



$$r_d = 1.2 \text{ fm} \quad r_p = 1.0 \text{ fm}$$

a) $\Delta E = ?$

$$\Delta E = (m_p + m_d - m_{\text{He}}) c^2$$

$$m_{\text{He}} = 3.0160293 \text{ u}$$

$$m_d = 2.0141018 \text{ u}$$

$$m_p = 1.0072765 \text{ u}$$

$$= (1.0072765 \text{ u} + 2.0141018 \text{ u} - 3.0160293 \text{ u}) \left(931.5 \frac{\text{MeV}}{\text{u}} \right) c^2$$

$$= (0.005349 \text{ u}) \left(931.5 \frac{\text{MeV}}{\text{u}} \right)$$

$$\boxed{\Delta E = 4.983 \text{ MeV}}$$

b) $V_c = \frac{k e^2}{r} \frac{Z_1 Z_2}{r}$ $r_p = 0.86 \text{ fm}$ $Z_p = 1$
 $r_d = 2.14 \text{ fm}$ $Z_d = 1$
 $V_c = \frac{192 \text{ MeV} \cdot \text{fm}}{137} \frac{(1)(1)}{(0.86 + 2.14) \text{ fm}}$

$$\boxed{V_c = 0.48 \text{ MeV}}$$

c) $KE_p = \frac{3}{2} kT = \frac{3}{2} (8.62 \times 10^{-11} \frac{\text{MeV}}{\text{K}}) (10^7 \text{ K})$
 $= 1.3 \times 10^{-3} \text{ MeV}$
 $= 1.3 \text{ eV}$

$$KE_p \ll V_c$$

$$d) \quad r_{\text{doca}} = ?$$

$$E = KE + V_c$$

$$\text{at } r_{\text{doca}} \quad KE = 0$$

$$E = V_c$$

$$E = KE_p$$

$$KE_p = \frac{\hbar c}{137} \frac{Z_1 Z_2}{r_{\text{doca}}}$$

$$r_{\text{doca}} = \frac{\hbar c}{137} \frac{Z_1 Z_2}{KE_p}$$

$$= \frac{197 \text{ MeV} \cdot \text{fm}}{137}$$

$$\frac{(1)(1)}{1.3 \times 10^{-3} \text{ MeV}}$$

$$r_{\text{doca}} = 1106 \text{ fm}$$

$$2) \quad p(v)dv = 4\pi \left(\frac{m}{2\pi kT} \right)^{3/2} N^2 e^{-mv^2/2kT} dv$$

a) To get most probable velocity set $p'(v) = 0$,

$$\begin{aligned} \frac{dp}{dv} &= 4\pi \left(\frac{m}{2\pi kT} \right)^{3/2} \left[2v e^{-mv^2/2kT} + \right. \\ &\quad \left. v^2 \left(-\frac{2mv}{2kT} \right) e^{-mv^2/2kT} \right] \\ &= 4\pi \left(\frac{m}{2\pi kT} \right)^{3/2} N e^{-mv^2/2kT} \left[2 - \frac{mv^2}{kT} \right] \end{aligned}$$

This will be zero at $v=0$, $v \rightarrow \infty$, and when

$$2 - \frac{mv^2}{kT} = 0$$

$$v_e = \sqrt{\frac{2kT}{m}}$$

$$b) \quad E = K + V \quad v_e = v_{\text{escape}} = ?$$

$$= \frac{1}{2} m v_e^2 - \frac{G m_1 m_e}{r_e} \quad \begin{array}{l} m_e \equiv \text{Earth mass} \\ r_e \equiv \text{Earth radius} \end{array}$$

To escape the Earth's gravity, one must have $E=0$

$$0 = \frac{1}{2} m_1 v_e^2 - \frac{G m_1 m_e}{r}$$

$$\frac{1}{2} m_1 v_e^2 = \frac{G m_1 m_e}{r}$$

$$v_e = \sqrt{\frac{2 G m_e}{r}}$$

$$v_e = \left[\frac{2 \left(6.67 \times 10^{-11} \frac{\text{Nm}^2}{\text{kg}^2} \right) \left(5.97 \times 10^{24} \text{ kg} \right)}{6.38 \times 10^6 \text{ m}} \right]^{1/2}$$

$$v_e = 1.1 \times 10^4 \text{ m/s}$$

- c) The Maxwellian velocity distribution describes the probability of a particle in a gas having a velocity in the range $v \rightarrow v+dv$. All the particles with a velocity above the escape velocity are candidates for escaping the Earth's gravitational field.

$$\text{answer} = \int_{v_e}^{\infty} P(v) dv$$

$$v_e = \sqrt{\frac{2kT}{m}}$$

$$T = \frac{m v_e^2}{2k}$$

$$= \frac{(2)(931.5 \times 10^6 \text{ eV}/c^2) (1.1 \times 10^4 \text{ m/s})^2 \left(\frac{c}{3 \times 10^8 \text{ m/s}} \right)^2}{2(8.62 \times 10^{-5} \text{ eV/K})}$$

$$T = 1.45 \times 10^4 \text{ K}$$

This result is far hotter than the average temperature in the upper atmosphere so we are OK.

3) Get velocity for two protons at the Coulomb barrier in the center of mass.

$$R = 1 \text{ fm} \quad E_{\text{coul}} = \frac{e^2}{2R} \quad e^2 = \frac{\hbar c}{137}$$

$$= \frac{197 \text{ MeV} \cdot \text{fm}}{137} \cdot \frac{1}{2 \text{ fm}}$$

$$E_{\text{coul}} = 0.718 \text{ MeV}$$

Get the lab energy of the protons for input to P(w)

$$E_{\text{cm}} = \frac{m_{\text{tgt}}}{m_{\text{proj}} + m_{\text{tgt}}} E_{\text{lab}}$$

$$E_{\text{cm}} = E_{\text{coul}}$$

$$m_{\text{proj}} = m_{\text{tgt}} = 1u$$

$$E_{\text{coul}} = \frac{1u}{1u + 1u} E_{\text{lab}}$$

$$E_{\text{lab}} = 2 E_{\text{coul}} = 2(0.718 \text{ MeV})$$

$$E_{\text{lab}} = 1.44 \text{ MeV}$$

$$E_{\text{lab}} = \frac{1}{2} m v_{\text{lab}}^2$$

$$v_{\text{lab}} = \sqrt{\frac{2 E_{\text{lab}}}{m}} = \sqrt{\frac{2(1.44 \text{ MeV})}{938 \text{ MeV}/c^2}}$$

$$v_{\text{lab}} = 0.055c$$

$$= 1.66 \times 10^5 \text{ m/s}$$

Put this into Maxwellian velocity

$$\text{answer} = \int_{v_{\text{lab}}}^{\infty} P(v) dv = 0 \quad \text{Mathematica}$$

Mathematica lower limit
2.2 x 10⁻³⁰⁸

$$4) \quad t_{11} = \frac{1}{4} \left[\left(1 + \frac{k_2}{k_1} \right) e^{-i k_2 2a} \left(1 + \frac{k_1}{k_2} \right) + \left(1 - \frac{k_2}{k_1} \right) e^{i k_2 2a} \left(1 - \frac{k_1}{k_2} \right) \right]$$

$$V_0 = 0.5 \text{ MeV}$$

$$a = 4 \text{ fm}$$

$$k_1 = \frac{\sqrt{2\mu E}}{\hbar c}$$

$$k_2 = \frac{\sqrt{2\mu (E - V_0)}}{\hbar c}$$

a. Get the velocity at the peak of $P(v)$. See 2.a.

$$T_0 = 10^7 \text{ K} \quad N_0 = \sqrt{\frac{2kT_0}{m}}$$

$$= \left[\frac{2 \left(8.617 \times 10^{-5} \frac{\text{eV}}{\text{K}} \right) (10^7 \text{ K})}{938 \times 10^6 \frac{\text{eV}}{c^2}} \right]^{1/2}$$

$$N_0 = 1.36 \times 10^{-3} c$$

b. Get the energy at the peak of $P(v)$

$$E_0 = \frac{1}{2} m v^2 = \frac{1}{2} \left(938 \frac{\text{MeV}}{c^2} \right) \left(1.36 \times 10^{-3} c \right)^2$$

$$E_0 = 8.67 \times 10^{-4} \text{ MeV} = 867 \text{ eV}$$

c. Get the transmission coefficient at E_0 .

$$k_1 = \frac{\sqrt{2 (938 \text{ MeV}/c^2) (8.67 \times 10^{-4} \text{ MeV})}}{197 \text{ MeV} \cdot \text{fm}}$$

$$= 6.47 \times 10^{-3} \text{ fm}^{-1}$$

$$k_2 = \frac{\sqrt{2 (938 \text{ MeV}/c^2) (8.67 \times 10^{-4} \text{ MeV} - 0.5 \text{ MeV})}}{197 \text{ MeV} \cdot \text{fm}}$$

$$= 0.155 i \quad \text{Imaginary}$$

Plug all this into t_{11} using Mathematica and

$$T_r = \frac{1}{|t_n|^2} = 0.0155$$