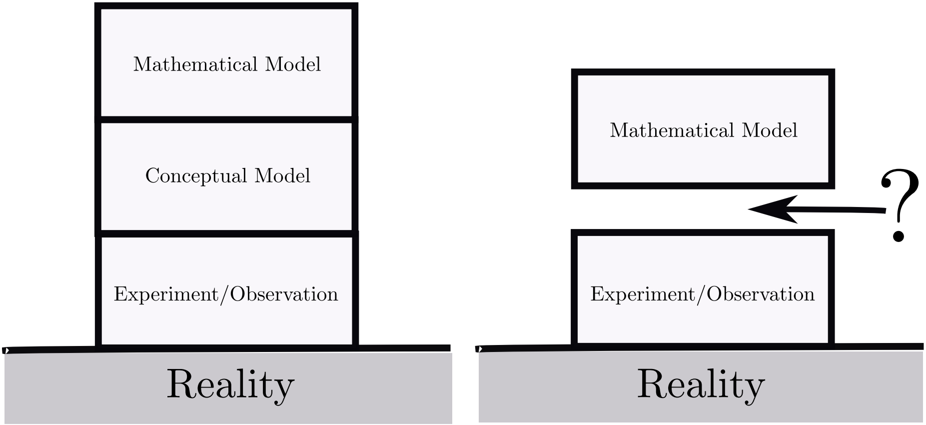
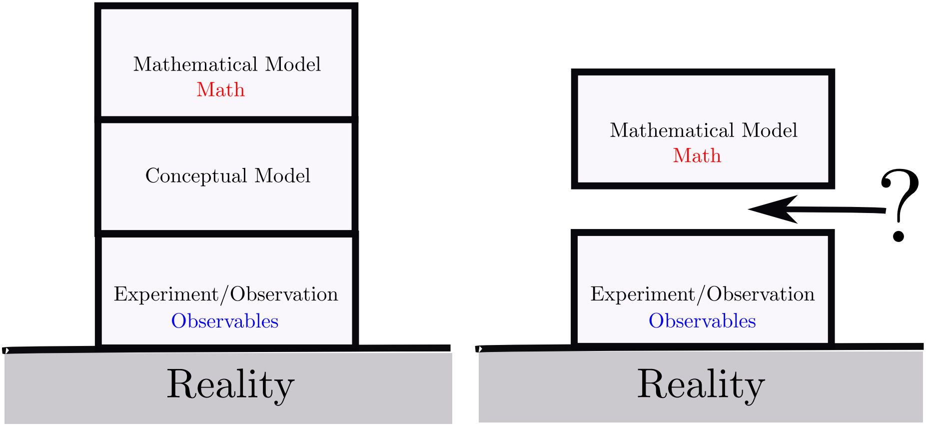
The Game of Quantum Reality - 1 1



The Game of Quantum Reality - 3 2



Rules of the Game - 1 (The Pieces) 3

For waves we need a wave function.

$$|\psi\rangle = \begin{bmatrix}\psi_1 \\ \psi_2\end{bmatrix} \qquad \langle\psi| = [\psi_1^* \;\; \psi_2^*] = [\psi_1 \;\; \psi_2]$$

And we need a dot product so the wave function is well-behaved.

$$\langle\psi|\psi\rangle = \begin{bmatrix}\psi_1 & \psi_2\end{bmatrix} \begin{bmatrix}\psi_1 \\ \psi_2\end{bmatrix} = \psi_1^2 + \psi_2^2$$

A more general form of the dot product.

$$\langle\alpha|\beta\rangle = \begin{bmatrix}\alpha_1 & \alpha_2 & \alpha_3 & \dots\end{bmatrix} \begin{bmatrix}\beta_1 \\ \beta_2 \\ \beta_3 \\ \vdots\end{bmatrix} = \alpha_1\beta_1 + \alpha_2\beta_2 + \alpha_3\beta_3 + \;\; \dots$$

And we need a normalization so we can compare different results.

$$\langle\psi|\psi\rangle = \psi_1^2 + \psi_2^2 = 1$$

The Rules 4

1. The State Vector Rule
2. The Eigenvector Rule
3. The Collapse Rule
4. The Outcome Probability Rule
5. The Sequence Rule
6. The Superposition Rule
7. The Time Evolution Rule.

The State Vector Rule 5

A quantum object (a quanton) can be represented by a normalized q-vector or

$$\text{The wave function} = |\psi\rangle = \sum_i a_i |\phi_i\rangle$$

For an electron with spin we observe only two components - parallel and anti-parallel to the axis of measurement.

$$|\psi\rangle = a_1|\phi_1\rangle + a_2|\phi_2\rangle = a_1 \begin{pmatrix} 1 \\ 0 \end{pmatrix} + a_2 \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

The vectors shown above are for measurements of S_z. They have switched from ψ to ϕ for reasons explained on the next page. Other measurements like S_x have different forms.

For every value of an observable there is a unique normalized q-vector called an eigenvector often represented as $|\phi\rangle$ (see previous slide). The observable's value associated with a particular eigenvector is the eigenvector's eigenvalue. The eigenvector's for S_z, S_x, and S_θ are shown below.

Eigenvalue	S_z	S_x	S_θ
$+\frac{1}{2}\hbar$	$\|+z\rangle = \begin{bmatrix}1\\0\end{bmatrix}$	$\|+x\rangle = \begin{bmatrix}\sqrt{1/2}\\\sqrt{1/2}\end{bmatrix}$	$\|+\theta\rangle = \begin{bmatrix}\cos\theta/2\\\sin\theta/2\end{bmatrix}$
$+\frac{1}{2}\hbar$	$\|-z\rangle = \begin{bmatrix}0\\1\end{bmatrix}$	$\|-x\rangle = \begin{bmatrix}\sqrt{1/2}\\-\sqrt{1/2}\end{bmatrix}$	$\|-\theta\rangle = \begin{bmatrix}-\sin\theta/2\\\cos\theta/2\end{bmatrix}$

The Collapse Rule 7

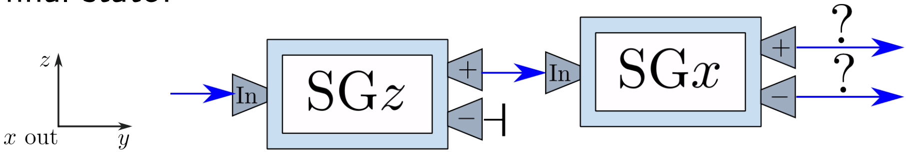
When we measure an observable we 'pluck' one of the components out of the state vector, collapsing the wave function into a single eigenstate.

$$|\Psi\rangle = a_1 \begin{pmatrix} 1 \\ 0 \end{pmatrix} + a_2 \begin{pmatrix} 0 \\ 1 \end{pmatrix} \xrightarrow[\textit{measurement}]{} \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

The Outcome Probability Rule 8

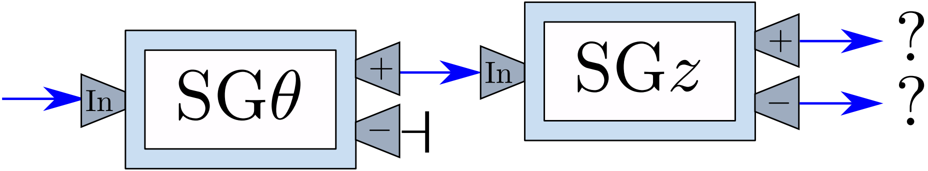
For a quantum state $|\psi\rangle$ the measurement of an observable has a quantum amplitude for a particular result $|\phi\rangle$ (one of the eigenfunctions) which is $\langle\phi|\psi\rangle$. The initial state is on the right and the final state is on the left. The probability of of obtaining this eigenvalue is $|\langle\phi|\psi\rangle|^2$.

What is the probability for the electron to have $+x$ in the final state?



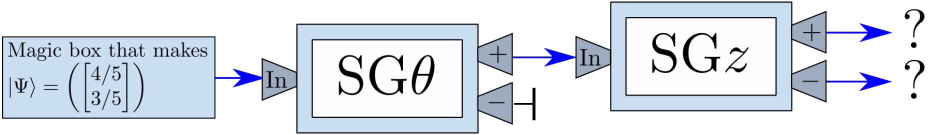
What is the probability for the electron to have $-x$ in the final state?

What is the probability for the electron to have $+z$ in the final state?

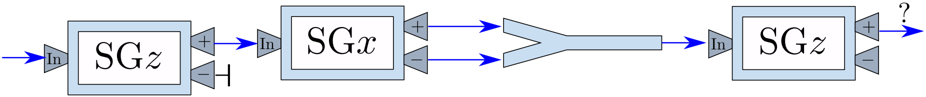


What is the probability for the electron to have $-z$ in the final state?

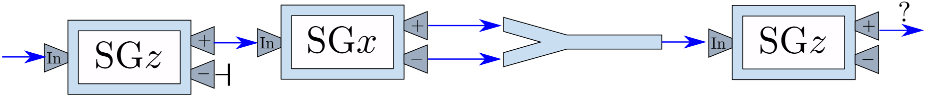
If you measure observable A (and get a_i) and then immediately measure observable B (and get b_j) the quantum amplitude is $\langle b_j|a_i\rangle\langle a_i|\psi_i\rangle$. The initial state is on the right and the final state on the left. What is the probability for the electron to have each of the final $\pm z$ states?



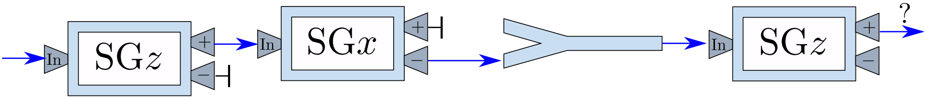
If a quanton can take more than one path to a detector and we cannot know which path is taken, then the quantum amplitude is the sum of the amplitudes of all the paths. What is the probability for the electron to have the final $+z$ state?



If a quanton can take more than one path to a detector and we cannot know which path is taken, then the quantum amplitude is the sum of the amplitudes of all the paths. What is the probability for the electron to have the final $+z$ state?



What is the probability for the electron to have the final $+z$ state now?



The Time Evolution Rule

- If the quantum system is in an eigenstate after a measurement, it will stay there until perturbed.

The Time Evolution Rule 15

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- If the system is not in an eigenstate it will vary smoothly and predictably with time.

- If the quantum system is in an eigenstate after a measurement, it will stay there until perturbed.
- If the system is not in an eigenstate it will vary smoothly and predictably with time.
- Time evolution is described using

$$|\Psi(t)\rangle = \sum_i a_i |\phi_i(t)\rangle \qquad .$$

The Rules 17

1. The State Vector Rule
2. The Eigenvector Rule
3. The Collapse Rule
4. The Outcome Probability Rule
5. The Sequence Rule
6. The Superposition Rule
7. The Time Evolution Rule.

1. The wave function $|\psi\rangle$ of a quanton consists of a superposition/sum of components $|\psi_n\rangle$ so $|\psi\rangle = \sum a_n|\psi_n\rangle$.
2. Each possible value of a measurement has a particular *q*-vector or $|\psi_n\rangle$ - an eigenvector.
3. Making a measurement collapses the wave function into a single eigenvector $|\psi_n\rangle$.
4. The probability of a measurement is $|a_n|^2$.
5. Successive measurements yield the product of the probabilities.
6. If a quanton has multiple paths to a detector, we cannot know which path it took.
7. A system in an eigenvector remains in that state. A system in a superposition of eigenvectors follows a smooth, predictible time evolution.

The Postulates of Quantum Mechanics 19

1. The wave function $|\psi\rangle$ of a quanton consists of a superposition/sum of components $|\psi_n\rangle$ which are often solutions of the Schroedinger equation, $|\psi\rangle = \sum a_n |\psi_n\rangle$.
2. Each possible value of a measurement has a particular *q*-vector or $|\psi_n\rangle$ associated with it (an eigenvector).
3. Making a measurement selects one of the eigenvectors in $|\psi\rangle$, collapsing the wave function into a single eigenvector $|\psi_n\rangle$.
4. The probability of getting a particular measurement is the square of the coefficient $|a_n|^2$.
5. Successive measurements will yield the product of the probabilities for each step.
6. If a quanton has multiple paths to reach a destination/detector, we cannot know which path the quanton took.
7. A system in an eigenvector will remain in that state. If a system is in a superposition of eigenvectors its time evolution will follow the evolution of the time dependent Schroedinger equation.