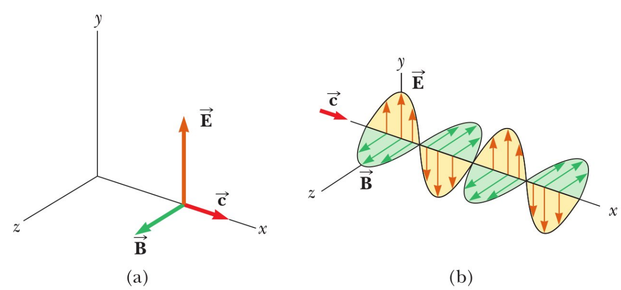
Homework 4 Wave Optics

1. (a) The distance to the North Star, Polaris, is approximately 6.44×10^{18} m. If Polaris were to burn out today, in what year would we see it disappear? (b) How long does it take for sunlight to reach the Earth? (c) How long does it take for a microwave radar signal to travel from the Earth to the Moon and back? (d) How long does it take for a radio wave to travel once around the Earth in a great circle, close to the planet's surface? (e) How long does it take for light to reach you from a lightning stroke 10.0 km away?

2. An electromagnetic wave in vacuum has an electric field amplitude of 220 V/m. Calculate the amplitude of the corresponding magnetic field.

3. The figure shows a plane electromagnetic sinusoidal wave propagating in the x direction. Suppose the wavelength is 50.0 m and the electric field vibrates in the xy plane with an amplitude of 22.0 V/m. Calculate (a) the frequency of the wave and (b) the magnitude and direction of B when the electric field has its maximum value in the negative y direction. (c) Write an expression for B with the correct unit vector, with numerical values for B_{max}, k, and ω, and with its magnitude in the form



$$\vec{B} = B_{max}\cos(kx - \omega t)$$

4. In SI units, the electric field in an electromagnetic wave propagating in the z-direction is described by

$$E_y = 100\sin(10^7 x - \omega t)$$

Find (a) the amplitude of the corresponding magnetic field oscillations, (b) the wavelength λ, and (c) the frequency f.

5. Verify by substitution that the following equations

$$E = E_{max}\cos(kx - \omega t) \qquad B = B_{max}\cos(kx - \omega t)$$

are solutions to

$$\frac{\partial^2 E}{\partial x^2} = \epsilon_0\mu_0\frac{\partial^2 E}{\partial t^2} \qquad \frac{\partial^2 B}{\partial x^2} = \epsilon_0\mu_0\frac{\partial^2 B}{\partial t^2}$$

6. A microwave oven is powered by an electron tube called a magnetron, which generates electromagnetic waves of frequency 2.45 GHz. The microwaves enter the oven and are reflected by the walls. The standing wave pattern produced in the oven can cook food unevenly, with hot spots in the food at antinodes and cool spots at nodes, so a turntable is often used to rotate the food and distribute the energy. If a microwave oven intended for use with a turntable is instead used with a cooking dish in a fixed position, the antinodes can appear as burn marks on foods such as carrot strips or cheese. The separation distance between the burns is measured to be 6 $cm \pm 5\%$. From these data, calculate the speed of the microwaves.

7. A community plans to build a facility to convert solar radiation to electrical power. The community requires 1.00 MW of power, and the system to be installed has an efficiency of 30.0% (that is, 30.0% of the solar energy incident on the surface is converted to useful energy that can power the community). What must be the effective area of a perfectly absorbing surface used in such an installation if sunlight has a constant intensity of 1000 W/m^2?

8. When a high-power laser is used in the Earth's atmosphere, the electric field can ionize the air, turning it into a conducting plasma that reflects the laser light. In dry air at 0° C and 1 atm, electric breakdown occurs for fields with amplitudes above about 3.00 MV/m. (a) What laser beam intensity will produce such a field? (b) At this maximum intensity, what power can be delivered in a cylindrical beam of diameter 5.00 mm?

9. In a region of free space, the electric field at an instant of time is $\vec{E} = (80.0\hat{\imath} + 32.0\hat{\jmath} - 64.0\hat{k})\ N/C$ and the magnetic field is $\vec{B} = (0.20\hat{\imath} + 0.080\hat{\jmath} + 0.290\hat{k})\ \mu T$. (a) Show that the two fields are perpendicular to each other. (b) Determine the magnitude of the Poynting vector for these fields.

10. An important news announcement is transmitted by radio waves to people sitting next to their radios 100 km from the station and by sound waves to people sitting across the newsroom 3.00 m from the newscaster. Who receives the news first? Explain. Take the speed of sound in air to be 343 m/s.

11. Compute an order-of-magnitude estimate for the frequency of an electromagnetic wave with wavelength equal to (a) your height and (b) the thickness of this sheet of paper. How is each wave classified on the electromagnetic spectrum?

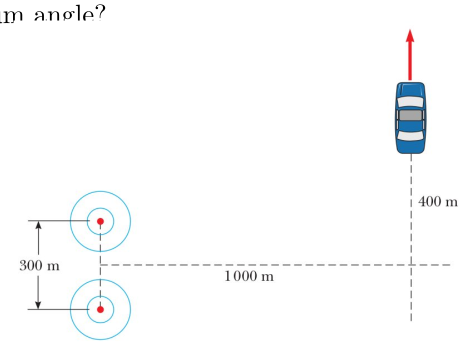
12. High-power lasers in factories are used to cut through cloth and metal. One such laser has a beam diameter of 1.00 mm and generates an electric field having an amplitude of 0.700 MV/m at the target. Find (a) the amplitude of the magnetic field produced, (b) the intensity of the laser, and (c) the power delivered by the laser.

13. An astronaut, stranded in space 10.0 m from her spacecraft and at rest relative to it, has a mass (including equipment) of 110 kg. Because she has a 100-W light source that forms a directed beam, she considers using the beam as a photon rocket to propel herself continuously toward the spacecraft. (a) Calculate the time interval required for her to reach the spacecraft by this method. (b) Assume, instead, that she throws the light source in the direction away from the spacecraft. The mass of the light source is 3.00 kg and, after being thrown, it moves at 12.0 m/s relative to the recoiling astronaut. After what time interval will the astronaut reach the spacecraft?

14. A Young's interference experiment is performed with monochromatic light. The separation between the slits is 0.500 mm, and the interference pattern on a screen 3.30 m away shows the first side maximum 3.40 mm from the center of the pattern. What is the wavelength?

15. In a location where the speed of sound is 354 m/s, a 2 000-Hz sound wave impinges on two slits 30.0 cm apart. (a) At what angle is the first maximum located? (b) If the sound wave is replaced by 3.00-cm microwaves, what slit separation gives the same angle for the first maximum? (c) If the slit separation is 1.0 μm, what frequency of light gives the same first maximum angle?

16. Two radio antennas separated by 300 m as shown in the figure simultaneously broadcast identical signals at the same wavelength. A radio in a car traveling due north receives the signals. (a) If the car is at the position of the second maximum, what is the wavelength of the signals? (b) How much farther must the car travel to encounter the next minimum in reception?

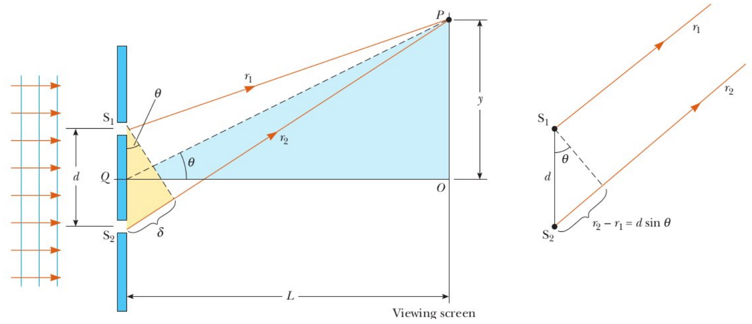


17. The two speakers of a boom box are 35.0 cm apart. A single oscillator makes the speakers vibrate in phase at a frequency of 2.00 kHz. At what angles, measured from the perpendicular bisector of the line joining the speakers, would a distant observer hear maximum sound intensity? Minimum sound intensity? (Take the speed of sound as 340 m/s.)

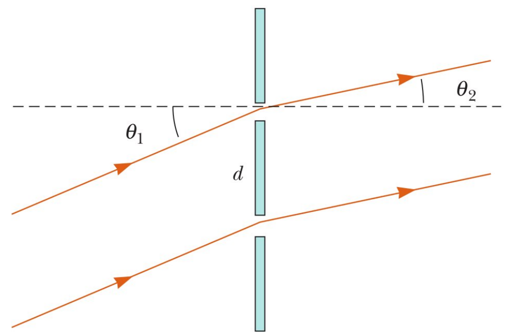
18. Young's double-slit experiment is performed with 589-nm light and a distance of 2.00 m between the slits and the screen. The tenth interference minimum is observed 7.26 mm from the central maximum. Determine the spacing of the slits.

19. Two slits are separated by 0.320 mm. A beam of 500-nm light strikes the slits, producing an interference pattern. Determine the number of maxima observed in the angular range $-30° < \theta < 30°$.

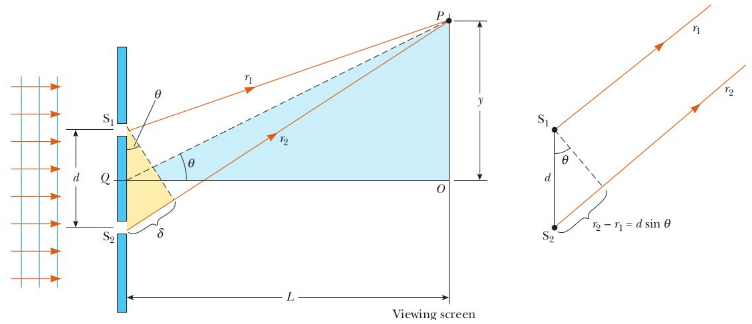
20. In the figure, let $L = 1.20\ m$ and $d = 0.120\ mm$ and assume that the slit system is illuminated with monochromatic 500-nm light. Calculate the phase difference between the two wave fronts arriving at P when (a) $\theta = 0.5°$ and (b) $y = 5.0\ mm$. (c) What is the value of θ for which the phase difference is 0.333 rad? (d) What is the value of θ for which the path difference is $\lambda/4$?



21. Coherent light rays of wavelength λ strike a pair of slits separated by distance d at an angle θ_1 as shown in the figure. Assume that an interference maximum is formed at an angle θ_2 a great distance from the slits. Show that $d(\sin\theta_2 - \sin\theta_1) = m\lambda$, where m is an integer.



22. In the figure, let $L = 120cm$ and $d = 0.250\ cm$. The slits are illuminated with coherent 600-nm light. Calculate the distance y above the central maximum for which the average intensity on the screen is 75.0% of the maximum.



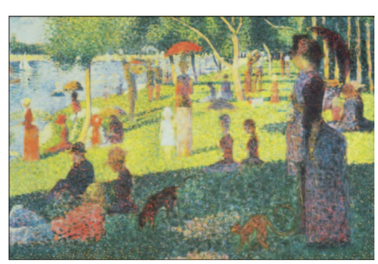
23. The intensity on the screen at a certain point in a double- slit interference pattern is 64.0% of the maximum value. (a) What minimum phase difference (in radians) between sources produces this result? (b) Express this phase difference as a path difference for 486.1-nm light.

24. Two slits are separated by 0.180 mm. An interference pattern is formed on a screen 80.0 cm away by 656.3-nm light. Calculate the fraction of the maximum intensity 0.600 cm above the central maximum.

25. Coherent microwaves of wavelength 5.00 cm enter a long, narrow window in a building otherwise essentially opaque to the microwaves. If the window is 36.0 cm wide, what is the distance from the central maximum to the first-order minimum along a wall 6.50 m from the window?

26. A helium-neon laser emits light that has a wavelength of 632.8 nm. The circular aperture through which the beam emerges has a diameter of 0.500 cm. Estimate the diameter of the beam 10.0 km from the laser.

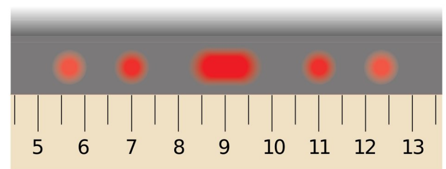
27. The Impressionist painter Georges Seurat created paintings with an enormous number of dots of pure pigment, each of which was approximately 2.00 mm in diameter. The idea was to have colors such as red and green next to each other to form a scintillating canvas (see figure). Outside what distance would one be unable to discern individual dots on the canvas? (Assume that $\lambda = 500\ nm$ and that the pupil diameter is 4.00 mm.)



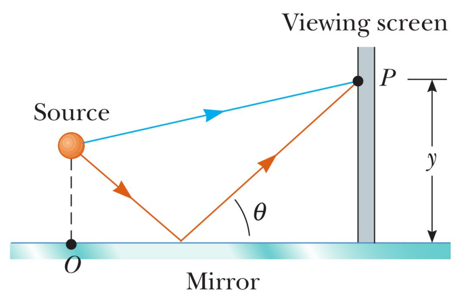
28. A spy satellite can consist essentially of a large-diameter concave mirror forming an image on a digital-camera detector and sending the picture to a ground receiver by radio waves. In effect, it is an astronomical telescope in orbit, looking down instead of up. Can a spy satellite read a license plate? Can it read the date on a dime? Argue for your answers by making an order-of-magnitude calculation, specifying the data you estimate.

29. A circular radar antenna on a Coast Guard ship has a diameter of 2.10 m and radiates at a frequency of 15.0 GHz. Two small boats are located 9.00 km away from the ship. How close together could the boats be and still be detected as two objects?

30. Laser light with a wavelength of 632.8 nm is directed through one slit or two slits and allowed to fall on a screen 2.60 m beyond. The figure shows the pattern on the screen, nearly actual size, with a centimeter rule below it. Did the light pass through one slit or two slits? If one, find its width. If two, find the distance between their centers.



31. Interference effects are produced at point P on a screen as a result of direct rays from a 500-nm source and reflected rays from the mirror as shown in the figure. There is a $\phi = \pi/2$ phase shift when light is reflected in the mirror. Assume that the source is 100 m to the left of the screen and 1.00 cm above the mirror. Find the distance y to the first dark band above the mirror.



32. The waves from a radio station can reach a home receiver by two paths. One is a straight-line path from transmitter to home, a distance of 30.0 km. The second path is by reflection from the ionosphere (a layer of ionized air molecules high in the atmosphere). Assume that this reflection takes place at a point midway between receiver and transmitter and that the wavelength broadcast by the radio station is 350 m. Find the minimum height of the ionospheric layer that produces destructive interference between the direct and reflected beams. (Assume that no phase change occurs on reflection.)

33. A screen is placed a distance $L = 50.0\ cm$ from a single slit, which is illuminated with light of wavelength $\lambda = 690\ nm$. If the distance between the first and third minima in the diffraction pattern is 3.0 mm, what is the width of the slit?

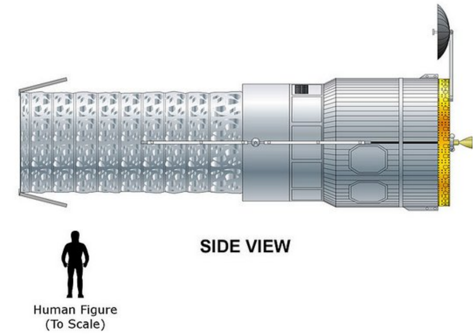
34. A beam of monochromatic green light is diffracted by a slit of width 0.550 mm. The diffraction pattern forms on a wall 2.06 m beyond the slit. The distance between the positions of zero intensity on both sides of the central bright fringe is 4.10 mm. Calculate the wavelength of the light.

35. A beam of laser light of wavelength 632.8 nm has a circular cross-section 2.00 mm in diameter. A rectangular aperture is to be placed in the center of the beam so that when the light falls perpendicularly on a wall 4.50 m away, the central maximum fills a rectangle 110 mm wide and 6.00 mm high. The dimensions are measured between the minima bracketing the central maximum. Find the required width and height of the aperture.

36. Sound with a frequency 650 Hz from a distant source passes through a doorway 1.10 m wide in a sound-absorbing wall. Find the number and approximate directions of the diffraction-maximum beams radiated into the space beyond.

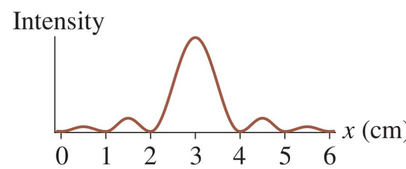
37. Light of wavelength $\lambda = 600\ nm$ passes though two slits separated by a distance $d = 0.20\ mm$ and is observed on a screen $L = 1.0\ m$ behind the slits. A very thin piece of glass is then placed in one slit. Light travels slower in glass than in air so the wave passing through the glass is delayed by a time $\Delta t = 5.0 \times 10^{-16}\ s$ in comparison to the wave going through the other slit. What fraction of the period of the light wave is this delay? The glass causes the interference fringe pattern on the screen to shift sideways. Which way does the central maximum move (toward or away from the slit with the glass) and by how far?

38. A spy satellite, military, commercial or otherwise, like the KH-11 satellite shown here (based on the Hubble Space Telescope) consists of a large-diameter concave mirror forming an image on a digital sensor. It is an astronomical telescope looking down instead of up. The KH-11 telescope may have an aperture of size $a = 0.70\ m$ and orbited the Earth at an altitude of $L = 681\ km$. Could the camera be used to read a license plate on Earth? The range of visible light is $\lambda = 400 - 700 \times 10^{-9}\ m$. Show your reasoning.

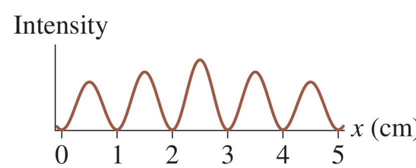


39. In a joint French-Soviet experiment in the 1980's to monitor the Moon's surface with a light beam, pulsed radiation from a ruby laser ($\lambda = 690\ nm = 6 \times 10^{-7}\ m$) was directed to the Moon through a reflecting telescope with a mirror radius of $r_1 = 1.3\ m$. A reflecting mirror on the Moon behaved like a circular, plane mirror with a radius $r_2 = 0.1\ m$, reflecting the light directly back toward the telescope on the Earth. What fraction of the original light energy was reflected back to the Earth by the mirror on the Moon? Assume that all the light energy is in the central diffraction peak and that it is uniformly spread out in that central diffraction peak. The Earth-Moon distance is $R_m = 3.84 \times 10^8\ m$.

40. The figure shows the light intensity on a screen a distance $D = 2.5\ m$ from an aperture. The aperture is illuminated with light of wavelength $\lambda = 600\ nm$. (a) Is the aperture a single slit or a double slit? Explain. (b) If the aperture is a single slit, what is its width? If it is a double slit, what is the spacing between the slits?

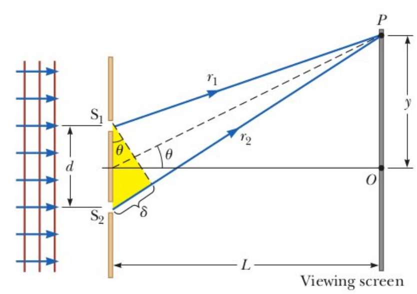


41. The figure shows the light intensity on a screen a distance $D = 2.5\ m$ from an aperture. The aperture is illuminated with light of wavelength $\lambda = 600\ nm$. (a) Is the aperture a single slit or a double slit? Explain. (b) If the aperture is a single slit, what is its width? If it is a double slit, what is the spacing between the slits?



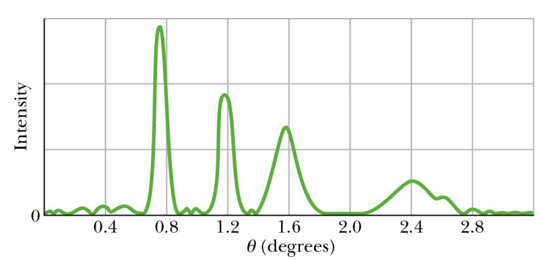
42. In the figure, let $L = 120\ cm$ and $d = 0.250\ cm$. The slits are illuminated with coherent light of wavelength $\lambda = 600\ nm$. Calculate the distance y above the central maximum for which the average intensity on the screen is 75.0% of the maximum.

43. In the figure, let $L = 160\ cm$, $y = 1\ mm$, and $d = 0.20\ mm$. What wavelength λ will result in an average intensity at P that is 36% of the maximum?

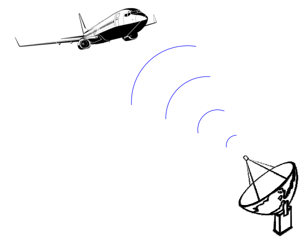


44. Two narrow parallel slits separated by a distance $d = 0.850\ mm$ are illuminated by $\lambda = 600 - nm$ light The viewing screen is $D = 2.80\ m$ away from the slits. (a) What is the phase difference between the two interfering waves on a screen at a point $y = 2.50\ mm$ from the central bright fringe? (b) What is the ratio of the intensity at this point to the intensity at the center of a bright fringe?

45. The figure shows a some data for intensity versus diffraction angle for the diffraction of an x-ray beam through a crystal acting like a diffraction grating. The beam consists of two wavelengths and the spacing between the planes of atoms is $d = 1.88\ nm$. What are the two wavelengths?



46. A radar for tracking aircraft must deliver an average intensity of $I_1 = 6\ J/s - m^2$ on a distant aircraft to detect it. It operates at a frequency $f = 1.2 \times 10^{10}\ s^{-1}$ from circular radar antenna of radius $r_a = 1\ m$ and emits a power $P_0 = 10^5\ J/s$. From a wave perspective, the antenna acts as a circular aperture. What is the range of the radar system?



47. In the figure two isotropic point sources of light (S_1 and S_2) are separated by distance $\Delta y = 2.70\ \mu m$ along the y axis and emit light in phase at wavelength $\lambda = 900\ nm$ and with the same amplitude. A light sensor is located at point P at x_P on the x axis. The distance x_p is not necessarily much larger than the separation Δy of the sources. What is the greatest value of x_P at which the detected light is a minimum due to destructive interference?