## Physics 205 Final

I pledge that I have neither given nor received unauthorized assistance during the completion of this work.

Name $\qquad$ Signature $\qquad$
Questions (10 for 4 pts. apiece) Answer in complete, well-written sentences WITHIN the spaces provided.

1. Imagine a small magnet with its North pole vertically upward passing near the North pole of an external magnet that is below the small magnet. In what direction does the net force acting on the magnet point? Explain.
2. Consider an ensemble of electrons with an initial spin state

$$
|\psi\rangle=\left[\begin{array}{l}
1 / \sqrt{2} \\
1 / \sqrt{2}
\end{array}\right]
$$

If we send these electrons into an $\operatorname{SG} x$ device, what is the probability that a given electron will be determined to have its spin antialigned with the $+x$ direction? Explain.
3. What is locality? What is reality? Answer in the context of Bell's Theorem and the EPR argument.
4. A spaceship departs from the solar system (Event A) and travels at a constant velocity to a distant star. It then returns at a constant velocity to the solar system (event B). A clock on the spaceship measures what kind of time interval - proper, coordinate, spacetime? Explain.
5. Consider the figure below. Suppose the marks on the ct axis are 1.0 cm apart. What should be the vertical separation of the corresponding marks on the $t^{\prime}$ axis? Explain.

6. Consider the figure above in Question 5. Which of the choices in the figure best correspond to the $x^{\prime}$ axis? Explain.
7. In a double-slit experiment how could you increase the spacing between the fringes? You can alter the experimental setup in any physically available way like moving the components, swapping out components, changing the light source, etc. Explain.
8. Does a long-wavelength photon, because it is larger, carry more energy than a shortwavelengh photon? Explain.
9. To create an interference pattern does the quanton beam's de Broglie wavelength have to be larger than an individual quanton? Explain.
10. The figure shows the measured current vs. $\Delta V$ for a photoelectric effect experiment with an unknown metal. How would this result change if the metal was replaced with a different metal with a smaller work function? Explain.


Problems (6). Clearly show all reasoning for full credit. Use a separate sheet for your work.

1. 8 pts. Consider the experiment shown below. If the probabilities that an electron entering the second SG device leaves by the plus and minus channels are $\frac{1}{4}$ and $\frac{3}{4}$, respectively, what are all the possible values of the angle $\theta$ of that device's axis? Are the results for the two channels consistent? Show how the probabilities are determined from the wave functions.

2. 8 pts. Consider the spacetime diagram shown below. The events at points $P$ and $Q$ are claimed to be causally connected, i.e. the event at $Q$ is caused by the event at $P$. Is that true? Show your reasoning graphically. There is a bigger version of the figure on page 5. Explain your procedure.


See large version on page 5 .

DO NOT WRITE BELOW THIS LINE.
3. $\quad 10$ pts. An electron of energy $E$ scatters elastically off a nucleon target as shown in the figure. The energy of the scattered electron $E^{\prime}$ is

$$
E^{\prime}=\frac{E}{1+\frac{2 E}{m_{n}} \sin ^{2} \frac{\theta}{2}}
$$

where $\theta$ is the angle the electron makes with the beam axis. Show the angle $\phi$ of the proton with respect to the beam direction is the following.

$$
\phi=\tan ^{-1}\left(\frac{E^{\prime} \sin \theta}{E-E^{\prime} \cos \theta}\right)
$$

The electron energies $E$ and $E^{\prime}$ are ultra-relativistic - they are far greater than the electron rest mass.

4. 10 pts. The nucleus of an atom is on the order of $10^{-14} \mathrm{~m}$ in diameter. For an electron to be confined to a nucleus, its de Broglie wavelength would have to be on this order of magnitude or smaller. (a) What would be the kinetic energy of an electron confined to this region? (b) Make an order-of-magnitude estimate of the electric potential energy of a system of an electron inside an atomic nucleus. (c) Would you expect to find an electron in a nucleus? Explain.
5. 12 pts. Imagine that we create a beam of electrons with an electron gun set at a voltage $\Delta V=55 J / C$. (a) What is the de Broglie wavelength of the electrons? (b) We are able to measure the $m=1$ bright spots in the double-slit interference pattern created by this electron beam down to $\theta=20^{\circ}$. What slit separation does this correspond to? (c) Do you think seeing the double-slit interference pattern with this electron beam is doable?
6. 12 pts. A photoelectric experiment is performed with an aluminum cathode. An electron inside the cathode has a speed $v_{i}=1.5 \times 10^{6} \mathrm{~m} / \mathrm{s}$. If the potential difference between the anode and the cathode is $\Delta V=$ $-2.0 V$, what is the highest possible speed $v_{f}$ the electron can have when it reaches the anode? The work function of aluminum is $W_{A l}=$ 2.16 eV .

For Problem 2


## Physics 205 Equations

$$
\begin{aligned}
& \vec{F}_{n e t}=\sum \vec{F}_{i}=m \vec{a}=\frac{d \vec{p}}{d t} \quad v=\frac{d x}{d t} \quad v=\frac{\Delta x}{\Delta t} \quad x=\frac{1}{2} a t^{2}+v_{0} t+x_{0} \quad v=a t+v_{0} \quad a_{g}=-g \quad a_{c}=\frac{v^{2}}{r} \\
& \vec{F}_{g}=-m g \hat{j} \quad K E=\frac{1}{2} m v^{2} \quad K E_{0}+P E_{0}=K E_{1}+P E_{1} P E_{g}=m g h \quad P E_{V}=q V P E_{C}=\frac{Z_{1} Z_{2} e^{2}}{r}=\frac{k_{e} q_{1} q_{2}}{r} \\
& \vec{p}_{i}=\vec{p}_{f} \quad \vec{p}=m \vec{v} \quad K E_{i}=K E_{f}(\text { elastic }) \quad \vec{F}_{C}=k_{e} \frac{q_{1} q_{2}}{r^{2}} \hat{r} \quad \vec{E} \equiv \frac{\vec{F}}{q_{0}} \quad d \vec{E}=\frac{1}{4 \pi \epsilon_{0}} \frac{d q}{r^{2}} \hat{r} \\
& \qquad \vec{B}=\frac{\mu_{0}}{4 \pi} \frac{I d \vec{s} \times \hat{r}}{r^{2}}=\frac{\mu_{0}}{4 \pi} \frac{q d \vec{v} \times \hat{r}}{r^{2}} \quad \vec{F}_{B}=q \vec{v} \times \vec{B} \quad\left|\vec{F}_{B}\right|=|q v B \sin \theta| \\
& \vec{E}_{B}=\vec{E}_{A}+\vec{v}_{B A} \times \vec{B}_{A} \quad \vec{B}_{B}=\vec{B}_{A}-\mu_{0} \epsilon_{0} \vec{v}_{B A} \times \vec{E}_{A} \quad \gamma=\frac{1}{\sqrt{1-v^{2} / c^{2}}} \\
& \qquad \begin{array}{ll|l|}
\hline \text { Galilean } \\
\text { Transformation } \\
\text { SI units } & \begin{array}{l}
\text { Lorentz } \\
\text { Transformation } \\
\text { SI units }
\end{array} & \begin{array}{l}
\text { Lorentz } \\
\text { Transformation } \\
\text { SR units }
\end{array} \\
\hline x^{\prime}=x-v t & \begin{array}{l}
x^{\prime}=\gamma(x-v t) \\
y^{\prime}=y \\
z^{\prime}=y \\
t^{\prime}=t \\
v_{x}^{\prime}=v_{x}-v_{O}
\end{array} & \begin{array}{l}
x^{\prime}=\gamma(x-\beta t) \\
z^{\prime}=y \\
z^{\prime}=z \\
t^{\prime}=\gamma\left(t-v x / c^{2}\right) \\
v_{x}^{\prime}=\frac{v_{x}-v_{0}}{1-v_{x} v / c^{2}}
\end{array} \\
\begin{array}{l}
z^{\prime}=z \\
t^{\prime}=\gamma(t-\beta x) \\
v_{x}^{\prime}=\frac{v_{x}-\beta}{1-v_{x} \beta}
\end{array} \\
\hline
\end{array}
\end{aligned}
$$

| Coordinate Time | Proper Time | Spacetime Interval |
| :---: | :---: | :---: |
| Time between two events in an inertial frame measured with synchronized clocks $c \Delta t, \Delta t$ <br> Frame dependent | Time between two events measured by the same clock at both events. <br> $\Delta \tau_{S I}, \Delta \tau_{S R}$ <br> Frame independent | Time between two events measured by the same, inertial clock at both events. $\Delta s_{S I}, \Delta s_{S R}$ <br> Frame independent |

$$
\begin{gathered}
\Delta s_{S I}^{2}=c^{2} \Delta t^{2}-\Delta d^{2}=\Delta s_{S I}^{\prime} \quad \text { or } \quad \Delta s_{S R}^{2}=\Delta t^{2}-\Delta d^{2}=\Delta s_{S R}^{\prime}{ }^{2} \\
\Delta \tau_{S I}=\int_{t_{A}}^{t_{B}} \sqrt{1-\frac{v^{2}}{c^{2}}} d t \quad \text { or } \quad \Delta \tau_{S R}=\int_{t_{A}}^{t_{B}} \sqrt{1-\beta^{2}} d t \\
\Delta \tau_{S I}=\sqrt{1-v^{2} / c^{2}} \Delta t \quad \text { or } \quad \Delta \tau_{S R}=\sqrt{1-\beta^{2}} \Delta t
\end{gathered}
$$

$$
\begin{aligned}
& L_{S I}=L_{R} \sqrt{1-v^{2} / c^{2}} \quad \text { or } \quad L_{S R}=L_{R} \sqrt{1-\beta^{2}} \\
& v_{x}^{\prime}=\frac{v_{x}-v}{1-v_{x} v / c^{2}} \quad v_{y}^{\prime}=\frac{v_{y} \sqrt{1-v_{x}^{2} / c^{2}}}{1-v_{x} v / c^{2}} \quad K E=E-m c^{2} \quad \text { SI units } \\
& v_{x}^{\prime}=\frac{v_{x}-\beta}{1-v_{x} \beta} \quad v_{y}^{\prime}=\frac{v_{y} \sqrt{1-\beta^{2}}}{1-v_{x} \beta} \quad K E=E-m \quad \text { SR units } \\
& {\underset{\sim}{x}}_{i}={\underset{\sim}{p}}_{f} \quad \underset{\sim}{p} 1 \cdot{\underset{\sim}{p}}_{2}^{p}={\underset{\sim}{p}}_{3} \cdot{\underset{\sim}{p}}_{4} \\
& \underset{\sim}{p}=m d \underset{\sim}{s} / d \tau=\left[m \frac{d t}{d \tau}, m \frac{d x}{d \tau}, m \frac{d y}{d \tau}, m \frac{d z}{d \tau}\right]=\frac{m}{\sqrt{1-|\vec{v}|^{2}}}[1, \vec{v}] \quad \underset{\sim}{p} \cdot \underset{\sim}{p}=E_{r}^{2}-|\vec{p}|^{2}=m^{2} \quad \text { SR units } \\
& \underset{\sim}{p}=m d \underset{\sim}{s} / d \tau=\left[m c \frac{d t}{d \tau}, m \frac{d x}{d \tau}, m \frac{d y}{d \tau}, m \frac{d z}{d \tau}\right]=\frac{m}{\sqrt{1-v^{2} / c^{2}}}[c, \vec{v}] \quad \underset{\sim}{p} c \cdot \underset{\sim}{p} c=E_{r}^{2}-|\vec{p} c|^{2}=\left(m c^{2}\right)^{2} \quad \text { SI units } \\
& y=A \sin (k x-\omega t+\phi) \quad k \lambda=\omega T=2 \pi \quad E=E_{m} \sin (k x-\omega t+\phi) \quad B=B_{m} \sin (k x-\omega t+\phi) \quad \phi=k \delta \\
& \vec{S}=\frac{1}{\mu_{0}} \vec{E} \times \vec{B} \quad E=c B \quad\langle | \vec{S}| \rangle=I=\frac{E^{2}}{2 \mu_{0} c}=\frac{\text { energy }}{\text { area } \cdot \text { time }} \quad c=\frac{\lambda}{T}=\lambda f \\
& \delta=d \sin \theta=m \lambda(m=0, \pm 1, \pm 2, \ldots) \quad \delta=a \sin \theta=m \lambda(m= \pm 1, \pm 2, \ldots) \quad \phi=k \delta \\
& I_{\text {int }}=I_{m} \cos ^{2}\left(\frac{\pi d}{\lambda} \sin \theta\right) \quad I_{\text {diff }}=I_{m}\left[\frac{\sin \left(\frac{\pi a}{\lambda} \sin \theta\right)}{\frac{\pi a}{\lambda} \sin \theta}\right]^{2} \quad I_{\text {total }}=I_{m} \cos ^{2}\left(\frac{\pi d}{\lambda} \sin \theta\right)\left[\frac{\sin \left(\frac{\pi a}{\lambda} \sin \theta\right)}{\frac{\pi a}{\lambda} \sin \theta}\right]^{2} \\
& E=h f \quad K E_{\max }=e V_{\text {stop }}=h f-W \quad c=\lambda f \quad p=\frac{h}{\lambda} \quad K E=\frac{p^{2}}{2 m} \\
& \frac{d \sigma}{d \Omega}=\left(\frac{Z_{1} Z_{2} e^{2}}{4 E}\right)^{2} \frac{1}{\sin ^{4}\left(\frac{\theta_{s}}{2}\right)}=\frac{\text { area }}{\text { steradian }} \quad d \Omega=r^{2} \sin \theta d \theta d \phi \quad M E=\frac{1}{2} m v^{2}+P E \quad M E_{i}=M E_{f}
\end{aligned}
$$

$$
\begin{gathered}
\frac{d}{d x}(f(u))=\frac{d f}{d u} \frac{d u}{d x} \quad \frac{d}{d x}(u \cdot v)=\frac{d u}{d x} \cdot v+\frac{d v}{d x} \cdot u \quad \int x^{n} d x=\frac{x^{n+1}}{n+1} \quad \int \frac{1}{x} d x=\ln x \\
\frac{d}{d x}\left(x^{n}\right)=n x^{n-1} \quad \frac{d e^{x}}{d x}=e^{x} \quad \frac{d}{d x}(\ln x)=\frac{1}{x} \quad \frac{d}{d x}(\cos a x)=-a \sin a x \quad \frac{d}{d x}(\sin a x)=a \cos a x \\
\langle x\rangle=\frac{1}{N} \sum_{i} x_{i} \quad \sigma=\sqrt{\frac{\sum_{i}\left(x_{i}-\langle x\rangle\right)^{2}}{N-1}} \quad C=2 \pi r \quad A=4 \pi r^{2} \quad V=A h \quad V=\frac{4}{3} \pi r^{3} \\
\frac{d f(x)}{d x}=\lim _{\Delta x \rightarrow 0} \frac{f(x+\Delta x)-f(x)}{\Delta x} \quad \int_{a}^{b} f(x) d x=\lim _{\Delta x \rightarrow 0} \sum_{n=1}^{N} f(x) \Delta x \\
\int \frac{1}{x} d x=\ln x \\
\int x^{n} d x=\frac{x^{n+1}}{n+1} \quad \int e^{a x} d x=\frac{e^{a x}}{a} \quad \int \frac{1}{\sqrt{x^{2}+a^{2}}} d x=\ln \left[x+\sqrt{x^{2}+a^{2}}\right] \\
\int \frac{x}{\sqrt{x^{2}+a^{2}}} d x=\sqrt{x^{2}+a^{2}} \quad \int \frac{x^{2}}{\sqrt{x^{2}+a^{2}}} d x=\frac{1}{2} x \sqrt{x^{2}+a^{2}}-\frac{1}{2} a^{2} \ln \left[x+\sqrt{x^{2}+a^{2}}\right] \\
\int \sqrt{1-a x^{2}} d x=\frac{x}{2} \sqrt{1-a x^{2}}+\frac{\arcsin (\sqrt{a} x)}{2 \sqrt{a}} \quad \int \frac{x^{3}}{\sqrt{x^{2}+a^{2}}} d x=\frac{1}{3}\left(-2 a^{2}+x^{2}\right) \sqrt{x^{2}+a^{2}} \\
\int x=\frac{-b \pm \sqrt{b^{2}-4 a c}}{2 a} \quad \vec{A} \cdot \vec{B}=A B \cos \theta \quad|\vec{A} \times \vec{B}|=|A B \sin \theta|
\end{gathered}
$$

## Physics 205 Constants and Conversions

| Avogadro's number $\left(N_{A}\right)$ | $6.022 \times 10^{23}$ | Speed of light $(c)$ | $3 \times 10^{8} \mathrm{~m} / \mathrm{s}$ |
| :--- | :--- | :--- | :--- |
| $k_{B}$ | $1.38 \times 10^{-23} \mathrm{~J} / \mathrm{K}$ | Elementary charge $(e)$ | $1.60 \times 10^{-19} \mathrm{C}$ |
| 1 u | $1.67 \times 10^{-27} \mathrm{~kg}$ | $g$ | $9.8 \mathrm{~m} / \mathrm{s}^{2}$ |
| Gravitation constant | $6.67 \times 10^{-11} \mathrm{~N}-\mathrm{m}^{2} / \mathrm{kg}^{2}$ | Earth's radius | $6.37 \times 10^{6} \mathrm{~m}$ |
| Coulomb constant $\left(k_{e}\right)$ | $8.99 \times 10^{9} \frac{\mathrm{N-m}^{2}}{\mathrm{C}^{2}}$ | Coulomb constant $\left(e^{2}\right)$ | $\hbar c / 137$ |
| Proton/Neutron mass | $938 \mathrm{MeV} / \mathrm{c}^{2} \mathrm{~g}$ | Proton/Neutron mass | $1.67 \times 10^{-27} \mathrm{~kg}$ |
| Electron mass | $9.11 \times 10^{-31} \mathrm{~kg}$ | Electron mass | $0.511 \mathrm{MeV} / \mathrm{c}^{2}$ |
| Permittivity constant $\left(\epsilon_{0}\right)$ | $8.85 \times 10^{-12} \frac{\mathrm{~kg}}{\mathrm{N-m}}$ | 1.0 eV | $1.6 \times 10^{-19} \mathrm{~J}$ |
| M MeV | $10^{6} \mathrm{eV}$ | atomic mass unit $(u)$ | $1.66 \times 10^{-27} \mathrm{~kg}$ |
| Planck's constant $(h)$ | $6.63 \times 10^{-34} \mathrm{Js}$ | Planck's constant $(h)$ | $4.14 \times 10^{-15} \mathrm{eVs}$ |
| Planck's constant $(\hbar c)$ | $197 \mathrm{MeV}-\mathrm{fm}$ | Planck's constant $(\hbar c)$ | $1970 \mathrm{eV}-\AA$ |
| Permeability constant $\left(\mu_{0}\right)$ | $1.26 \times 10^{-6} \mathrm{Tm} / \mathrm{A}$ | Rydberg constant $\left(R_{H}\right)$ | $1.097 \times 10^{7} \mathrm{~m}^{-1}$ |
| Becquerel $(B q)$ | $1 \mathrm{decay/s}$ | Curie $(C i)$ | $3.7 \times 10^{10} \mathrm{~Bq}$ |



