## Now It Starts to Get Weird - SG Devices 1

What fraction of the input beam comes out of each, final output?


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$$
\begin{aligned}
& A_{+}=\langle+z \mid+x\rangle\langle+x \mid+z\rangle=\left[\left(\begin{array}{ll}
1 & 0
\end{array}\right)\binom{\sqrt{1 / 2}}{\sqrt{1 / 2}}\right]\left[(\sqrt{1 / 2} \sqrt{1 / 2})\binom{1}{0}\right]=\frac{1}{2} \\
& A_{-}=\langle-z \mid+x\rangle\langle+x \mid+z\rangle=\left[\left(\begin{array}{ll}
0 & 1
\end{array}\right)\binom{\sqrt{1 / 2}}{\sqrt{1 / 2}}\right]\left[(\sqrt{1 / 2} \sqrt{1 / 2})\binom{1}{0}\right]=\frac{1}{2} \\
& P_{+}=\left|A_{+}\right|^{2}=\left(\frac{1}{2}\right)^{2}=\frac{1}{4} \quad P_{-}=\left|A_{-}\right|^{2}=\left(\frac{1}{2}\right)^{2}=\frac{1}{4}
\end{aligned}
$$

## Now It Starts to Get Weird - SG Devices 2

What is the probability for the electron to have the final $+z$ state?


| Observable | $S_{z}$ | $S_{x}$ | $S_{\theta}$ |
| :---: | :---: | :---: | :---: |
| $+\frac{1}{2} \hbar$ | $\|+z\rangle=\left[\begin{array}{l}1 \\ 0\end{array}\right]$ | $\|+x\rangle=\left[\begin{array}{c}\sqrt{1 / 2} \\ \sqrt{1 / 2}\end{array}\right]$ | $\|+\theta\rangle=\left[\begin{array}{c}\cos \frac{\theta}{2} \\ \sin \frac{\theta}{2}\end{array}\right]$ |
| $-\frac{1}{2} \hbar$ | $\|-z\rangle=\left[\begin{array}{l}0 \\ 1\end{array}\right]$ | $\|-x\rangle=\left[\begin{array}{c}\sqrt{1 / 2} \\ -\sqrt{1 / 2}\end{array}\right]$ | $\|-\theta\rangle=\left[\begin{array}{c}-\sin \frac{\theta}{2} \\ \cos \frac{\theta^{2}}{2}\end{array}\right]$ |

## Now It Starts to Get Weird - SG Devices 2

What is the probability for the electron to have the final $+z$ state now?


| Observable | $S_{z}$ | $S_{x}$ | $S_{\theta}$ |
| :---: | :---: | :---: | :---: |
| $+\frac{1}{2} \hbar$ | $\|+z\rangle=\left[\begin{array}{l}1 \\ 0\end{array}\right]$ | $\|+x\rangle=\left[\begin{array}{c}\sqrt{1 / 2} \\ \sqrt{1 / 2}\end{array}\right]$ | $\|+\theta\rangle=\left[\begin{array}{c}\cos \frac{\theta}{2} \\ \sin \frac{\theta}{2}\end{array}\right]$ |
| $-\frac{1}{2} \hbar$ | $\|-z\rangle=\left[\begin{array}{l}0 \\ 1\end{array}\right]$ | $\|-x\rangle=\left[\begin{array}{c}\sqrt{1 / 2} \\ -\sqrt{1 / 2}\end{array}\right]$ | $\|-\theta\rangle=\left[\begin{array}{c}-\sin \frac{\theta}{2} \\ \cos \frac{\theta}{2}\end{array}\right]$ |

## Summary of Spin Measurements



Not a surprise.


Electrons with a definite $S_{z}$ value have an equal chance for either $S_{X}$ eigenvalue.


Measurement alters the properties/components of the spin.


Removing information 'removes' a measurement (see second measurement).


Blocking a channel destroys the 'interference' pattern.

## The Einstein, Podolsky, Rosen (EPR) Paradox



## The Einstein, Podolsky, Rosen (EPR) Paradox 7


(1) Fire two electrons in a joint quantum state where the electrons have opposite momentum and zero total spin angular momentum.

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## The Einstein, Podolsky, Rosen (EPR) Paradox 9


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(2) The detectors are at $A$ and $B$ equidistant from the source.
(3) Once the electron is detected at $A$ we instantly know something about the electron at $B$.

## The Einstein, Podolsky, Rosen (EPR) Paradox 10


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(2) The detectors are at $A$ and $B$ equidistant from the source.
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(9) The information about state $B$ arrives at $A$ faster than a signal could travel from $B$ to $A$ at light speed - violating special relativity.

## The Einstein, Podolsky, Rosen (EPR) Paradox 11


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(6) The electron states must have predetermined, objective, real properties when the button was pushed.

## The Einstein, Podolsky, Rosen (EPR) Paradox 12


(1) Fire two electrons in a joint quantum state where the electrons have opposite momentum and zero total spin angular momentum.
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(3) Once the electron is detected at $A$ we instantly know something about the electron at $B$.
(9) The information about state $B$ arrives at $A$ faster than a signal could travel from $B$ to $A$ at light speed - violating special relativity.
(6) The electron states must have predetermined, objective, real properties when the button was pushed.
(0) Quantum mechanics is incomplete. There must be hidden variables.

## Bell's Theorem - 1

(1) Consider firing two electrons in a joint quantum state. The electrons have opposite momentum and zero total spin angular momentum.


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## Bell's Theorem - 1

(1) Consider firing two electrons in a joint quantum state. The electrons have opposite momentum and zero total spin angular momentum.

position A


position B
(2) The red and green colors flash when an electron is detected.
(3) If the two $\mathrm{SG} \theta$ devices have the same orientation, they flash the same color when each detects an electron - the color links opposite spins in the two detectors.
(4) Let $\theta_{A}=0^{\circ}, \theta_{B}=120^{\circ}$, and $A$ flashes red. (a) What is the state of the other electron? (b) For the $B$ device to flash red, what state must it be in? (c) What is the probability of the lights both flashing red?

| Observable | $S_{z}$ | $S_{x}$ | $S_{\theta}$ |
| :---: | :---: | :---: | :---: |
| $+\frac{1}{2} \hbar$ | $\|+z\rangle=\left[\begin{array}{l}1 \\ 0\end{array}\right]$ | $\|+x\rangle=\left[\begin{array}{c}\sqrt{1 / 2} \\ \sqrt{1 / 2}\end{array}\right]$ | $\|+\theta\rangle=\left[\begin{array}{c}\cos \frac{\theta}{2} \\ \sin \frac{\theta}{2}\end{array}\right]$ |
| $-\frac{1}{2} \hbar$ | $\|-z\rangle=\left[\begin{array}{l}0 \\ 1\end{array}\right]$ | $\|-x\rangle=\left[\begin{array}{c}\sqrt{1 / 2} \\ -\sqrt{1 / 2}\end{array}\right]$ | $\|-\theta\rangle=\left[\begin{array}{c}-\sin \frac{\theta}{2} \\ \cos \frac{\theta}{2}\end{array}\right]$ |

## Bell's Theorem - 2

Do a series of runs where you press the EPR button, select $\theta_{A}$ and $\theta_{B}$ in flight, at random, and independently. Set the orientations of each SG devices to one of the values $\theta_{1}=0^{\circ}, \theta_{2}=120^{\circ}$, and $\theta_{3}=-120^{\circ}$.
(1) When the orientations are the same what fraction have the same color?
(2) When the orientations are different what fraction have the same color?
(3) Ignoring the orientations what is the fraction of events with the same color?

## Bell's Theorem - 2

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| 31 RG | 23 RG | 23 RG | 23 GR | 22 GG | 33 GG | 22 GG |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 23 RR | 31 GR | 12 GR | 33 RR | 33 GG | 32 RG | 11 RR |
| 13 GR | 22 GG | 23 GR | 13 RG | 33 RR | 21 GR | 21 RG |
| 32 RG | 12 RR | 11 RR | 33 GG | 13 GR | 12 GG | 23 RR |
| 13 GR | 13 RG | 33 GG | 32 GG | 33 GG | 23 GR | 21 GR |
| 23 RR | 33 GG | 23 GG | 31 RG | 13 RR | 21 RG | 12 RG |
| 22 RR | 21 RG | 31 GR | 11 RR | 11 GG | 32 RG | $13 R \mathrm{R}$ |
| 31 RG | 11 GG | 33 GG | 11 GG | 23 GR | 31 GR | 23 GR |

(1) When the orientations are the same what fraction have the same color?
(2) When the orientations are different what fraction have the same color?
(3) Ignoring the orientations what is the fraction of events with the same color?

## Bell's Theorem - 3

Consider a classical model for the electron that includes hidden variables consistent with locality and reality. Add this 'observable' to the electron that will identify the color for for different orientations or values of $\theta$ for the SG devices. Call it the electron 'gizmo'.

Assign the colors:
(1) $0^{\circ} \rightarrow$ red. $\quad$ gizmo $=R G R$.
(2) $120^{\circ} \rightarrow$ green.
(3) $-120^{\circ} \rightarrow$ red.

What are all the different combinations for the gizmos?

What are all the different combinations for orientations of the SG devices?

## Bell's Theorem - 3

Consider the classical model where electrons carry additional information (hidden variables that tell which colors to flash for the different orientations). In the table below the columns represent all the combinations of orientations. The rows represent the 'hidden variable'. The 'Pr' is the probability of getting the same color. 'S' means the orientations are the same and ' D ' means they are different.

|  | 11 | 12 | 13 | 21 | 22 | 23 | 31 | 32 | 33 | $\operatorname{Pr}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| RRR |  |  |  |  |  |  |  |  |  |  |
| RRG |  |  |  |  |  |  |  |  |  |  |
| RGR | S | D | S | D | S | D | S | D | S |  |
| RGG |  |  |  |  |  |  |  |  |  |  |
| GRR |  |  |  |  |  |  |  |  |  |  |
| GRG |  |  |  |  |  |  |  |  |  |  |
| GGR |  |  |  |  |  |  |  |  |  |  |
| GGG |  |  |  |  |  |  |  |  |  |  |

## Bell's Theorem - 3

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|  | 11 | 12 | 13 | 21 | 22 | 23 | 31 | 32 | 33 | Pr |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| RRR | S | S | S | S | S | S | S | S | S | 1 |
| RRG | S | S | D | S | S | D | D | D | S | $5 / 9$ |
| RGR | S | D | S | D | S | D | S | D | S | $5 / 9$ |
| RGG | S | D | D | S | S | S | D | S | S | $5 / 9$ |
| GRR | S | D | D | D | S | S | D | S | S | $5 / 9$ |
| GRG | S | D | S | D | S | D | S | D | S | $5 / 9$ |
| GGR | S | S | D | S | S | D | D | D | S | $5 / 9$ |
| GGG | S | S | S | S | S | S | S | S | S | 1 |

## Bell's Theorem - 3

Consider the classical model where electrons carry additional information (hidden variables that tell which colors to flash for the different orientations). In the table below the columns represent all the combinations of orientations. The rows represent the 'hidden variable'. The 'Pr' is the probability of getting the same color. 'S' means the orientations are the same and ' D ' means they are different.

|  | 11 | 12 | 13 | 21 | 22 | 23 | 31 | 32 | 33 | Pr |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| RRR | S | S | S | S | S | S | S | S | S | 1 |
| RRG | S | S | D | S | S | D | D | D | S | $5 / 9$ |
| RGR | S | D | S | D | S | D | S | D | S | $5 / 9$ |
| RGG | S | D | D | S | S | S | D | S | S | $5 / 9$ |
| GRR | S | D | D | D | S | S | D | S | S | $5 / 9$ |
| GRG | S | D | S | D | S | D | S | D | S | $5 / 9$ |
| GGR | S | S | D | S | S | D | D | D | S | $5 / 9$ |
| GGG | S | S | S | S | S | S | S | S | S | 1 |

What is the average probability $\langle\operatorname{Pr}\rangle$ for getting the same color from the EPR apparatus?

## Summary

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(1) EPR argument - for joint, entangled states there is no apparent delay when one partner/electron/photon is measured and the wave function collapse is instantaneous violating relativity.
(2) Bell's Theorem (1) - Using standard quantum mechanical methods we found the EPR apparatus would detect events with the same color lights a fraction $f=0.50$ of the time.
(3) Bell's Theorem (2) - A 'hidden variable' model predicts the fraction $f$ of events with the same color lights is the following.

$$
f>5 / 9 \quad \text { and } \quad\langle P r\rangle=\frac{2}{3}
$$

## Bell's Theorem - 4

## 24

## WHO WINS? WHY?

(1) We found that in the quantum case $50 \%$ of the SG events should flash the same color.
(2) How does that fraction compare with the classical model?

## Bell's Theorem - 4

## WHO WINS? WHY?

(1) We found that in the quantum case $50 \%$ of the SG events should flash the same color.
(2) How does that fraction compare with the classical model?

## Quantum wins.

## Nature is non-local and unreal.

## Aspect's Results

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## Experimental Tests of Realistic Local Theories via Bell's Theorem

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## (Received 30 March 1981)

We have measured the linear polarization correlation of the photons emitted in a radiative atomic cascade of calcium. A high-efficiency source provided an improved statistical accuracy and an ability to perform new tests. Our results, in excellent agreement with the quantum mechanical predictions, strongly violate the generalized Bell's inequalities, and rule out the whole class of realistic local theories. No significant change in results was observed with source-polarizer separations of up to 6.5 m .


FIG. 4. Normalized coincidence rate as a function of the relative polarizer orientation. Indicated errors are $\pm 1$ standard deviation. The solid curve is not a fit to the data but the prediction of quantum mechanics.

## Precision Measurements and Theory Comparison 27

The $z$ component of the muon magnetic moment is

$$
\left(\mu_{s}\right)_{z}=-g_{s} \mu_{B} m_{s}
$$

where $m_{s}=1 / 2$ is the spin quantum number, $\mu_{B}=e \hbar / 2 m_{e}$ is the Bohr magneton, and the spin $g$-factor is

$$
\begin{gathered}
g_{\mu}=-2.00233184131(82)(\exp ) \\
g_{\mu}=-2.00233183620(86)(\mathrm{th})
\end{gathered}
$$

where the relative standard uncertainty for both values is about $4 \times 10^{-10}$. This difference may be due to physics Beyond the Standard Model (BSM).

## Some Properties of Spin Measurements



Not a surprise.


Electrons with a definite value of $S_{z}$ have an equal chance for either eigenvalue of $S_{x}$.


Measurement alters the properties/components of the spin.


Removing information 'removes' a measurement (see second measurement.

