frac{\partial^2}{\partial\phi^2} \right] \frac{1}{4}\left(\frac{5}{12}\right)^{1/2} \sin^2\theta\, e^{2i\phi}$

let $A = \frac{1}{4}\left(\frac{5}{12}\right)^{1/2} \sin^2\theta\, e^{2i\phi}$

$$\frac{\partial Y_2^2}{\partial\phi} = \frac{\partial}{\partial\phi}\left(A\sin^2\theta\, e^{2i\phi}\right) = A\sin^2\theta\, 2i\, e^{2i\phi}$$

$$\frac{\partial^2 Y_2^2}{\partial\phi^2} = A\sin^2\theta\,(-4)\,e^{2i\phi}$$

$$\frac{\partial Y_2^2}{\partial\theta} = A e^{2i\phi}\, 2\sin\theta\cos\theta$$

$$\frac{\partial}{\partial\theta}\left(\sin\theta \frac{\partial Y_2^2}{\partial\theta}\right) = \frac{\partial}{\partial\theta}\left(A e^{2i\phi}\, 2\sin^2\theta\cos\theta\right)$$

$$= 2Ae^{2i\phi}\left[2\sin\theta\cos\theta\cos\theta + - \sin^2\theta\sin\theta\right]$$

$$= 2Ae^{2i\phi}\left[2\sin\theta\cos^2\theta - \sin^3\theta\right]$$

$$\hat{L}^2 Y_2^2 = -\hbar^2\left[\frac{1}{\cancel{\sin\theta}}\left(2Ae^{2i\phi}\right)\left(\cancel{\sin}\,\theta\right)\left(2\cos^2\theta - \sin^2\theta\right) + \frac{1}{\sin^2\theta} A e^{2i\phi}(-4)\sin^2\theta\right]$$

$$= -\hbar^2 2Ae^{2i\phi}\left[\left(2\cos^2\theta - \sin^2\theta\right) + -2\right]$$

$$\hat{L}^2 Y_2^2 = -\hbar^2 2A e^{2i\phi} \left[2\cos^2\theta - \sin^2\theta - 2 \right]$$

$$= 2\hbar^2 A e^{2i\phi} \left(2 - 2\cos^2\theta + \sin^2\theta \right)$$

$$= 2\hbar^2 A e^{2i\phi} \left(2(\underbrace{1 - \cos^2\theta}_{\sin^2\theta}) + \sin^2\theta \right)$$

$$= 2\hbar^2 A e^{2i\phi} \left(2\sin^2\theta + \sin^2\theta \right)$$

$$= 6\hbar^2 A e^{2i\phi} \sin^2\theta$$

$$= 6\hbar^2 Y_2^2$$

$$\hat{L}_z Y_2^2 = -i\hbar \frac{\partial}{\partial\phi} \left(A\sin^2\theta\, e^{2i\phi} \right)$$

$$= -i\hbar A\sin^2\theta\,(2i) e^{2i\phi}$$

$$= 2\hbar A\sin^2\theta\, e^{2i\phi}$$

$$= 2\hbar Y_2^2$$

9.23) $l_z = \hbar m \qquad l^2 = l(l+1)\hbar^2$

a. show $\langle L_x \rangle = \langle L_y \rangle = 0$

$$\hat{L}_x = \frac{\hat{L}_+ + \hat{L}_-}{2} \qquad L_y = \frac{L_+ - L_-}{2i}$$

$$L_+ |\phi_{lm}\rangle = |\phi_{lm+1}\rangle$$

$$L_- |\phi_{lm}\rangle = |\phi_{lm-1}\rangle$$