Homework 5 Quanta

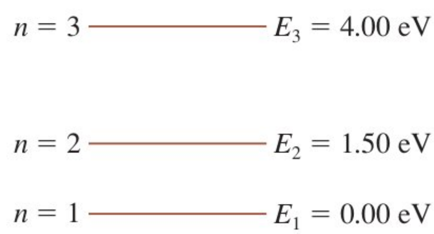
1. What is the wavelength, in nm, of a photon with energy (a) 0.30 eV, (b) 3.0 eV, and (c) 30 eV? For each, is this wave- length visible, ultraviolet, or infrared light?

2. For what wavelength of light does a 100 mW laser deliver 2.50×10^{17} photons per second?

3. Through what potential difference must an electron be accelerated from rest to have a de Broglie wavelength of 500 nm?

4. What is the de Broglie wavelength of a neutron that has fallen 1.0 m in a vacuum chamber, starting from rest?

5. The figure is an energy-level diagram for a simple atom. What wavelengths appear in the atom's (a) emission spectrum and (b) absorption spectrum?



6. The allowed energies of a simple atom are 0.00 eV, 4.00 eV, and 6.00 eV.
   (a) Draw the atom's energy-level diagram. Label each level with the energy and the quantum number.
   (b) What wavelengths appear in the atom's emission spectrum?
   (c) What wavelengths appear in the atom's absorption spectrum?

7. How much energy does it take to ionize a hydrogen atom that is in its first excited state?

8. What is the third-longest wavelength in the absorption spectrum of hydrogen?

9. Two hydrogen atoms collide head-on. The collision brings both atoms to a halt. Immediately after the collision, both atoms emit a 121.6 nm photon. What was the speed of each atom just before the collision?

10. A beam of electrons is incident upon a gas of hydrogen atoms.
    (a) What minimum speed must the electrons have to cause the emission of 656 nm light from the $3 \to 2$ transition of hydrogen?
    (b) Through what potential difference must the electrons be accelerated to have this speed?

11. The wave function for a particle is
$$\psi(x) = \sqrt{\frac{a}{\pi(x^2 + a^2)}}$$
for $a > 0$ and $-\infty < x < +\infty$. Determine the probability that the particle is located somewhere between $x = -a$ and $x = +a$.

12. An electron is contained in a one-dimensional box of length 0.100 nm. (a) Draw an energy level diagram for the electron for levels up to $n = 4$. (b) Find the wavelengths of all photons that can be emitted by the electron in making downward transitions that could eventually carry it from the $n = 4$ state to the $n = 1$ state.

13. The nuclear potential energy that binds protons and neutrons in a nucleus is often approximated by a square well. Imagine a proton confined in an infinitely high square well of length 10.0 fm, a typical nuclear diameter. Calculate the wavelength and energy associated with the photon emitted when the proton moves from the $n = 2$ state to the ground state. In what region of the electromagnetic spectrum does this wavelength belong?

14. A photon with wavelength λ is absorbed by an electron confined to a box. As a result, the electron moves from state $n = 1$ to $n = 4$. (a) Find the length of the box. (b) What is the wavelength of the photon emitted in the transition of that electron from the state $n = 4$ to the state $n = 2$?

15. The wave function of a particle is given by

$$\psi(x) = A\cos(kx) + B\sin(kx)$$

where A, B, and k are constants. Show that ψ is a solution of the Schrödinger equation

$$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} + U\psi = E\psi$$

assuming the particle is free ($U = 0$), and find the corresponding energy E of the particle.

16. The wave function of an electron is

$$\psi(x) = \sqrt{\frac{2}{L}}\sin\left(\frac{2\pi x}{L}\right)$$

Calculate the probability of finding the electron between $x = 0$ and $x = L/4$.

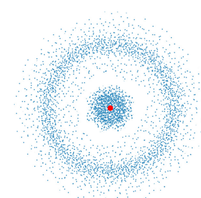
17. A particle of mass $m = 2.0 \times 10^{-28}$ kg is confined to a one-dimensional box of length $\ell = 1.0 \times 10^{10}$ m. For $n = 1$, what are (a) the particle's wavelength, (b) its momentum, and (c) its ground-state energy?

18. Imagine that a particle has a wave function

$$\psi(x) = \begin{cases} \sqrt{\frac{2}{a}}e^{-x/a} & \text{for} > 0 \\ 0 & \text{for} < 0 \end{cases}$$

19. An atom (not a hydrogen atom) absorbs a photon whose frequency is $f = 6.2 \times 10^{14}$ Hz. How much does the energy of the atom increase?

20. What is the energy of the hydrogen atom electron whose probability density is represented by the dot plot shown below? What minimum energy is needed to remove this electron from the atom?



21. What are the energy and wavelength of a photon emitted when a hydrogen atom undergoes a transition from the $n = 3$ state to the $n = 1$ state?

22. What is the wavelength of the least energetic photon emitted in the Lyman series (where $n_f = 1$) of the hydrogen atom spectrum lines?

23. What is the series limit for the Lyman series (where $n_f = 1$)?

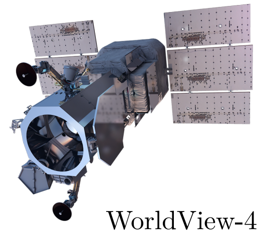
24. A Russian Arktica satellite that monitors polar weather follows an elliptical orbit around the Earth at an altitude of $h = 300$ km above the surface (radius $r_s = 6.67 \times 10^6$ m) at a velocity

$$\vec{v} = 4.1 \times 10^3 \ m/s \ \hat{r} + 7.5 \times 10^3 \ m/s \ \hat{\theta} \qquad .$$

What is the angular momentum? What is the total energy? What is the distance of closest approach to the Earth? The satellite mass is $m_s = 600$ kg.

Arktica-M

25. The WorldView-4 satellite is in a circular orbit around the Earth at an altitude of $h = 681\ km$ above the surface at a speed $v_0 = 7.52 \times 10^3\ m/s$. It quickly burns it rockets pointing in a direction away from the Earth and parallel to the direction of the gravitational force. It reaches a new mechanical energy $ME_1 = -1.14 \times 10^{10}\ J$. What will be its new angular momentum and new distance of closest approach to the Earth? Is this rocket burn a good idea? The satellite mass is $m_s = 726\ kg$.


WorldView-4

26. A satellite of mass $m_s = 726\ kg$ is orbiting the Earth and has a perigee (closest point to the center of the Earth) $r_p = 1.31 \times 10^7\ m$ with a velocity $v_p = 7.5 \times 10^3\ m/s$ at that point. How far away is apogee r_a (farthest point from the center of the Earth)?

27. A satellite of mass $m_s = 800\ kg$ is orbiting the Earth with a velocity $\vec{v}_i = 5.3 \times 10^3\ m/s\ \hat{r} + 6.8 \times 10^3\ m/s\ \hat{\theta}$ at a distance $r_s = 10^7\ m$ from the Earth. It has to burn its rocket so the apogee r_a (farthest point from the center of the Earth) is $r_a = 1.5 \times 10^8\ m$. How much does its speed have to increase?

28. A monochromatic source of light radiates with power $P = 25$ W at a wavelength $\lambda = 5000$ Å. A plate of metal is placed 100 cm from the source. Atoms in the metal have a radius of 1 Å. Assume the atom can continually absorb light. The work function of the metal is $W = 4$ eV. How long is it before an electron is emitted from the metal?

29. The work function of zinc is 3.6 eV. What is the energy of the most energetic electron emitted by ultraviolet light of wavelength 1900 Å?

30. Photoelectrons are observed when a metal is illuminated by light with a wavelength less than 388 nm. What is the metal's work function?

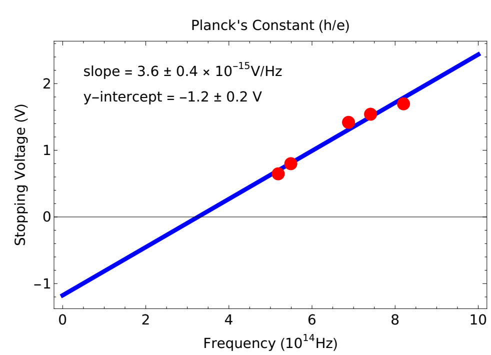
31. Electrons in a photoelectric-effect experiment emerge from an aluminum surface with a maximum kinetic energy of 1.30 eV. What is the wavelength of the light? The table to the right has a list of work functions of different metals.

32. A photoelectric-effect experiment finds a stopping potential of 1.56 V when light of 200 nm is used to illuminate the cathode. (a) From what metal is the cathode made? (b) What is the stopping potential if the intensity of the light is doubled? The table to the right has a list of work functions of different metals.

Element	$E_0(eV)$
Potassium	2.30
Sodium	2.75
Aluminum	4.28
Tungsten	4.55
Copper	4.65
Iron	4.70
Gold	5.10

33. A helium laser pointer has a power $P = 1\ mW$ with a beam spot of radius $w = 1\ cm$. You shine the laser on a piece of tungsten which has atoms of radius $r = 1.41$ Å and work function $E_{ej} = 4.55\ eV$. Use the 'swimming pool' model of the photoelectric effect. Assume all of the light striking a single atom gets absorbed and funneled into the kinetic energy of a single electron. What is the rate of electrons ejected from the surface? How long does it take for the first electron to get ejected?

34. The plot shows some results from Intermediate Lab on measuring Planck's constant and the work function of a metal. What is the relationship between the stopping voltage and the frequency f of the light striking the metal? What is the value of Planck's constant from the fit to the data? What is the work function of the metal? What is the metal?



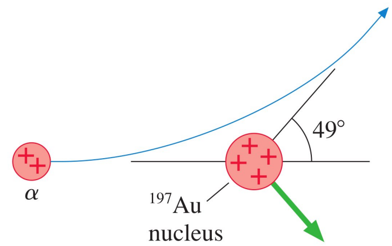
35. Through what potential difference would you need to accelerate an alpha particle (a ^{4}He nucleus), starting from rest, so that it will just reach the surface of a 15-fm-diameter ^{238}U nucleus?

36. The oxygen nucleus ^{16}O has a radius of 3.0 fm. (a) With what speed must a proton be fired toward an oxygen nucleus to have a turning point 1.0 fm from the surface? Assume the nucleus remains at rest. (b) What is the proton's kinetic energy in MeV?

37. In a head-on collision, the closest approach of a 6.24 MeV ^{4}He nucleus to the center of a nucleus is 6.0 fm. The nucleus is an atom of what element? Assume the nucleus remains at rest.

38. A 100-eV hydrogen atom ^{1}H collides head-on in a perfectly elastic collision with a ^{28}Si atom at rest. What is the kinetic energy of each atom after the collision?

39. An alpha particle (a ^{4}He nucleus) approaches a ^{197}Au nucleus with a speed of $1.50 \times 10^{7}\ m/s$ as shown in the figure. The alpha particle is scattered at a 49° angle at the slower speed of $1.49 \times 10^{7}\ m/s$. In what direction does the ^{197}Au nucleus recoil, and with what speed?



40. (a) An electron has a kinetic energy of 3.00 eV. Find its wavelength. (b) Suppose a photon has energy 3.00 eV. Find its wavelength.

41. The nucleus of an atom is on the order of $10^{-14}\ m$ in diameter. For an electron to be confined to a nucleus, its de Broglie wavelength would have to be on this order of magnitude or smaller. (a) What would be the kinetic energy of an electron confined to this region? (b) Make an order-of-magnitude estimate of the electric potential energy of a system of an electron inside an atomic nucleus. (c) Would you expect to fi nd an electron in a nucleus? Explain.

42. In the Davisson–Germer experiment, 54.0-eV electrons were diffracted from a nickel lattice. If the first maximum in the diffraction pattern was observed at $\phi = 50°$, what was the lattice spacing a between the vertical columns of atoms in the figure?