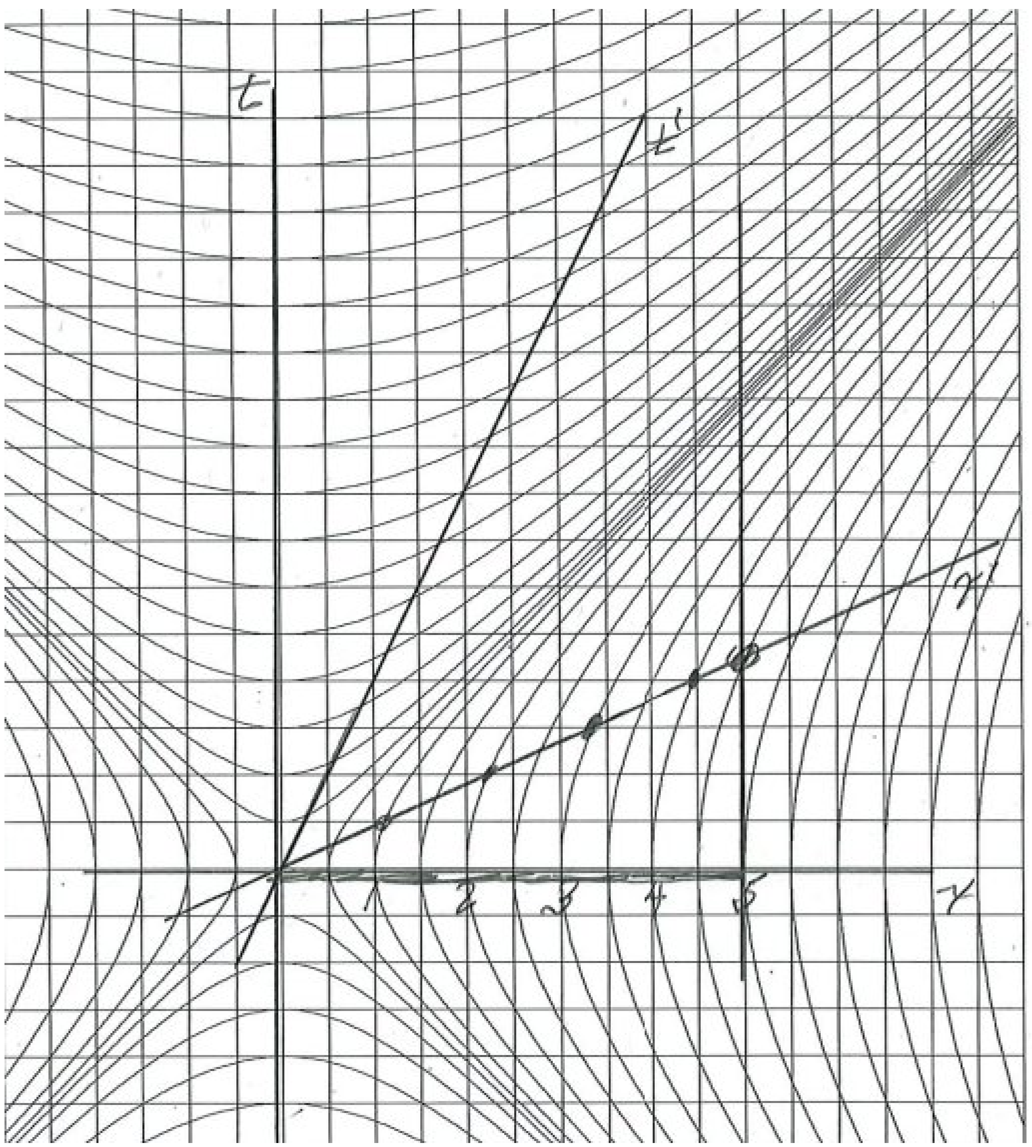
1) $L_p = 5\,ns$

$\beta = 0.5c$

$L \cong 4.45\,ns$ from plot

$$L = L_p\sqrt{1 - v^2/c^2} = (5\,ns)\sqrt{1-(0.5)^2} = 4.33\,ns$$



1)

$r = 8m$

$v = 0.9994c$

a. $\Delta t = \dfrac{\pi R}{v}$ $\qquad \Delta d = 2R = 16m$

$= \dfrac{\pi(8m)}{(0.9994)(3\times10^8 m/s)}$

$\underline{= 8.38\times10^{-8}s}$

b. $\Delta s = \left[c^2\Delta t^2 - \Delta d^2\right]^{1/2}$

$= \sqrt{(3\times10^8 \frac{m}{s}\times 8.38\times10^{-8}s)^2 - (16m)^2}$

$\boxed{\Delta s = 19.4m} = 64.7\,ns$

c. $\Delta\tau = \Delta t\sqrt{1-v^2/c^2}$

$= (8.38\times10^{-8}s)\sqrt{1-(0.9994)^2}$

$\underline{\Delta\tau = 2.9\times10^{-9}s}$

3) $L = 10\,ly$

$v_S = 0.9c$

$v_G = 0.7c$

$\Delta t = ?$

$$\Delta t_{G1} = \frac{L}{v_G} = \frac{10\,ly}{0.7c} = 14.29\,y$$

$$\Delta t_{G2} = \Delta t_{G1}\sqrt{1-\beta_G^2}$$

$$= 14.29\,y\sqrt{1-(0.7)^2}$$

$$= 10.21\,y$$ Goslo's aging

$$\Delta t_{S1} = \frac{L}{v_S} = \frac{10\,ly}{0.9c} = 11.11\,y$$

$$\Delta t_{S2} = \Delta t_{S1}\sqrt{1-\beta_S^2}$$

$$= 11.11\,y\sqrt{1-(0.9)^2}$$

$$= 4.843\,y$$ Speedo's aging

Time spent waiting for Goslo to arrive $\longrightarrow$ $\Delta t_{SG} = \Delta t_{G1} - \Delta t_{S1}$

$$= 14.29\,y - 11.11\,y$$

$$= 3.18\,y$$

$$\text{answer} = \Delta t_{G2} - (\Delta t_{SG} + \Delta t_{S2})$$

$$= 10.21 - (3.18\,y + 4.843\,y)$$

$$\boxed{\text{answer} = 2.187\,y}$$

4) $v = \frac{2}{5} c$

$t' = 4s \quad x' = 2s$

$t = 5.25s \quad x = 3.8s$

$x' = \gamma(x - \beta t)$

$\frac{x'}{\gamma} + \beta t = x$

$\gamma = \frac{1}{\sqrt{1-\beta^2}}$

$= \frac{1}{\sqrt{1-(0.4)^2}}$

$= 1.091$

$t' = \gamma(t - \beta x)$

$\frac{t'}{\gamma} + \beta x = t$

$\therefore x = \frac{x'}{\gamma} + \beta\left(\frac{t'}{\gamma} + \beta x\right)$

$x - \beta^2 x = \frac{x'}{\gamma} + \frac{\beta}{\gamma} t'$

$x(1-\beta^2) = \quad ''$

$$x = \frac{(x' + \beta t')\frac{1}{\gamma}}{1-\beta^2} = \gamma(x' + \beta t')$$

$$= \frac{(2s + (0.4)(4s))\frac{1}{1.091}}{1-(0.4)^2}$$

$x = 3.93s$

$t = \beta x + \frac{t'}{\gamma} = (0.4)(3.93s) + \frac{4s}{1.091}$

$t = 5.238s$

Or do it this way.

4) another way for analytic part

$$x = \gamma(x' + \beta t') \qquad t = \gamma[t' + \beta x']$$

$$\gamma = \frac{1}{\sqrt{1-\beta^2}} = \frac{1}{\sqrt{1-(0.4)^2}} = 1.091$$

$$x = 1.091(2s + (0.4)(4s)) = \underline{3.93s}$$

$$t = (1.091)(4s + (0.4)(2s)) = \underline{5.237s}$$