## Physics 205 Test 3

I pledge that I have neither given nor received unauthorized assistance during the completion of this work.

Name $\qquad$ Signature $\qquad$
Questions ( 7 for 7 pts. apiece) Answer in complete, well-written sentences WITHIN the spaces provided.

1. If the momentum of a particle going close to the speed of light doubles does the velocity double? Explain.
2. If you move from one bright fringe in a double-slit interference pattern to the next one out from the central maximum, does the path length difference $\delta$ increase, decrease, or stay the same? If it changes, by how much in terms of wavelength $\lambda$ ? Explain.
3. A system consists of two photons moving in opposite directions, one with energy $E$ and the other with energy $4 E$. What is the system's total mass? Explain.
4. The figure below shows a two-observer, spacetime diagram. It is claimed that the points $P$ and $Q$ are causally connected. Is that true or false? Explain.

5. A sealed cup of water is placed inside a microwave oven. The water absorbs microwave energy which causes its atoms to vibrate more vigorously making the water warmer. What happens to the mass of the water in the cup? Explain.
6. Answer this question non-relativistically. An observer on the (roughly) stationary Earth watches a spaceship traveling away and towards the galactic core at a speed $v_{r}$. The craft emits a burst of light from a beacon that is later observed by the Earthbound observer. After a proper time interval $\Delta t_{\text {rocket }}$ as measured by the captain of the spaceship, another burst of light is emitted. A schematic drawing of the two events is shown below. The ship travels a proper distance $\Delta d$ between pulses as measured by the Earthbound observer. How long is the time interval between light pulses as measured by the Earthbound observer in terms of $v_{r}, \Delta t_{\text {rocket }}$, and $\Delta d$ ?

7. From the figure below how would you determine the size of the slits and their separation in a double slit experiment? You should use the plot to define the numerical inputs to your calculation, but don't actually do any calculations.


Problems (4). Clearly show all reasoning for full credit. Use a separate sheet for your work.

1. 8 pts. A particle has 4 -momentum $\underset{\sim}{p}=\left[p_{t}, p_{x}, p_{y}, p_{z}\right]=[5 \mathrm{~kg}, 0,3 \mathrm{~kg}, 0]$ in SR units. What is the particle's mass?
2. 12 pts. A spaceship, at rest in the reference frame $S$, is given a speed increment of $0.5 c$. Relative to its new rest frame it is given a further $0.5 c$ increment. This process is continued until its speed with respect to its original frame $S$ exceeds $0.96 c$. How many increments does this process require?
3. 15 pts. When a high-power laser is used in the Earths atmosphere, the electric field can ionize the air, turning it into a conducting plasma that reflects the laser light. In dry air at $0^{\circ} \mathrm{C}$ and 1 atm , electric breakdown occurs for fields with amplitudes above about $2.0 \times 10^{5} \mathrm{~V} / \mathrm{m}$. (a) What average laser beam intensity will produce such a field? (b) At this maximum intensity, what power can be delivered in a cylindrical beam of diameter 4.0 mm ? (c) What is the size of the magnetic field?
4. 16 pts. A particle of mass $m$ decays into two identical particles that move in opposite directions, each with a speed of $\frac{12}{13} c$. What is the mass of each of the product particles expressed as a fraction of $m$ ?

DO NOT WRITE BELOW THIS LINE.

## Physics 205 Equations

$$
\begin{aligned}
& \vec{F}_{n e t}=\sum \vec{F}_{i}=m \vec{a}=\frac{d \vec{p}}{d t} \quad v=\frac{d x}{d t} \quad v=\frac{\Delta x}{\Delta t} \quad x=\frac{1}{2} a t^{2}+v_{0} t+x_{0} \quad v=a t+v_{0} \quad a_{g}=-g \quad a_{c}=\frac{v^{2}}{r} \\
& \vec{F}_{\text {Earth }}=-m g \hat{j} \quad K E=\frac{1}{2} m v^{2} \quad K E_{0}+P E_{0}=K E_{1}+P E_{1} \quad P E_{E a r t h}=m g h \quad P E_{V}=q V \\
& \qquad \vec{p}_{i}=\vec{p}_{f} \quad \vec{p}=m \vec{v} \quad \vec{F}_{C}=k_{e} \frac{q_{1} q_{2}}{r^{2}} \hat{r} \quad \vec{E} \equiv \frac{\vec{F}}{q_{0}} \quad d \vec{E}=\frac{1}{4 \pi \epsilon_{0}} \frac{d q}{r^{2}} \hat{r} \\
& d \vec{B}=\frac{\mu_{0}}{4 \pi} \frac{I d \vec{s} \times \hat{r}}{r^{2}}=\frac{\mu_{0}}{4 \pi} \frac{q d \vec{v} \times \hat{r}}{r^{2}} \quad \vec{F}_{B}=q \vec{v} \times \vec{B} \quad\left|\vec{F}_{B}\right|=|q v B \sin \theta| \\
& \qquad \begin{array}{ll|l|l|}
\hline \text { Galilean } \\
\text { Transformation } & \begin{array}{l}
\text { Lorentz } \\
\text { Transformation } \\
\text { SI units } \\
\text { SI units }
\end{array} & \begin{array}{l}
\text { Lorentz } \\
\text { Transformation } \\
\text { SR units }
\end{array} \\
\hline x^{\prime}=x-v t & \begin{array}{l}
x^{\prime}=\gamma(x-v t) \\
y^{\prime}=y \\
y^{\prime}=y \\
z^{\prime}=z \\
z^{\prime}=z \\
t^{\prime}=t \\
v_{x}^{\prime}=v_{x}-v_{O}
\end{array} & \begin{array}{l}
x^{\prime}=\gamma(x-\beta t) \\
y^{\prime}=y \\
v_{x}^{\prime}=\frac{v_{x}-v x}{1-v_{x} v / c^{2}}
\end{array} & \begin{array}{l}
z^{\prime}=z \\
t^{\prime}=\gamma(t-\beta x) \\
v_{x}^{\prime}=\frac{v_{x}-\beta}{1-v_{x} \beta}
\end{array} \\
\hline
\end{array}
\end{aligned}
$$

| Coordinate Time | Proper Time | Spacetime Interval |
| :--- | :--- | :--- |
| Time between two | Time between two | Time between two |
| events in an in- | events measured by | events measured by |
| ertial frame mea- |  |  |
| sured with syn- |  |  |
| chronized clocks | bame clock at | the same, inertial |
| both events. | clock at both events. |  |
| $c \Delta t, \Delta t$ | $\Delta \tau_{S I}, \Delta \tau_{S R}$ | $\Delta s_{S I}, \Delta s_{S R}$ |
| Frame dependent | Frame independent | Frame independent |

$$
\begin{gathered}
\Delta s_{S I}^{2}=c^{2} \Delta t^{2}-\Delta d^{2}=\Delta s_{S I}^{\prime}{ }^{2} \quad \text { or } \quad \Delta s_{S R}^{2}=\Delta t^{2}-\Delta d^{2}=\Delta s_{S R}^{\prime}{ }^{2} \\
\Delta \tau_{S I}=\int_{t_{A}}^{t_{B}} \sqrt{1-\frac{v^{2}}{c^{2}}} d t \quad \text { or } \quad \Delta \tau_{S R}=\int_{t_{A}}^{t_{B}} \sqrt{1-\beta^{2}} d t \\
\Delta \tau_{S I}=\sqrt{1-v^{2} / c^{2}} \Delta t \quad \text { or } \quad \Delta \tau_{S R}=\sqrt{1-\beta^{2}} \Delta t \\
L_{S I}=L_{R} \sqrt{1-v^{2} / c^{2}} \quad \text { or } \quad L_{S R}=L_{R} \sqrt{1-\beta^{2}}
\end{gathered}
$$

$$
\begin{gathered}
v_{x}^{\prime}=\frac{v_{x}-v}{1-v_{x} v / c^{2}} \quad v_{y}^{\prime}=\frac{v_{y} \sqrt{1-v_{x}^{2} / c^{2}}}{1-v_{x} v / c^{2}} \quad K E=E-m c^{2} \quad \text { SI units } \\
v_{x}^{\prime}=\frac{v_{x}-\beta}{1-v_{x} \beta} \quad v_{y}^{\prime}=\frac{v_{y} \sqrt{1-\beta^{2}}}{1-v_{x} \beta} \quad K E=E-m \quad \text { SR units } \\
{\underset{\sim}{p}}_{i}=\underset{\sim}{p} \quad \underset{\sim}{p}{\underset{\sim}{p}}_{1}^{p_{2}}=\underset{\sim}{p} \cdot{\underset{\sim}{p}}_{4}^{p}
\end{gathered}
$$

$$
\begin{aligned}
& \underset{\sim}{p}=m d \underset{\sim}{s} / d \tau=\left[m \frac{d t}{d \tau}, m \frac{d x}{d \tau}, m \frac{d y}{d \tau}, m \frac{d z}{d \tau}\right]=\frac{m}{\sqrt{1-|\vec{v}|^{2}}}[1, \vec{v}] \quad \underset{\sim}{p} \cdot \underset{\sim}{p}=E_{r}^{2}-|\vec{p}|^{2}=m^{2} \quad \text { SR units } \\
& \underset{\sim}{p}=m d \underset{\sim}{s} / d \tau=\left[m c \frac{d t}{d \tau}, m \frac{d x}{d \tau}, m \frac{d y}{d \tau}, m \frac{d z}{d \tau}\right]=\frac{m}{\sqrt{1-v^{2} / c^{2}}}[c, \vec{v}] \quad \underset{\sim}{p} c \cdot \underset{\sim}{p} c=E_{r}^{2}-|\vec{p} c|^{2}=\left(m c^{2}\right)^{2} \quad \text { SI units } \\
& y=A \sin (k x-\omega t+\phi) \quad k \lambda=\omega T=2 \pi \quad E=E_{m} \sin (k x-\omega t+\phi) \quad B=B_{m} \sin (k x-\omega t+\phi) \quad \phi=k \delta \\
& \vec{S}=\frac{1}{\mu_{0}} \vec{E} \times \vec{B} \quad E=c B \quad\langle | \vec{S}| \rangle=I=\frac{E^{2}}{2 \mu_{0} c}=\frac{\text { energy }}{\text { area } \cdot \operatorname{time}} \quad c=\frac{\lambda}{T}=\lambda f \\
& \delta=d \sin \theta=m \lambda(m=0, \pm 1, \pm 2, \ldots) \quad \delta=a \sin \theta=m \lambda(m= \pm 1, \pm 2, \ldots) \quad \phi=k \delta
\end{aligned}
$$

$$
I_{\text {int }}=I_{m} \cos ^{2}\left(\frac{\pi d}{\lambda} \sin \theta\right) \quad I_{d i f f}=I_{m}\left[\frac{\sin \left(\frac{\pi a}{\lambda} \sin \theta\right)}{\frac{\pi a}{\lambda} \sin \theta}\right]^{2} \quad I_{\text {total }}=I_{m} \cos ^{2}\left(\frac{\pi d}{\lambda} \sin \theta\right)\left[\frac{\sin \left(\frac{\pi a}{\lambda} \sin \theta\right)}{\frac{\pi a}{\lambda} \sin \theta}\right]^{2}
$$

$$
\frac{d}{d x}(f(u))=\frac{d f}{d u} \frac{d u}{d x} \quad \int x^{n} d x=\frac{x^{n+1}}{n+1} \quad \int \frac{1}{x} d x=\ln x \quad \vec{A} \cdot \vec{B}=A B \cos \theta \quad|\vec{A} \times \vec{B}|=|A B \sin \theta|
$$

$$
\frac{d}{d x}\left(x^{n}\right)=n x^{n-1} \quad \frac{d e^{x}}{d x}=e^{x} \quad \frac{d}{d x}(\ln x)=\frac{1}{x} \quad \frac{d}{d x}(\cos a x)=-a \sin a x \quad \frac{d}{d x}(\sin a x)=a \cos a x
$$

$$
\langle x\rangle=\frac{1}{N} \sum_{i} x_{i} \quad \sigma=\sqrt{\frac{\sum_{i}\left(x_{i}-\langle x\rangle\right)^{2}}{N-1}} \quad C=2 \pi r \quad A=4 \pi r^{2} \quad V=A h \quad V=\frac{4}{3} \pi r^{3}
$$

$$
\frac{d f(x)}{d x}=\lim _{\Delta x \rightarrow 0} \frac{f(x+\Delta x)-f(x)}{\Delta x} \int_{a}^{b} f(x) d x=\lim _{\Delta x \rightarrow 0} \sum_{n=1}^{N} f(x) \Delta x
$$

$$
\int \frac{1}{x} d x=\ln x \quad \int x^{n} d x=\frac{x^{n+1}}{n+1} \quad \int e^{a x} d x=\frac{e^{a x}}{a} \quad \int \frac{1}{\sqrt{x^{2}+a^{2}}} d x=\ln \left[x+\sqrt{x^{2}+a^{2}}\right]
$$

$$
\begin{aligned}
& \int \frac{x}{\sqrt{x^{2}+a^{2}}} d x=\sqrt{x^{2}+a^{2}} \int \frac{x^{2}}{\sqrt{x^{2}+a^{2}}} d x=\frac{1}{2} x \sqrt{x^{2}+a^{2}}-\frac{1}{2} a^{2} \ln \left[x+\sqrt{x^{2}+a^{2}}\right] \\
& \int \sqrt{1-a x^{2}} d x=\frac{x}{2} \sqrt{1-a x^{2}}+\frac{\arcsin (\sqrt{a} x)}{2 \sqrt{a}} \int \frac{x^{3}}{\sqrt{x^{2}+a^{2}}} d x=\frac{1}{3}\left(-2 a^{2}+x^{2}\right) \sqrt{x^{2}+a^{2}}
\end{aligned}
$$

## Physics 205 Constants and Conversions

| Avogadro's number $\left(N_{A}\right)$ | $6.022 \times 10^{23}$ | Speed of light $(c)$ | $3 \times 10^{8} \mathrm{~m} / \mathrm{s}$ |
| :--- | :--- | :--- | :--- |
| $k_{B}$ | $1.38 \times 10^{-23} \mathrm{~J} / \mathrm{K}$ | proton/neutron mass | $1.67 \times 10^{-27} \mathrm{~kg}$ |
| 1 u | $1.67 \times 10^{-27} \mathrm{~kg}$ | $g$ | $9.8 \mathrm{~m} / \mathrm{s}^{2}$ |
| Gravitation constant | $6.67 \times 10^{-11} \mathrm{~N}-\mathrm{m}^{2} / \mathrm{kg}^{2}$ | Earth's radius | $6.37 \times 10^{6} \mathrm{~m}$ |
| Coulomb constant $\left(k_{e}\right)$ | $8.99 \times 10^{9} \frac{\mathrm{N-m}^{2} \mathrm{C}^{2}}{}$ | Electron mass | $9.11 \times 10^{-31} \mathrm{~kg}$ |
| Elementary charge $(e)$ | $1.60 \times 10^{-19} \mathrm{C}$ | Proton/Neutron mass | $1.67 \times 10^{-27} \mathrm{~kg}$ |
| Permittivity constant $\left(\epsilon_{0}\right)$ | $8.85 \times 10^{-12} \frac{\mathrm{~kg}^{2}}{\mathrm{N-m}}$ | 1.0 eV | atomic mass unit $(\mathrm{u})$ |
| 1 MeV | $10^{6} \mathrm{eV}$ | $1.66 \times 10^{-27} \mathrm{~kg}$ |  |
| Planck's constant $(\mathrm{h})$ | $6.63 \times 10^{-34} \mathrm{Js}$ | Planck's constant $(\mathrm{h})$ | $4.14 \times 10^{-15} \mathrm{eV} \mathrm{s}$ |
| Permeability constant $\left(\mu_{0}\right)$ | $1.26 \times 10^{-6} \mathrm{Tm} / \mathrm{A}$ | Rydberg constant $\left(R_{H}\right)$ | $1.097 \times 10^{7} \mathrm{~m}$ |
| Becquerel $(B q)$ | $1 \mathrm{decay} / \mathrm{s}$ | Curie $(\mathrm{Ci})$ | $3.7 \times 10^{10} \mathrm{~Bq}$ |

