9.26) $|\varphi\rangle = \sqrt{\frac{3}{4\pi}} \sin\varphi \sin\theta$

a. Get $\sin\varphi \sin\theta$ in terms of Y_ℓ^m's the sneaky way using Table 9.1 in Liboff.

$$|\varphi\rangle = \sqrt{\frac{3}{4\pi}} \left(\frac{e^{i\varphi} - e^{-i\varphi}}{2i}\right) \sin\theta$$

$$= \sqrt{\frac{3}{4\pi}} \frac{1}{2i} \left[\sin\theta e^{i\varphi} - \sin\theta e^{-i\varphi} \right]$$

$$= \sqrt{\frac{\cancel{3}}{\cancel{4\pi}_{2}}} \frac{1}{\cancel{2}i} \left[\left(-\cancel{2}\sqrt{\frac{\cancel{2\pi}}{\cancel{3}}}\, Y_1^1\right) - \left(+\cancel{2}\sqrt{\frac{\cancel{2\pi}}{\cancel{3}}}\, Y_1^{-1}\right) \right]$$

$$|\varphi\rangle = \frac{-i}{\sqrt{2}} \left(Y_1^1 + Y_1^{-1}\right)$$

$\therefore$ $b_{11} = \langle Y_1^1 | \varphi\rangle = -\frac{i}{\sqrt{2}}$ $\qquad$ $b_{1-1} = \langle Y_1^{-1} | \varphi \rangle = +\frac{i}{\sqrt{2}}$

$|b_{11}|^2 = \frac{1}{2} = |b_{1-1}|^2$ so the probality of each m state is 1/2.

b. $\langle \hat{L}_y \rangle = \langle \varphi | \hat{L}_y | \varphi \rangle$

$$= \frac{1}{2} \left(\langle 11| + \langle 1-1|\right)\left(\frac{\hbar}{2}\right)\left(\sqrt{2}\,|10\rangle + \sqrt{2}\,|10\rangle\right)$$

$$= \frac{1}{2}\frac{\hbar}{\cancel{2}}\sqrt{2}\left(\langle 11| + \langle 1-1|\right)\left(\cancel{2}\,|10\rangle\right)$$

$$= \frac{\hbar}{\sqrt{2}}\left(\cancelto{0}{\langle 11|10\rangle} + \cancelto{0}{\langle 1-1|10\rangle}\right)$$

$= 0$

c. $\langle \hat{L}^2 \rangle = \frac{1}{\cancel{2}} \left(\langle 11| + \langle 1\,-1| \right) \left(\hbar^2 \right) \left(\cancel{2}|11\rangle + \cancel{2}|1\,-1\rangle \right)$

$= \hbar^2 \left(\cancelto{1}{\langle 11|11\rangle} + \cancelto{0}{\langle 11|1\,-1\rangle} + \cancelto{0}{\langle 1\,-1|11\rangle} + \cancelto{1}{\langle 1\,-1|1\,-1\rangle} \right)$

$= 2\hbar^2$