Another Paradox 2

Consider the two twins again. One sets out at the age of 25 on a spaceship from Earth at a speed of $0.99c$ where c is the speed of light. The Earthbound twin goes on about her/his business accumulating the normal accouterments of advancing age (gray hair, drooping body parts, *etc.*). After twenty years have passed for the Earthbound twin, the spacefaring one returns. In the frame of the Earthbound twin what distance did the traveling twin cover during his/her trip? What distance did the traveling twin measure in his/her frame?

The Pole and Barn Paradox 3

A ridiculously fast pole vaulter named Anna runs with her 10-meter long pole at $v_A = 0.8c$ in the positive x direction. She runs into an 8-meter-long barn, which has doors on both sides. Here's what happens, according to farmer Bob, standing inside the barn:

- The pole is length-contracted so that it's shorter than the barn.
- When the pole is centered inside the barn, Bob pushes a button and both doors shut very quickly with the pole entirely inside the barn.
- Bob says, 'Ah-ha! I've closed Anna's pole inside the barn.'

But here's what happens according to Anna:

- Her pole is 10 meters long.
- The barn is length-contracted to less than 8 meters long.
- Anna says, 'My pole couldn't possibly have been closed inside the barn!'

Which one is right?

What is Length? 4

1. Recall one of our definitions of time.

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What is Length? 6

1. Recall one of our definitions of time. What you measure with a clock.
2. What is Length?

What is Length?

1. Recall one of our definitions of time. What you measure with a clock.
2. What is Length? What you measure with a ruler.

What is Length? 8

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3. Length is the distance between two simultaneous events at the ends of the object.

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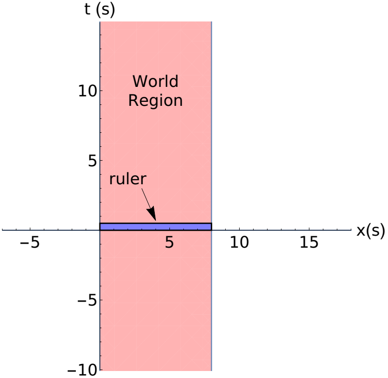
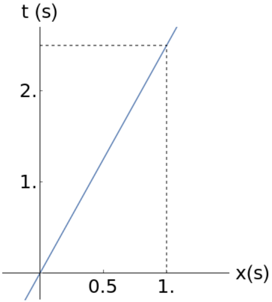


Coordinates For Length Contraction



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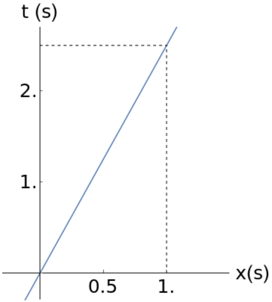
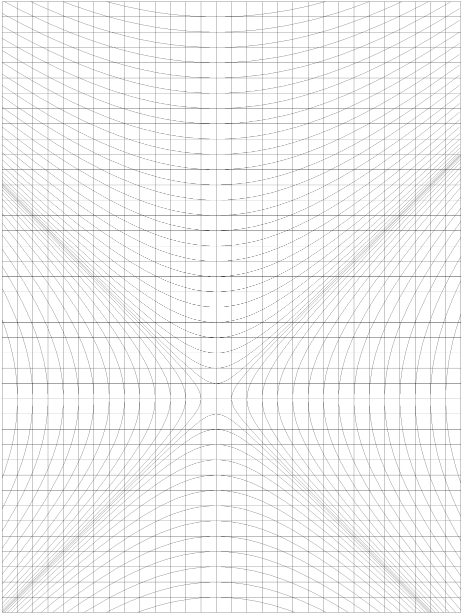


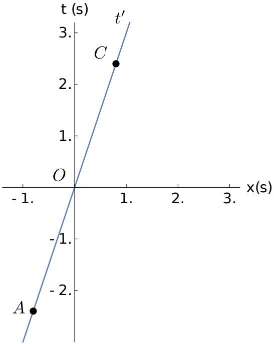
Hyperbolic Graph Paper 12



Consider 3 events:

1. A - pulse leaves origin at $t'_o = -T$.
2. B - reflects off a mirror at (x'_1, t'_1).
3. C - arrives at the origin at $t' = +T$.

Blue line - world line Dashed line - light pulse



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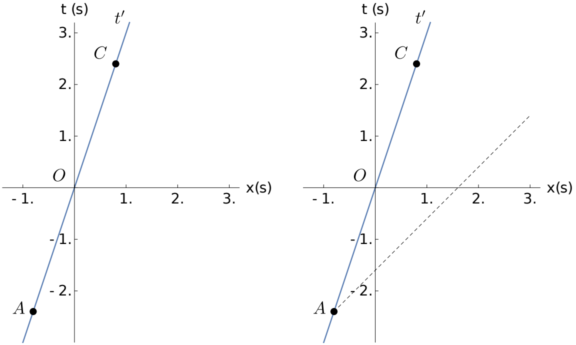

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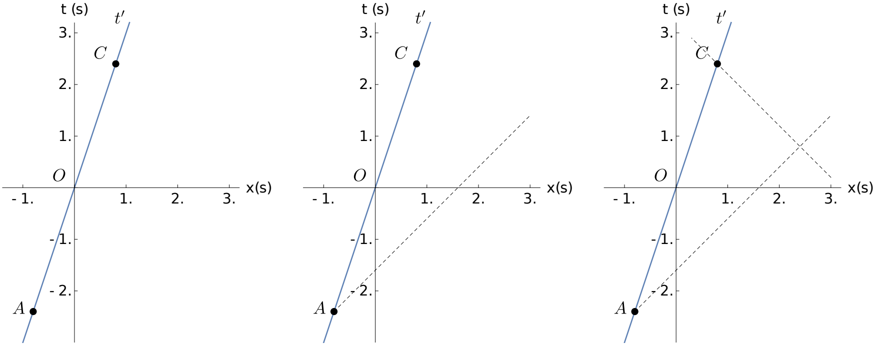

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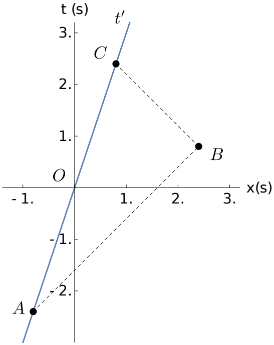

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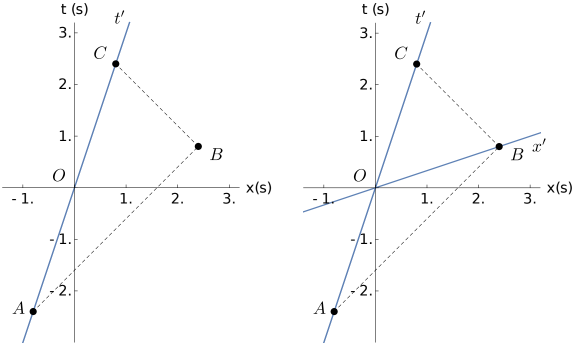

Drawing the Diagram x' Axis - 2 18

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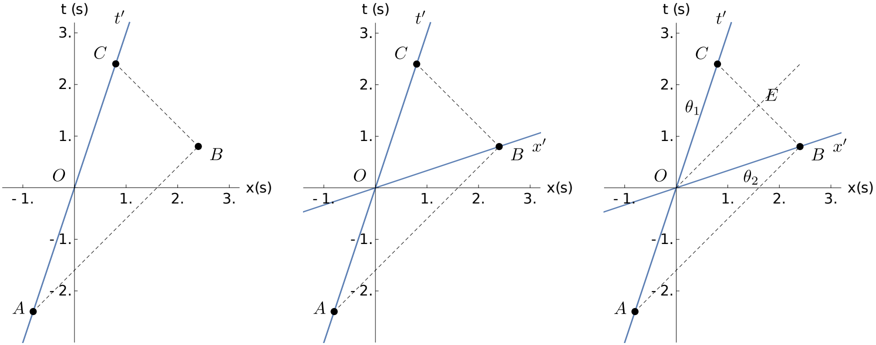

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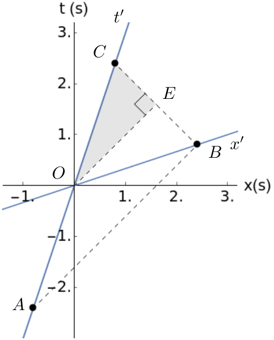

Drawing the Diagram x' Axis - 3

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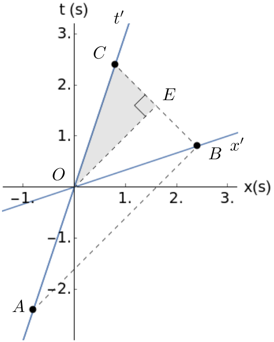
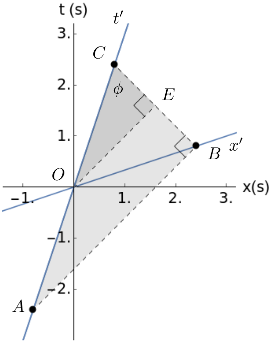
Dashed line - light pulse





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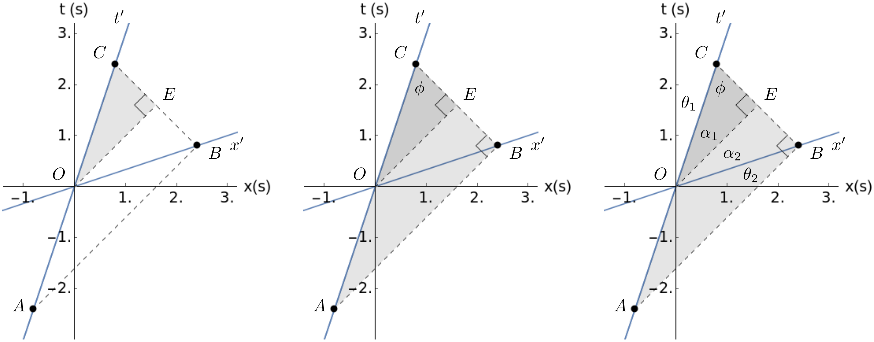
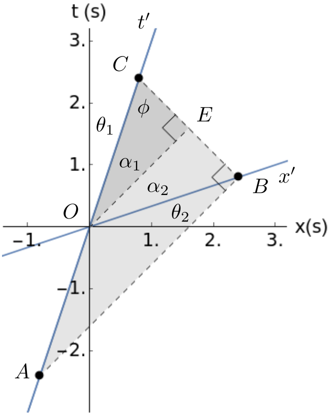
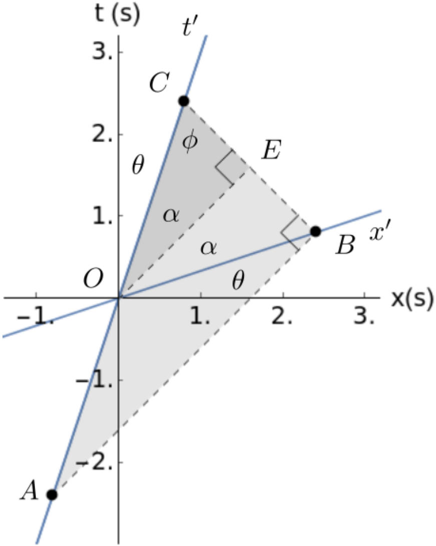


- Events A, O, and C are at the origin so they define the t' axis.
- Events A and C must be symmetric to O so $\overline{AO} = \overline{OC}$.
- Events O and B occur at $t' = 0$ so they define the x' axis.
- Let a light pulse be emitted from the origin going in the positive x' direction.
- $\triangle ABC$ and $\triangle OEC$ have the same internal angles so they are similar.
- The hypotenus of $\triangle ABC$ is twice as long as $\triangle OEC$ because $\overline{AO} = \overline{OC}$.
- $\therefore\ \overline{BC} = 2 \times \overline{EC}$.
- $\therefore\ \triangle OEC$ similar to $\triangle OEB$
- $\triangle OEC$ and $\triangle OEB$ share $\overline{OE}$.
- $\therefore \overline{CE} = \overline{BE}$.
- $\therefore\ \alpha_1 = \alpha_2$
- $\overline{OE}$ has a slope of 1 so $\theta_1 = \theta_2$




t (s)
t′
3.
2.
1.
−1.
−2.
C
E
B
x′
O
A
θ
ϕ
α
α
θ
x(s)
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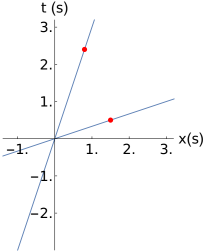
1.
2.
3.

Plotting A Point

Given the values (x_0', t_0'), where do you draw them?

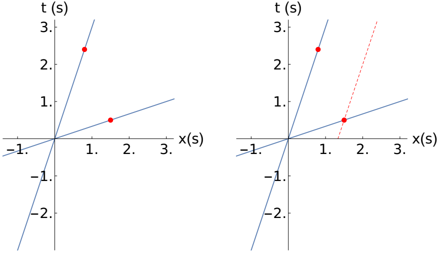


Blue line - origin world line

Red Dashed line - parallel world lines

Plotting A Point 25

Given the values (x'_0, t'_0), where do you draw them?



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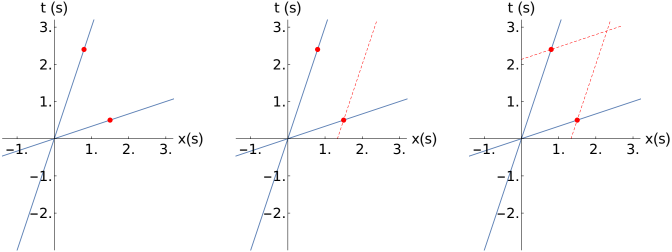

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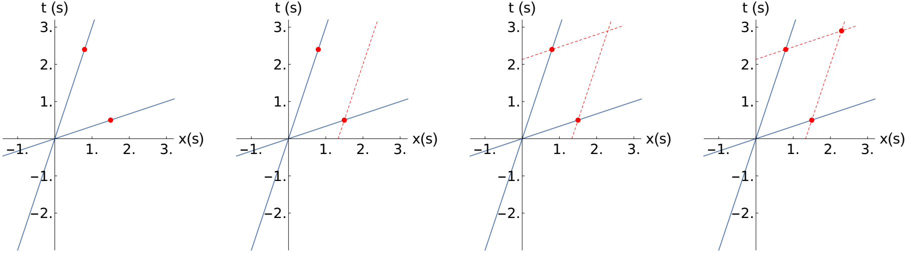
Plotting A Point 27

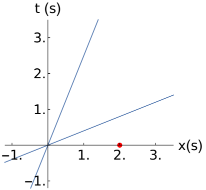
Given the values (x'_0, t'_0), where do you draw them?



Blue line - origin world line   Red Dashed line - parallel world lines

1. Mark x scale.
2. Mark t scale.
3. Draw t' with $slope = 1/v$.
4. Draw x', $slope = v$.
5. Mark point in *Home*.
6. Draw x_0' line.
7. Trace closest hyperbola to x scale and get x'.
8. Draw t_0' line.
9. Trace closest hyperbola to t scale and get t'.

What are the coordinates in the Other frame moving with $\beta = 2/5$ for a point $(x_0, t_0) = (2s, 0)$ in the Home frame?



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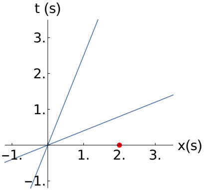

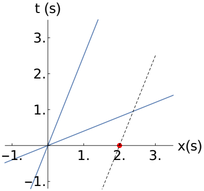

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An Example

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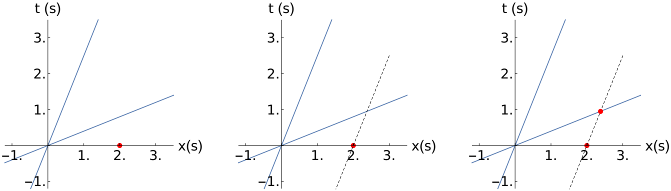

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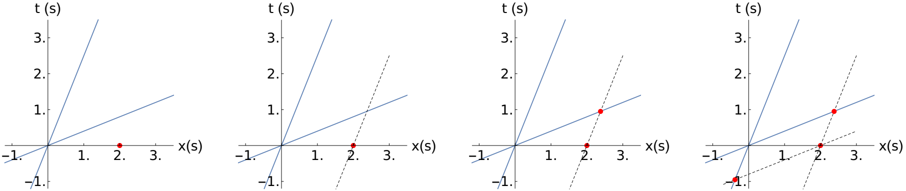

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Galilean	Lorentz
$x' = x - vt$	
$y' = y$	
$z' = z$	
$t' = t$	
$u'_x = u_x - v$	
$u'_y = u_y$	
$u'_z = u_z$	

primes refer to the frame moving with velocity v.

v - velocity of moving frame.

u_i - i^{th} component of the velocity of an object in the stationary frame.

u'_i - i^{th} component of the velocity of an object in the moving frame.

$\gamma = \frac{1}{\sqrt{1-v^2/c^2}}$ where c is the speed of light.

Galilean	Lorentz
$x' = x - vt$	$x' = \gamma(x - vt)$
$y' = y$	$y' = y$
$z' = z$	$z' = z$
$t' = t$	$t' = \gamma(t - vx/c^2)$
$u'_x = u_x - v$	$u'_x = \frac{u_x - v}{1 - u_x v/c^2}$
$u'_y = u_y$	$u'_y = u_y$
$u'_z = u_z$	$u'_z = u_z$

primes refer to the frame moving with velocity v.

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u_i - i^{th} component of the velocity of an object in the stationary frame.

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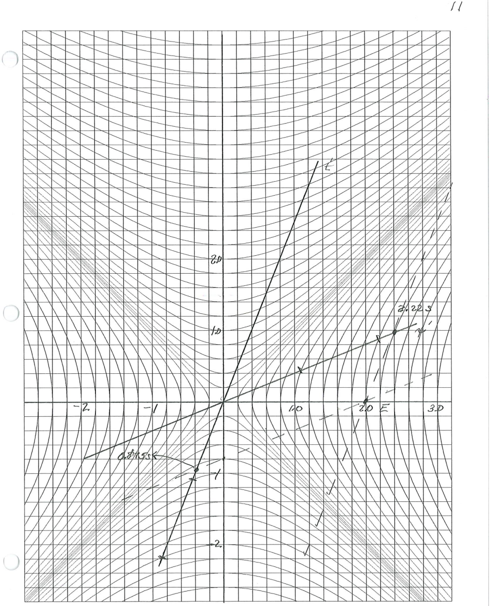
Worldlines for the Example



Another Example

What are the Other coordinates if $t = 4.2\ s$ and $x = 7.0\ s$ in the Home frame and $\beta = 2/5$?

Another Example 37

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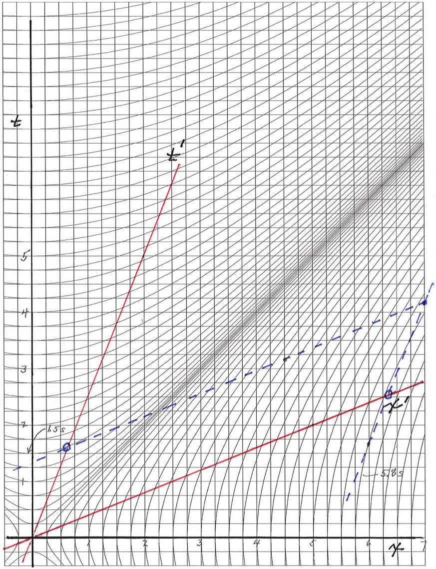

Yet Another Example

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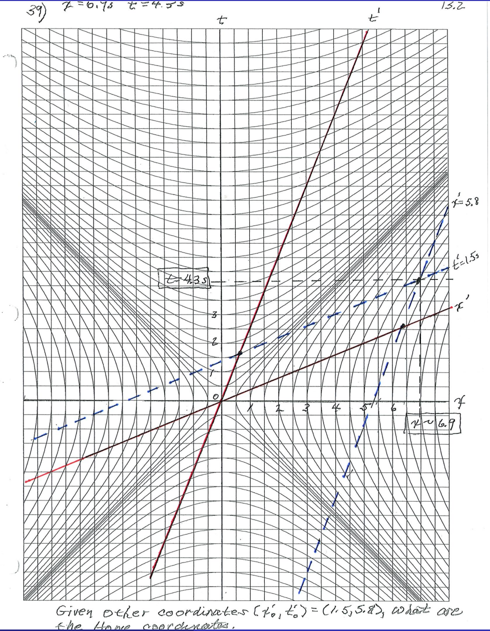

- You own a barn, 8.0 *m* long, with automatic doors at either end, that can be opened and closed simultaneously . You also have a pole, also 10.0 *m* long, which does not fit in the barn.

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The Barn and Pole Paradox - 1 42

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- As the pole passes through the barn, there is a time when it is completely within the barn. You close both doors simultaneously and, at least momentarily, you have the contracted pole shut up in your barn.

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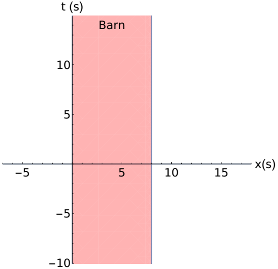
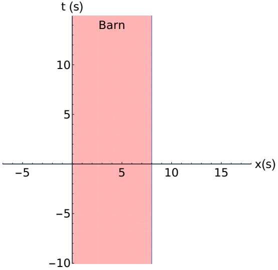
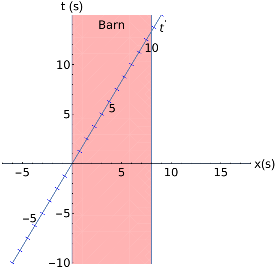
- As the pole passes through the barn, there is a time when it is completely within the barn. You close both doors simultaneously and, at least momentarily, you have the contracted pole shut up in your barn.
- Now consider the point of view of the runner. She sees the pole as stationary and the barn approaching at high speed. The pole is still 10 m long, and the barn is now less than $(8\ m)\sqrt{1 - 0.8^2} = 4.8\ m$ long. Surely the runner is in trouble if the doors close while she is inside. Is she?

The Barn and Pole Paradox - 2 44



$$v = \frac{3}{5}c$$
$$L_{pole} = 10\ m$$
$$L_{barn} = 8\ m$$





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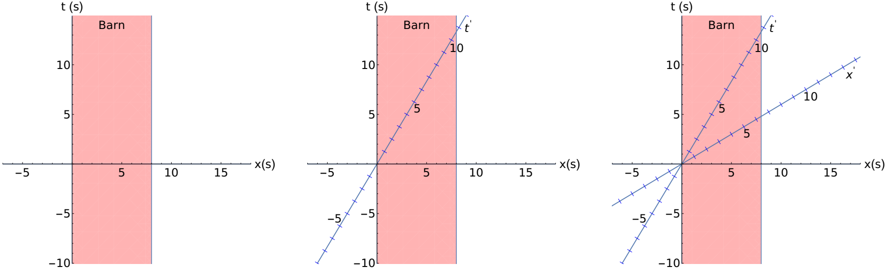
The Barn and Pole Paradox - 2 46



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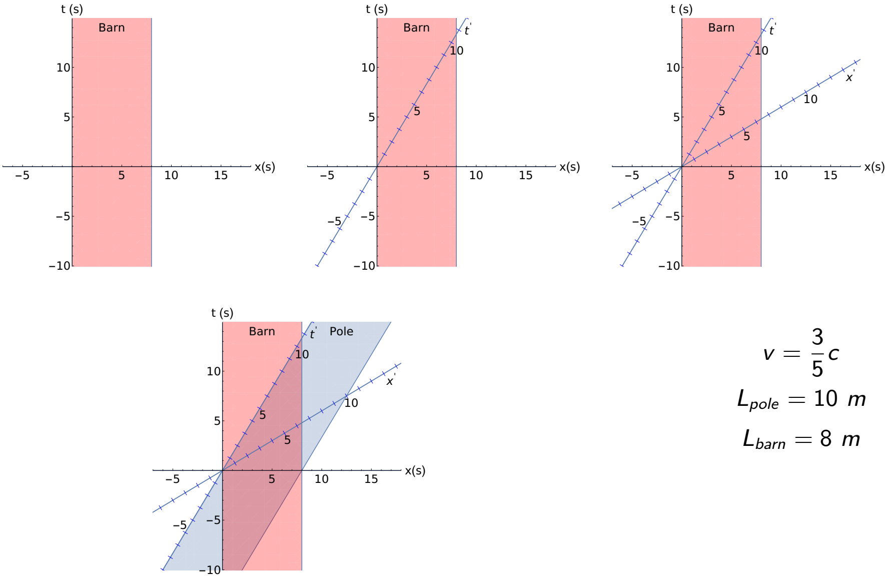

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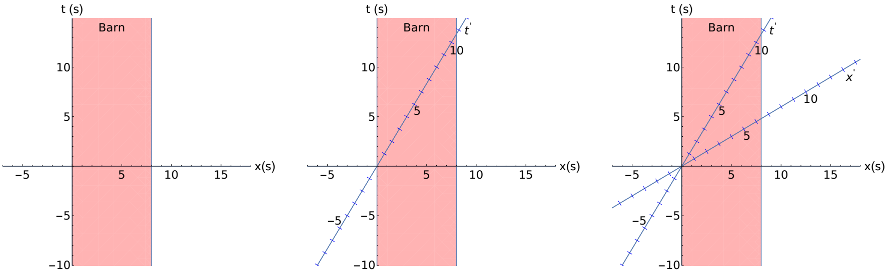

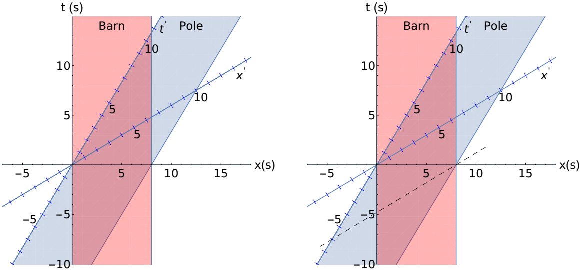

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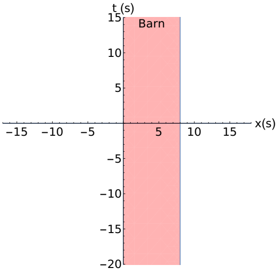

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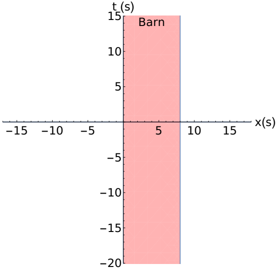

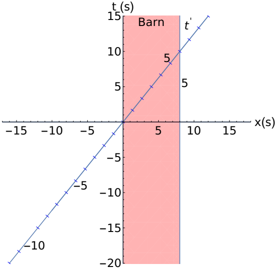

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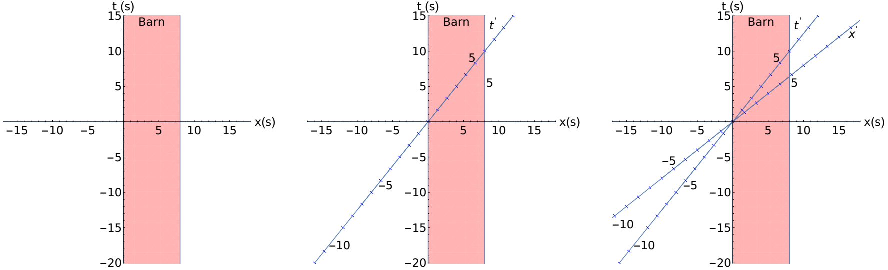

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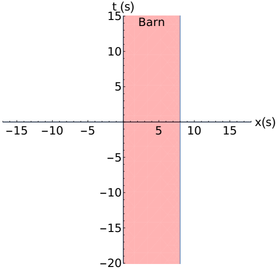
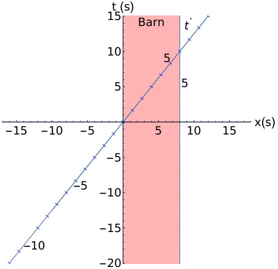
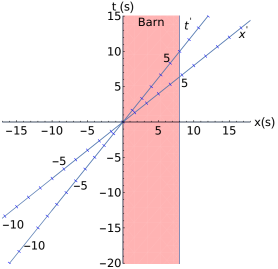
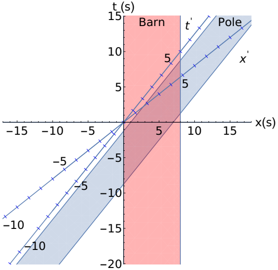
The Barn and Pole Paradox - 2









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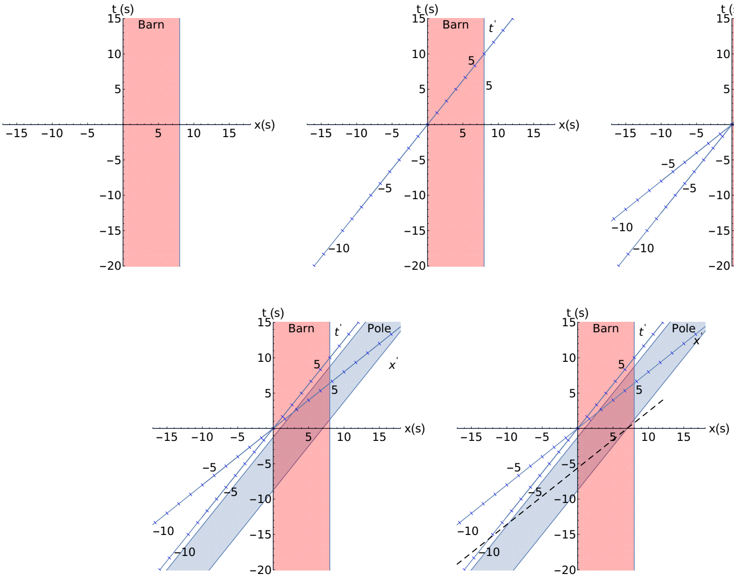
$$L_{pole} = 10\ m$$

$$L_{barn} = 8\ m$$



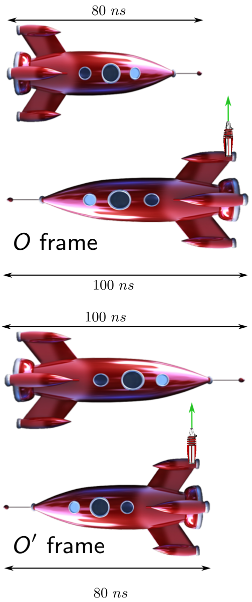
$$v = \frac{4}{5}c$$

$$L_{pole} = 10\ m$$

$$L_{barn} = 8\ m$$

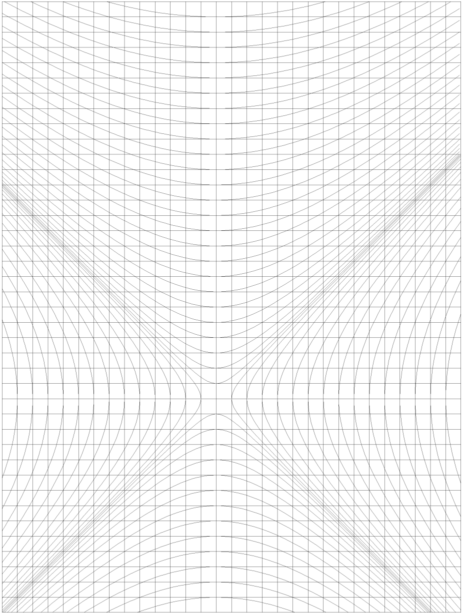
Two spacecraft of equal rest length $L_R = 100$ *ns* pass very close to each other as they travel in opposite directions at a relative speed $|\beta| = \frac{3}{5}$. See figure. The captain of ship O intends to fire her laser cannon at the instant her bow is lined up with the tail of ship O'. Since O' is Lorentz-contracted to a length of 80 *ns* in the frame of ship O, she expects the laser burst to miss the other ship by 20 *ns*. However, to the captain on ship O', it is ship O that has contracted to 80 *ns* so the laser burst will strike ship O' about 20 *ns* behind the bow

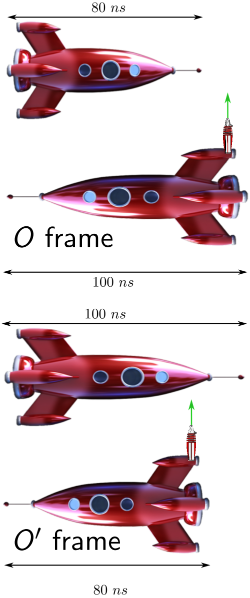
What really happens if O carries out her plan? Construct a two-observer spacetime diagram. Define event A to be coincidence of the bow of ship O and the tail of O' and event B to be the firing of the laser. Let A define the origin event in both frames. When and where does this event occur as measured in the O' frame? The travel time of the laser from one ship to another is negligible. Verify your results with the Lorentz transformations.

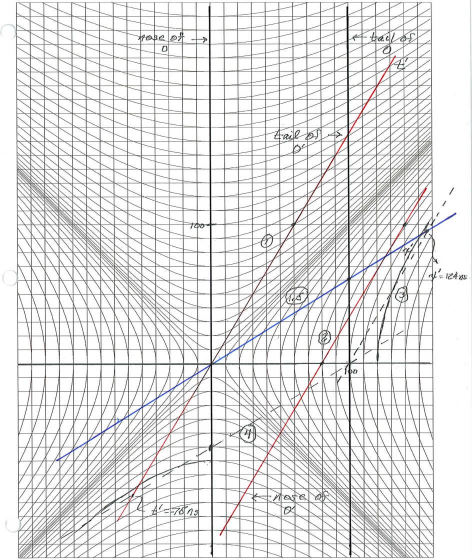


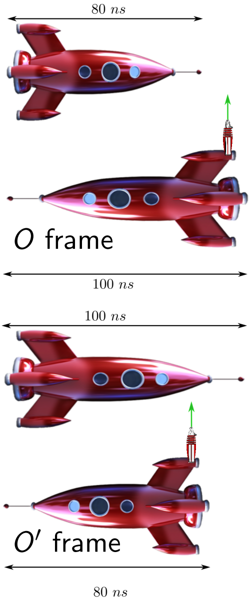
Adapted from *Spacetime Physcs* by Taylor and Wheeler, 1966.

Hyperbolic Graph Paper

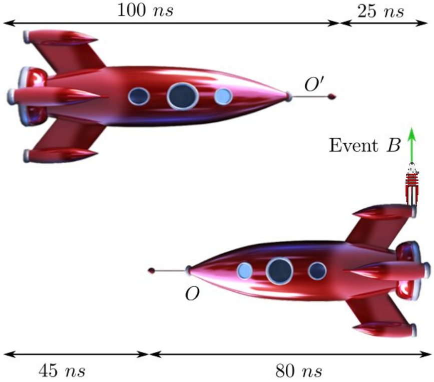





nose of O
tail of O
t'
tail of O'
100
①
1.5
②
③
④
x' = 124 ns
100
t' = -76 ns
nose of O'


80 ns
O frame
100 ns
100 ns
O' frame
80 ns

What O' sees at $t' = -75ns$.



What O' sees at $t' = 0ns$.