9.1) φ is an eigenstate of $\hat{L}_x, \hat{L}_y$

$\therefore \quad \hat{L}_x \varphi = l_x \varphi \qquad \hat{L}_y = l_y \varphi$

$$[\hat{L}_x, \hat{L}_y]\varphi = (\hat{L}_x \hat{L}_y - \hat{L}_y \hat{L}_x)\varphi$$
$$= (l_x l_y - l_y l_x)\varphi$$
$$= 0$$

$$[\hat{L}_x, \hat{L}_y]\varphi = i\hbar \hat{L}_z \varphi = i\hbar\, l_z \varphi$$

$\therefore \quad l_z = 0 \qquad \hat{L}_z = 0$

$[\hat{L}_x, \hat{L}_z]\varphi = -i\hbar \hat{L}_y \varphi = 0$ since φ is an eigenstate of $\hat{L}_x, \hat{L}_y$.

$\therefore \quad \Delta L_x \, \Delta L_z \geq \frac{\hbar}{2} |\langle \hat{L}_y \rangle|$

$\Delta L_x \, \Delta L_z \geq 0$

$\therefore \quad \langle \hat{L}_y \rangle = 0 = \hat{L}_y$

$[\hat{L}_y, \hat{L}_z]\varphi = i\hbar \hat{L}_x \varphi = 0$ since φ is an eigenfunction of $\hat{L}_y, \hat{L}_z$

$\therefore \quad \Delta L_y \, \Delta L_z \geq \frac{\hbar}{2} |\langle \hat{L}_x \rangle|$

$\Delta L_y \, \Delta L_z \geq 0$

$\therefore \quad \langle \hat{L}_x \rangle = \hat{L}_x = 0$

9.3) $L = \sqrt{56}\,\hbar \implies l = 7$

$L_x \leq \hbar l$

The angle will be smallest when L_x is at the maximum value.

$$\cos\theta = L_x / L = 7\hbar / \sqrt{56}\,\hbar \implies \underline{\theta = 21^\circ}$$

9.5)

Show $\hbar\omega = \frac{\hbar^2}{I}(\ell+1)$

$$E_\ell = \frac{\hbar^2}{2I}\ell(\ell+1)$$

$$E_{\ell+1} - E_\ell = \frac{\hbar^2}{2I}(\ell+1)(\ell+2) - \frac{\hbar^2}{2I}\ell(\ell+1)$$

$$= \frac{\hbar^2}{2I}\left[\cancel{\ell^2} + 3\ell + 2 - \cancel{\ell^2} - \ell\right]$$

$$= \frac{\hbar^2}{2I}(2\ell + 2)$$

$$= \frac{\hbar^2}{I}(\ell+1)$$

9.6) 7.48

vibrational modes: $E_n = (n + \frac{1}{2})\hbar\omega$

rotational modes: $E_l = \frac{\hbar^2}{2I} l(l+1)$

only $l \to l \pm 1$ transitions take place

$\frac{\hbar\omega}{k_B} = 4150\ K$ vibrational

$\frac{\hbar^2}{2Ik_B} = 15.2\ K$ rotational

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energy differend between vibrational levels $= E_{n+1} - E_n$

$$= (n+1+\tfrac{1}{2})\hbar\omega - (n+\tfrac{1}{2})\hbar\omega$$

$$= \hbar\omega$$

energy differend between rotational levels $= E_{l+1} - E_l$

$$= \frac{\hbar^2}{2I}(l+1)(l+2) - \frac{\hbar^2}{2I} l(l+1)$$

$$= \frac{\hbar^2}{2I}\left[\cancel{l^2} + 3l + 3 - \cancel{l^2} - l\right]$$

$$= \frac{\hbar^2}{I}\left[l + 1\right]$$

Emission from vibrational states will consist of high energy photons ($\hbar\omega \gg \hbar^2/2I$) of a single frequency. Emission from rotational states will cover a range of values.