## Physics 205 Test 2

I pledge that I have neither given nor received unauthorized assistance during the completion of this work.

Name $\qquad$ Signature $\qquad$
Questions ( 7 for 7 pts. apiece) Answer in complete, well-written sentences WITHIN the spaces provided.

1. A spacecraft zooms past the Earth at constant velocity. An observer on Earth measures a clock on the spacecraft is ticking at one-third the rate as an identical clock on the Earth. What does an observer see on the spacecraft for the Earth-based clock's ticking rate? Explain.
2. The Other frame is moving in the $+x$ direction with $x$-velocity $0.5 c$. What lines in the figure correspond to the $t^{\prime}$ and and $x^{\prime}$ axes for this frame? Explain.

3. Two lights are 500 light - ns apart along a straight road. In the ground frame, the west light flashes 600 ns before the east light flashes. Could these flashes be simultaneous in the frame of a car moving along the road at an appropriate speed? (Hint: Use a qualitative spacetime diagram.)
4. An object's length would be negative in a frame where it travels faster than the speed of light. True or False? Explain.
5. The figure shows two clocks in Home frame $S$ that are synchronized and one clock in the Other frame $S^{\prime}$. Clocks $C_{1}$ and $C_{1}^{\prime}$ both read zero when they pass each other. When clocks $C_{1}^{\prime}$ and $C_{2}$ pass each other which clock has a smaller reading and which clock measures a proper time? Explain.

6. The figure shows a rocket traveling from left to right. At the instant it is halfway between two trees, lightning simultaneously (in the rocket's frame) hits both trees. Do the light flashes reach the rocket pilot simultaneously? If not, which reaches her first? Explain.

7. As the meter stick in the figure flies past you in the Home frame, you simultaneously measure the positions of both ends and determine that it is less than $1-m$ long. To an experimenter in the Other frame, the meter stick's frame, did you make your two measurements simultaneously? Explain.


Problems (4). Clearly show all reasoning for full credit. Use a separate sheet for your work.

1. 10 pts. Suppose an object with a rest length of $5 n s$ is at rest in the Home frame. The Other frame is moving with a speed $\beta=0.5$ relative to the Home frame. (a) Draw a two-observer diagram of this situation and use it to determine the length of the object in the Other frame. Also calculate the same length using the appropriate equation.
2. 12 pts. The designers of particle accelerators use electromagnetic fields to boost particles to relativistic speeds while at the same time constraining them to move in a circle inside a donut-shaped evacuated cavity. Imagine a particle traveling in such an accelerator in a circular path of radius 8 m at a constant speed of $0.9994 c$ as measured by laboratory observers. Let event $A$ be the particle passing a certain point on its circular path and let event $B$ be the particle passing the point on the circle directly opposite event A as shown in the figure. (a) What are the coordinate time $\Delta t$ and the distance $|\Delta \vec{d}|$ in the lab frame? (b) What is the spacetime interval between the events? (c) What is the proper time $\Delta \tau_{S R}$ or $\Delta \tau_{S I}$ (SI units) as measured by a clock traveling with the particle?

3. 14 pts. The identical twins Speedo and Goslo join a migration from the Earth to Planet X. It is 10.0 ly away in a reference frame in which both planets are at rest. The twins, of the same age, depart at the same time on different spacecraft. Speedos craft travels steadily at 0.90c, and Goslos travels at 0.70c. What is the age difference between the twins after Goslos spacecraft lands on Planet X. Which twin is the older?
4. 15 pts. An Other frame moves in the positive $x$-direction with a velocity $v=\frac{2}{5} c$ relative to the Home frame. Other frame observers see an event at $t^{\prime}=4 s$ and $x^{\prime}=2 s$ in SR units. (a) Use a two-observer spacetime diagram to determine when and where this event occurs in the Home Frame. (b) Determine when and where this event occurs in the Home Frame using the Lorentz transformations.

## Physics 205 Equations

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\begin{gathered}
v=\frac{d x}{d t} \quad v=\frac{\Delta x}{\Delta t} \quad x=\frac{1}{2} a t^{2}+v_{0} t+x_{0} \quad v=a t+v_{0} \quad a_{g}=-g \\
\vec{F}_{n e t}=\sum \vec{F}_{i}=m \vec{a}=\frac{d \vec{p}}{d t} \quad \vec{F}_{E a r t h}=-m g \hat{j} \quad a_{c}=\frac{v^{2}}{r} \\
K E=\frac{1}{2} m v^{2} \quad K E_{0}+P E_{0}=K E_{1}+P E_{1} \quad P E_{E a r t h}=m g h \quad P E_{V}=q V \\
\vec{p}_{i}=\vec{p}_{f} \quad \vec{p}=m \vec{v} \quad \vec{F}_{C}=k_{e} \frac{q_{1} q_{2}}{r^{2}} \hat{r} \quad \vec{E} \equiv \frac{\vec{F}}{q_{0}} \quad d \vec{E}=\frac{1}{4 \pi \epsilon_{0}} \frac{d q}{r^{2}} \hat{r} \\
d \vec{B}=\frac{\mu_{0}}{4 \pi} \frac{I d \vec{s} \times \hat{r}}{r^{2}}=\frac{\mu_{0}}{4 \pi} \frac{q d \vec{v} \times \hat{r}}{r^{2}} \quad \vec{F}_{B}=q \vec{v} \times \vec{B} \quad\left|\vec{F}_{B}\right|=|q v B \sin \theta|
\end{gathered}
$$

| Galilean | Lorentz | Lorentz |
| :--- | :--- | :--- |
| Transformation | Transformation | Transformation |
| SI units | SI units | SR units |
| $x^{\prime}=x-v t$ | $x^{\prime}=\gamma(x-v t)$ | $x^{\prime}=\gamma(x-\beta t)$ |
| $y^{\prime}=y$ | $y^{\prime}=y$ | $y^{\prime}=y$ |
| $z^{\prime}=z$ | $z^{\prime}=z$ | $z^{\prime}=z$ |
| $t^{\prime}=t$ | $t^{\prime}=\gamma\left(t-v x / c^{2}\right)$ | $t^{\prime}=\gamma(t-\beta x)$ |
| $v_{x}^{\prime}=v_{x}-v_{O}$ | $v_{x}^{\prime}=\frac{v_{x}-v_{O}}{1-v_{x} v / c^{2}}$ | $v_{x}^{\prime}=\frac{v_{x}-\beta}{1-v_{x} \beta}$ |


| Coordinate Time | Proper Time | Spacetime Interval |
| :--- | :--- | :--- |
| Time between two <br> events in an in- <br> ertial frame mea- <br> sured with syn- <br> chronized clocks | events measured by <br> the same clock at <br> both events. | Time between two <br> events measured by <br> the same, inertial <br> clock at both events. |
| $c \Delta t, \Delta t$ | $\Delta \tau_{S I}, \Delta \tau_{S R}$ | $\Delta s_{S I}, \Delta s_{S R}$ |
| Frame dependent | Frame independent | Frame independent |

$$
\begin{gathered}
\Delta s_{S I}^{2}=c^{2} \Delta t^{2}-\Delta d^{2}=\Delta s_{S I}^{\prime}{ }^{2} \quad \text { or } \quad \Delta s_{S R}^{2}=\Delta t^{2}-\Delta d^{2}=\Delta s_{S R}^{\prime 2} \\
\Delta \tau_{S I}=\int_{t_{A}}^{t_{B}} \sqrt{1-\frac{v^{2}}{c^{2}}} d t \quad \text { or } \quad \Delta \tau_{S R}=\int_{t_{A}}^{t_{B}} \sqrt{1-\beta^{2}} d t \\
\Delta \tau_{S I}=\sqrt{1-v^{2} / c^{2}} \Delta t \quad \text { or } \quad \Delta \tau_{S R}=\sqrt{1-\beta^{2}} \Delta t \\
L_{S I}=L_{R} \sqrt{1-v^{2} / c^{2}} \quad \text { or } \quad L_{S R}=L_{R} \sqrt{1-\beta^{2}}
\end{gathered}
$$

$$
\begin{gathered}
\frac{d}{d x}(f(u))=\frac{d f}{d u} \frac{d u}{d x} \quad \int x^{n} d x=\frac{x^{n+1}}{n+1} \quad \int \frac{1}{x} d x=\ln x \quad \vec{A} \cdot \vec{B}=A B \cos \theta \quad|\vec{A} \times \vec{B}|=|A B \sin \theta| \\
\frac{d}{d x}\left(x^{n}\right)=n x^{n-1} \quad \frac{d e^{x}}{d x}=e^{x} \quad \frac{d}{d x}(\ln x)=\frac{1}{x} \quad \frac{d}{d x}(\cos a x)=-a \sin a x \quad \frac{d}{d x}(\sin a x)=a \cos a x \\
\langle x\rangle=\frac{1}{N} \sum_{i} x_{i} \quad \sigma=\sqrt{\frac{\sum_{i}\left(x_{i}-\langle x\rangle\right)^{2}}{N-1}} \quad C=2 \pi r \quad A=4 \pi r^{2} \quad V=A h \quad V=\frac{4}{3} \pi r^{3} \\
\frac{d f(x)}{d x}=\lim _{\Delta x \rightarrow 0} \frac{f(x+\Delta x)-f(x)}{\Delta x} \quad \int_{a}^{b} f(x) d x=\lim _{\Delta x \rightarrow 0} \sum_{n=1}^{N} f(x) \Delta x \\
\int \frac{1}{x} d x=\ln x \\
\int x^{n} d x=\frac{x^{n+1}}{n+1} \quad \int e^{a x} d x=\frac{e^{a x}}{a} \quad \int \frac{1}{\sqrt{x^{2}+a^{2}}} d x=\ln \left[x+\sqrt{x^{2}+a^{2}}\right] \\
\int \frac{x}{\sqrt{x^{2}+a^{2}}} d x=\sqrt{x^{2}+a^{2}} \int \frac{x^{2}}{\sqrt{x^{2}+a^{2}}} d x=\frac{1}{2} x \sqrt{x^{2}+a^{2}}-\frac{1}{2} a^{2} \ln \left[x+\sqrt{x^{2}+a^{2}}\right] \\
\int \sqrt{1-a x^{2}} d x=\frac{x}{2} \sqrt{1-a x^{2}}+\frac{\arcsin (\sqrt{a} x)}{2 \sqrt{a}} \quad \int \frac{x^{3}}{\sqrt{x^{2}+a^{2}}} d x=\frac{1}{3}\left(-2 a^{2}+x^{2}\right) \sqrt{x^{2}+a^{2}}
\end{gathered}
$$

## Physics 205 Constants and Conversions

| Avogadro's number $\left(N_{A}\right)$ | $6.022 \times 10^{23}$ | Speed of light $(c)$ | $3 \times 10^{8} \mathrm{~m} / \mathrm{s}$ |
| :--- | :--- | :--- | :--- |
| $k_{B}$ | $1.38 \times 10^{-23} \mathrm{~J} / \mathrm{K}$ | proton/neutron mass | $1.67 \times 10^{-27} \mathrm{~kg}$ |
| 1 u | $1.67 \times 10^{-27} \mathrm{~kg}$ | $g$ | $9.8 \mathrm{~m} / \mathrm{s}^{2}$ |
| Gravitation constant | $6.67 \times 10^{-11} \mathrm{~N}-\mathrm{m}^{2} / \mathrm{kg}^{2}$ | Earth's radius | $6.37 \times 10^{6} \mathrm{~m}$ |
| Coulomb constant $\left(k_{e}\right)$ | $8.99 \times 10^{9} \frac{\mathrm{N-m}^{2} \mathrm{C}^{2}}{}$ | Electron mass | $9.11 \times 10^{-31} \mathrm{~kg}$ |
| Elementary charge $(e)$ | $1.60 \times 10^{-19} \mathrm{C}$ | Proton/Neutron mass | $1.67 \times 10^{-27} \mathrm{~kg}$ |
| Permittivity constant $\left(\epsilon_{0}\right)$ | $8.85 \times 10^{-12} \frac{\mathrm{~kg}^{2}}{\mathrm{N-m}}$ | 1.0 eV | $1.6 \times 10^{-19} \mathrm{~J}$ |
| 1 MeV | $10^{6} \mathrm{eV}$ | atomic mass unit $(u)$ | $1.66 \times 10^{-27} \mathrm{~kg}$ |
| Planck's constant $(h)$ | $6.63 \times 10^{-34} \mathrm{Js}$ | Planck's constant $(\mathrm{h})$ | $4.14 \times 10^{-15} \mathrm{eVs}$ |
| Permeability constant $\left(\mu_{0}\right)$ | $1.26 \times 10^{-6} \mathrm{Tm} / \mathrm{A}$ | Rydberg constant $\left(R_{H}\right)$ | $1.097 \times 10^{7} \mathrm{~m}$ |
| Becquerel $(B q)$ | $1 \mathrm{decay} / \mathrm{s}$ | Curie $(C i)$ | $3.7 \times 10^{10} \mathrm{~Bq}$ |

