9.27) $|\psi\rangle = Y_1^{-1}$

What values of L_x will a measurement find and with what probabilities?

$$L_x X = \hbar\alpha X$$

let $L_x = \frac{L_+ + L_-}{2}$

$X = aY_1^1 + bY_1^0 + cY_1^{-1}$

$$\left(\frac{L_+ + L_-}{2}\right)\left(aY_1^1 + bY_1^0 + cY_1^{-1}\right) = \hbar\alpha X$$

$$\frac{\hbar}{2}\left(0 + b\sqrt{2}\,Y_1^1 + c\sqrt{2}\,Y_1^0 + a\sqrt{2}\,Y_1^0 + b\sqrt{2}\,Y_1^{-1}\right) =$$

$$\hbar\alpha\left(aY_1^1 + bY_1^0 + cY_1^{-1}\right)$$

$$\frac{1}{\sqrt{2}}\left[bY_1^1 + (a+c)Y_1^0 + bY_1^{-1}\right] =$$

$$\alpha\left(aY_1^1 + bY_1^0 + cY_1^{-1}\right)$$

$$Y_1^1\left(\frac{b}{\sqrt{2}} - \alpha a\right) + Y_1^0\left(\frac{a+c}{\sqrt{2}} - \alpha b\right) + Y_1^{-1}\left(\frac{b}{\sqrt{2}} - \alpha c\right)$$

$$= 0$$

$$Y_1^1\left(\sqrt{2}\,\alpha a - b\right) + Y_1^0\left(\sqrt{2}\,\alpha b - a - c\right) + Y_1^{-1}\left(\sqrt{2}\,\alpha c - b\right) = 0$$

$$\begin{vmatrix} \sqrt{2}\,\alpha & -1 & 0 \\ -1 & \sqrt{2}\,\alpha & -1 \\ 0 & -1 & \sqrt{2}\,\alpha \end{vmatrix} = 0$$

$$\sqrt{2}\,\alpha\left(2\alpha^2 - 1\right) - (+1)\left(+\sqrt{2}\,\alpha\right) + 0 = 0$$

$$\cancel{\sqrt{2}}\,2\alpha^3 - \cancel{\sqrt{2}}\,\alpha - \cancel{\sqrt{2}}\,\alpha = 0$$

$$2\alpha(\alpha^2-1)=0$$

$$\therefore \quad \alpha = 0 \quad \text{or} \quad \pm 1$$

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now

$$\begin{pmatrix} \sqrt{2}\alpha & -1 & 0 \\ -1 & \sqrt{2}\alpha & -1 \\ 0 & -1 & \sqrt{2}\alpha \end{pmatrix}\begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

for $\alpha = 0$

$$\begin{pmatrix} 0 & -1 & 0 \\ -1 & 0 & -1 \\ 0 & -1 & 0 \end{pmatrix}\begin{pmatrix} a \\ b \\ c \end{pmatrix} = 0$$

$$\begin{pmatrix} -b \\ -a-c \\ -b \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\therefore \quad b = 0 \quad a = -c$$

$$\therefore \quad X_0 = \left(Y_1^1 - Y_1^{-1} \right) A_0$$

Normalize

$$\langle X_0 | X_0 \rangle = 1 = \left(\langle Y_1^1 | - \langle Y_1^{-1} | \right)\left(| Y_1^1 \rangle - | Y_1^{-1} \rangle \right) A_0^2$$

$$= (1+1) A_0^2$$

$$A_0^2 = \frac{1}{2}$$

$$\boxed{\therefore \quad X_0 = \frac{1}{\sqrt{2}} \left(Y_1^1 - Y_1^{-1} \right)}$$

for $\alpha = \pm 1$

$$\begin{pmatrix} \pm\sqrt{2} & -1 & 0 \\ -1 & \pm\sqrt{2} & 0 \\ 0 & -1 & \pm\sqrt{2} \end{pmatrix} \begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} \pm\sqrt{2}\, a - b \\ -a \pm \sqrt{2}\, b \\ -b \pm \sqrt{2}\, c \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\therefore \quad b = \pm\sqrt{2}\, a = \pm\sqrt{2}\, c$$

$$\therefore \quad a = c$$

$$X_\pm = A_\pm \left(Y_1^1 \pm \sqrt{2}\, Y_1^0 + Y_1^{-1} \right)$$

Normalize

$$\langle X_\pm | X_\pm \rangle = 1 = A_\pm^2 \left(\langle Y_1^1| \pm \sqrt{2} \langle Y_1^0| + \langle Y_1^{-1}| \right) \left(|Y_1^1\rangle \pm \sqrt{2}\, |Y_1^0\rangle + |Y_1^{-1}\rangle \right)$$

$$= A_\pm^2 \left(1 + 2 + 1 \right)$$

$$A_\pm = \frac{1}{2}$$

$$\boxed{X_\pm = \frac{1}{2} \left(Y_1^1 \pm \sqrt{2}\, Y_1^0 + Y_1^{-1} \right)}$$

By inspection $\quad |\psi(t=0)\rangle = Y_1^{-1} = \dfrac{X_+ + X_- - \sqrt{2}\, X_0}{2}$

$\therefore$ Measurement of $\hat{L}_x$ will find values $0\hbar, +\hbar, -\hbar$ with relative probabilities of $\frac{1}{2}, \frac{1}{4}, \frac{1}{4}$.