9.25) D_2 at 30K at $t=0$

a. $$|\psi(\theta,\phi,t=0)\rangle = \frac{3Y_1^1 + 4Y_7^3 + Y_7^1}{\sqrt{26}}$$

$$\hat{L}^2|\psi\rangle = \frac{3\cdot 1\cdot 2\hbar^2 Y_1^1 + 4\, 7\cdot 8\hbar^2 Y_7^3 + 7\cdot 8\hbar^2 Y_7^1}{\sqrt{26}}$$

for $l=1$

$$\langle Y_1^1|\hat{L}^2\psi\rangle = \frac{3}{\sqrt{26}}\,2\hbar^2$$

for $l=7$

$$\langle Y_7^3|\hat{L}^2\psi\rangle = \frac{4}{\sqrt{26}}\,56\hbar^2$$

$$\langle Y_7^1|\hat{L}^2\psi\rangle = \frac{1}{\sqrt{26}}\,56\hbar^2$$

$\therefore$ probability of $l=7$ $= \left(\frac{4}{\sqrt{26}}\right)^2 + \left(\frac{1}{\sqrt{26}}\right)^2 = 0.65$

probability of $l=1$ $= \left(\frac{3}{\sqrt{26}}\right)^2 = .35$

$$\hat{L}_z|\psi\rangle = \frac{3\hbar Y_1^1 + 4\,3\hbar Y_7^3 + 1\,Y_7^1}{\sqrt{26}}$$

for $m=1$

$$\langle Y_1^1|\hat{L}_z\psi\rangle = \frac{3}{\sqrt{26}}\hbar$$

$$\langle Y_7^1|\hat{L}_z\psi\rangle = \frac{1}{\sqrt{26}}\hbar$$

for $m = 3$

$$\langle Y_7^3 | \hat{L}_z \psi \rangle = \frac{4}{\sqrt{26}} 3\hbar$$

$\therefore$ probability of $m = 1$ $= \left(\frac{3}{\sqrt{26}}\right)^2 + \left(\frac{1}{\sqrt{26}}\right)^2 = 0.38$

probability of $m = 3$ $= \left(\frac{4}{\sqrt{26}}\right)^2 = 0.62$

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b.

$$|\psi(\theta, \varphi, t)\rangle = \sum b_\ell |\varphi_\ell\rangle e^{-i\omega_\ell t}$$

$$E_\ell = \hbar\omega_\ell = \frac{\hbar^2 \ell(\ell+1)}{2I}$$

$$\omega_\ell = \frac{E_\ell}{\hbar} = \frac{\ell(\ell+1)\hbar}{2I}$$

$$= \frac{3 Y_1^1 e^{-i\omega_1 t} + (4 Y_7^3 + Y_7^1) e^{-i\omega_7 t}}{\sqrt{26}}$$

c. For purely rotational states

$$\hat{H} |\varphi_\ell\rangle = \frac{\ell(\ell+1)}{2I}\hbar^2 |\varphi_\ell\rangle$$

$$\langle E \rangle = \langle \psi | \hat{H} \psi \rangle$$

$$= \frac{1}{26}\left[3\langle \varphi_{11}| e^{+i\omega_1 t} + (4\langle\varphi_{73}| + \langle\varphi_{71}|) e^{+i\omega_7 t} \right]$$

$$\hat{H}\left[3|\varphi_{11}\rangle e^{-i\omega_1 t} + (4|\varphi_{73}\rangle + |\varphi_{71}\rangle) e^{-i\omega_7 t} \right]$$

7.56

$$\langle E \rangle = \frac{1}{26}\left[3\langle \varphi_{11}| e^{+i\omega_1 t} + \left(4\langle \varphi_{73}| + \langle \varphi_{71}|\right) e^{i\omega_7 t}\right]$$

$$\left[\frac{2\hbar^2}{2I} 3|\varphi_{11}\rangle e^{-i\omega_1 t} + \frac{56\hbar^2}{2I}\left(4|\varphi_{73}\rangle + |\varphi_{71}\rangle\right) e^{-i\omega_7 t}\right]$$

$$= \frac{1}{26}\left\{ 9 \frac{2\hbar^2}{2I} + 16 \frac{56\hbar^2}{2I} + 1 \frac{56\hbar^2}{2I} \right\}$$

$$= \frac{9}{26}\frac{\hbar^2}{I} + \frac{17}{26}\frac{28\hbar^2}{I}$$

$$= 0.35 \frac{\hbar^2}{I} + 0.65 \frac{28\hbar^2}{I}$$

$$= 18.7 \frac{\hbar^2}{I}$$

$$\frac{\hbar}{4\pi I c} = 30.4\ cm^{-1} = B = 30.4\ cm^{-1}\left(\frac{100\ cm}{1\ m}\right) = 3.04 \times 10^3\ m^{-1}$$

$$I = \frac{\hbar}{4\pi B c}$$

$$\langle E \rangle = 18.7 \frac{\hbar^2}{\hbar / 4\pi B c} = 18.7\ \hbar 4\pi B c$$

$$= 39.4\ \hbar B c (4\pi)$$

$$= 39.4\ \hbar c\ B (4\pi)$$

$$= 39.4 \overset{\times 4\pi}{\wedge} (197 \times 10^6\ eV\text{-}fm)\left(10^{-15} \frac{m}{fm}\right)(3.04 \times 10^3\ m^{-1})$$

$$\boxed{\langle E \rangle = .15\ eV}$$

Terry, Terry, Terry!