Homework 3 Relativity

1. A sprinter crosses the finish line of a race. The roar of the crowd in front approaches her at a speed of $v_f = 360\ m/s$. The roar from the crowd behind her approaches at $v_f = 330\ m/s$. What are the speed of sound and the speed of the sprinter?

2. A baseball pitcher can throw a ball with a speed of $v_p = 40\ m/s$. He is in the back of a pickup truck that is driving away from you. He throws the ball in your direction, and it floats toward you at a lazy $v_t = 10\ m/s$. What is the speed of the truck?

3. A 2000-kg car moving at 20.0 m/s collides and locks together with a 1 500-kg car at rest at a stop sign. Using Newtonian relativity show that momentum is conserved in a reference frame moving at 10.0 m/s in the direction of the moving car.

4. In a laboratory frame of reference, an observer notes that Newton's second law is valid. Using Newtonian relativity show that it is also valid for an observer moving at a constant speed, small compared with the speed of light, relative to the laboratory frame.

5. Show that Newton's Second Law is not valid in a reference frame moving past the laboratory frame of Problem 4 above with a constant acceleration.

6. Using special relativity, at what speed does a clock move if it is measured to run at a rate that is one-half the rate of a clock at rest with respect to an observer?

7. You wish to make a round trip from Earth in a spaceship traveling at constant speed in a straight line for 1 year and then returning at the same constant speed. You wish, further, on your return to find Earth as it will be 100 years in the future. How fast must you travel?

8. An astronomer on the Earth observes a meteoroid in the southern sky approaching the Earth at a speed $v_0 = 0.85c$. At the time of its discovery the meteoroid is a distance $x_0 = 1.0\ ly$ from Earth. What is the time interval for the meteoroid to reach the Earth as measured by the Earthbound astronomer? What is the time interval as measured by an alien tourist traveling on the meteoroid?

9. An astronaut is traveling in a space vehicle that has a speed of 0.500c relative to the Earth. The astronaut measures her pulse rate at 75.0 beats per minute. Signals generated by the astronaut's pulse are radioed to the Earth when the vehicle is moving in a direction perpendicular to the line that connects the vehicle with an observer on the Earth. (a) What pulse rate does the Earth observer measure? (b) What would be the pulse rate if the speed of the space vehicle were increased to 0.990c?

10. A muon formed high in the Earth's atmosphere travels at speed $v = 0.990c$ for a distance of 4.60 km before it decays into an electron, a neutrino, and an antineutrino ($\mu^- \to e^- + \nu + \nu$). (a) How long does the muon live, as measured in its reference frame? (b) How far does the Earth travel, as measured in the frame of the muon?

11. An atomic clock moves at 1000 km/h for 1.00 h as measured by an identical clock on the Earth. How many nanoseconds slow will the moving clock be compared with the Earth clock at the end of the $1.00 - h$ interval?

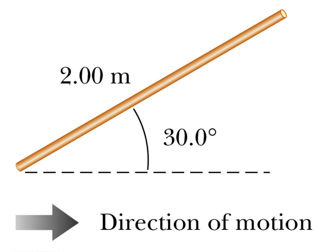
12. A spacecraft with a proper length of 300 m takes 0.750 μs to pass an Earth observer. Determine the speed of the spacecraft as measured by the Earth observer.

13. The identical twins Speedo and Goslo join a migration from the Earth to Planet X. It is 20.0 ly away in a reference frame in which both planets are at rest. The twins, of the same age, depart at the same time on different spacecraft. Speedo's craft travels steadily at 0.950c, and Goslo's travels at 0.750c. Calculate the age difference between the twins after Goslo's spacecraft lands on Planet X. Which twin is the older?

14. A friend passes by you in a spacecraft traveling at a high speed. He tells you that his craft is $\ell_1 = 20.0\ m$ long and that the identically constructed craft you are sitting in is $\ell_2 = 19.0\ m$ long. According to your observations, (a) how long is your spacecraft, (b) how long is your friend's craft, and (c) what is the speed of your friend's craft?

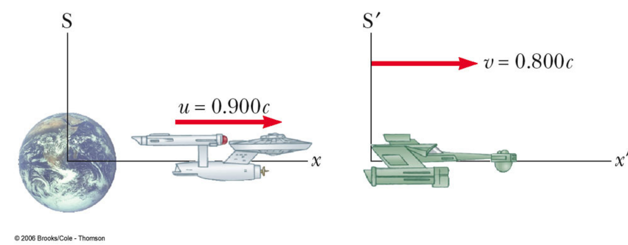
15. An interstellar space probe is launched from the Earth. After a brief period of acceleration it moves with a constant velocity, with a magnitude of 70.0% of the speed of light. Its nuclear-powered batteries supply the energy to keep its data transmitter active continuously. The batteries have a lifetime of 15.0 yr as measured in a rest frame. (a) How long do the batteries on the space probe last as measured by Mission Control on the Earth? (b) How far is the probe from the Earth when its batteries fail as measured by Mission Control? (c) How far is the probe from the Earth when its batteries fail as measured by its built-in trip odometer? (d) For what total time interval after launch are data received from the probe by Mission Control? Note that radio waves travel at the speed of light and fill the space between the probe and the Earth at the time of battery failure.

16. A moving rod is measured to have a length of 2.00 m and to be oriented at an angle of 30.0° with respect to the direction of motion as shown in the figure. The rod has a speed of $0.995c$. (a) What is the proper length of the rod? (b) What is the orientation angle in the proper frame?



17. Galaxy A is reported to be receding from us with a speed of $0.5c$. Galaxy B, located in precisely the opposite direction, is found to be receding from us at a velocity of $0.6c$. (a) What recessional velocity would an observer on Galaxy A find for our galaxy? (b) What recessional velocity would an observer on Galaxy A find for Galaxy B?

18. A Klingon spacecraft moves away from the Earth at a speed of 0.800c (see figure). The starship Enterprise pursues at a speed of 0.900c relative to the Earth. Observers on the Earth measure the Enterprise overtaking the Klingon craft at a relative speed of 0.100c. With what speed is the Enterprise overtaking the Klingon craft as measured by the crew of the Enterprise?



19. A spacecraft is launched from the surface of the Earth with a velocity of $0.600c$ at an angle of 50.0° above the horizontal positive x axis. Another spacecraft is moving past with a velocity of $0.700c$ in the negative x direction. Determine the magnitude and direction of the velocity of the first spacecraft as measured by the pilot of the second spacecraft.

20. A spaceship, at rest in the reference frame S, is given a speed increment of $0.60c$. Relative to its new rest frame it is given a further $0.6c$ increment. This process is continued until its speed with respect to its original frame S exceeds $0.99c$. How many increments does this process require?

21. A physics professor on the Earth gives an exam to her students, who are in a spacecraft traveling at speed v relative to the Earth. The moment the craft passes the professor, she signals the start of the exam. She wishes her students to have a time interval T_0 (spacecraft time) to complete the exam. Show that she should wait a time interval (Earth time) of

$$T = T_0\sqrt{\frac{1 - v/c}{1 + v/c}}$$

before sending a light signal telling them to stop. (*Suggestion*: Remember that it takes some time for the second light signal to travel from the professor to the students.)

22. To preserve some of the useful ideas of physics we developed in this course even at relativistic speeds consider a re-definition of the momentum $\vec{p}$ from $\vec{p} = m\vec{v}$ to this.

$$\vec{p} = \frac{m\vec{v}}{\sqrt{1 - v^2/c^2}}$$

Now suppose we use the expression for the force that uses the momentum $\vec{p}$

$$\vec{F} = \frac{d\vec{p}}{dt}$$

and then take the derivative of the relativistic momentum by applying the chain rule and the product rule to show the following is true in one dimension.

$$\frac{dp}{dt} = \frac{d}{dt}\left(\frac{mv}{\sqrt{1 - v^2/c^2}}\right) = m\frac{1}{\left(1 - \frac{v^2}{c^2}\right)^{3/2}}\frac{dv}{dt}$$

23. A proton is accelerated to 0.999c. By what factor does the proton's momentum exceed its Newtonian momentum?

24. At what speed is a particle's momentum twice its Newtonian value?

25. A 1.0 g particle has momentum 400,000 kg m/s. What is the particle's speed?

26. What is the speed of a particle whose momentum is mc?

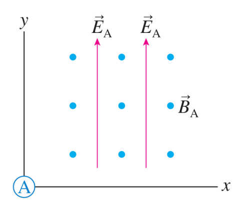
27. A quarter-pound hamburger with all the fixings has a mass of 200 g. The food energy of the hamburger (480 food calories) is 2 MJ. (a) What is the energy equivalent of the mass of the hamburger? (b) By what factor does the energy equivalent exceed the food energy?

28. At what speed is a particle's total energy twice its rest energy?

29. A typical nuclear power plant generates electricity at the rate of 1000 MW. The efficiency of transforming thermal energy into electrical energy is 1/3 and the plant runs at full capacity for 80% of the year. (Nuclear power plants are down about 20% of the time for maintenance and refueling.) (a) How much thermal energy does the plant generate in one year? (b) What mass of uranium is transformed into energy in one year?

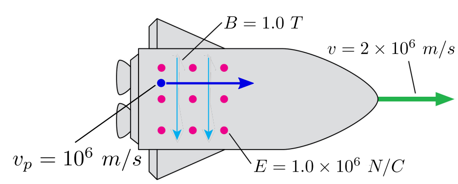
30. A rocket cruises past a laboratory at $v_O = 1.0 \times 10^6\ m/s$ in the positive x-direction just as a proton is launched with velocity (in the laboratory frame) $\vec{v} = (1.41 \times 10^6\hat{\imath} + 1.41 \times 10^6\hat{\jmath}\,)\ m/s$. What are the proton's speed and its angle from the y-axis in (a) the laboratory frame and (b) the rocket frame?

31. The figure shows the electric and magnetic field in frame A. A rocket in frame B travels parallel to one of the axes of the A coordinate system. Along which axis must the rocket travel, and in which direction, in order for the rocket scientists to measure (a) $B_B > B_A$, (b) $B_B = B_A$, and (c) $B_B < B_A$.



32. Scientists in the laboratory create a uniform electric field $\vec{E} = 1.0 \times 10\hat{k}\ \ V/m$ in a region of space where $B = 0$. What are the fields in the reference frame of a rocket traveling in the positive x-direction at $1.0 \times 10^6\ m/s$?

33. A rocket flies past the Earth at a velocity $v = 2 \times 10^6\ m/s$ (in green) as shown in the figure. The astronauts on board have set up electric and magnetic fields on the rocket. A proton (in blue) is injected into the field region with the velocity $v_p = 2.5 \times 10^6\ m/s$ as measured by the astronauts on the ship. What is the force on the proton as measured by the astronauts? What is the force on the proton as measured by someone on the Earth? Does this make sense?



34. Suppose the speed of a spaceship relative to an inertial frame fixed to the Sun is given by $|\vec{v}| = v = |\vec{a}|t = at$ where $a = 9.8\ m/s$. Where would the proper time be measured? How long does it take to accelerate from rest to $v_f = 0.5c$ in the frame of the Sun? In the frame on the spaceship?

35. An electron of mass $m_e = 0.511$ MeV and energy E scatters elastically off a stationary proton of mass $m_n = 938$ MeV. Show the energy of the scattered electron E' in terms of the incident/beam energy of the electron and the electron angle θ with respect to the beam direction is the following.

$$E' = \frac{E}{1 + \frac{2E}{m_n}\sin^2\frac{\theta}{2}}$$

What is the scattered electron energy for $E = 10.6$ GeV and $\theta = 10°$?

36. Use the previous result to show the angle ϕ of the proton with respect to the beam direction is

$$\phi = \tan^{-1}\left(\frac{E'\sin\theta}{E - E'\cos\theta}\right)$$

and calculate ϕ for the previous problem. Then show the magnitude of the scattered neutron 3-momentum is

$$p_{n'} = E'\frac{\sin\theta}{\sin\phi}$$

and calculate it for the previous problem.

37. In the previous two problems an electron of energy E scattered elastically off a stationary proton of mass m_n. Starting from the conservation of 4-momentum get the magnitude of the scattered proton momentum p_n' in terms of the beam energy E and the proton angle ϕ and show the following.

$$p_n' = \frac{2m_n\cos\phi(1 + m_n/E)}{1 + \frac{m_n^2}{E^2} + \frac{2m_n}{E} - \cos^2\phi}$$

Calculate p_n' for $\phi = 42.9°$ and $E = 10.6$ GeV. Compare your result for p_n' with what you obtained in the previous problem.