

Consider the two twins again. One sets out at the age of 25 on a spaceship from Earth at a speed of $0.99c$ where c is the speed of light. The Earthbound twin goes on about her/his business accumulating the normal accouterments of advancing age (gray hair, drooping body parts, *etc.*). After twenty years have passed for the Earthbound twin, the spacefaring one returns. In the frame of the Earthbound twin what distance did the traveling twin cover during his/her trip? What distance did the traveling twin measure in his/her frame?

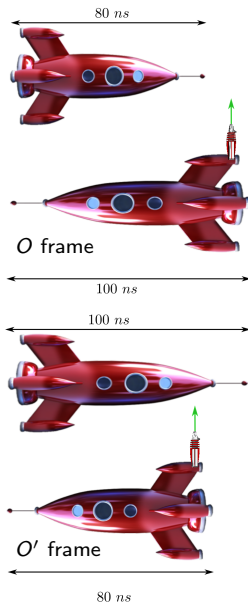


The Space Wars Paradox

3

Two spacecraft of equal rest length $L_R = 100 \text{ ns} = 30 \text{ m}$ pass very close to each other as they travel in opposite directions at a relative speed $|\beta| = \frac{3}{5}$ ($0.6c$). See figure. The captain of ship O intends to fire a warning shot across the bow of ship O' with her laser cannon at the instant her bow is lined up with the tail of ship O' . Since O' is Lorentz-contracted to a length of $80 \text{ ns} = 24 \text{ m}$ in the frame of ship O , she expects the laser burst to miss the other ship by a distance $20 \text{ ns} = 6 \text{ m}$. However, to the captain on ship O' , it is ship O that has contracted to 80 ns so the laser burst will strike ship O' about 20 ns behind the bow

What really happens if O carries out her plan? Construct a two-observer spacetime diagram. Define event A to be coincidence of the bow of ship O and the tail of O' and event B to be the firing of the laser. Let A define the origin event in both frames. When and where does this event occur as measured in the O' frame? The travel time of the laser from one ship to another is negligible. Verify your results with the Lorentz transformations.



Adapted from *Spacetime Physics* by Taylor and Wheeler, 1966.

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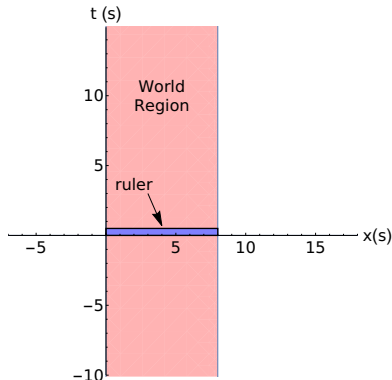
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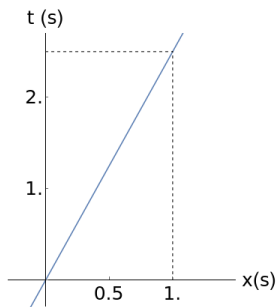
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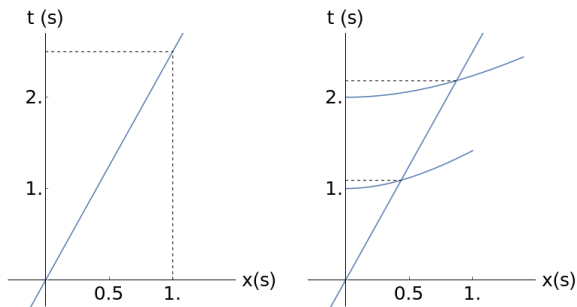
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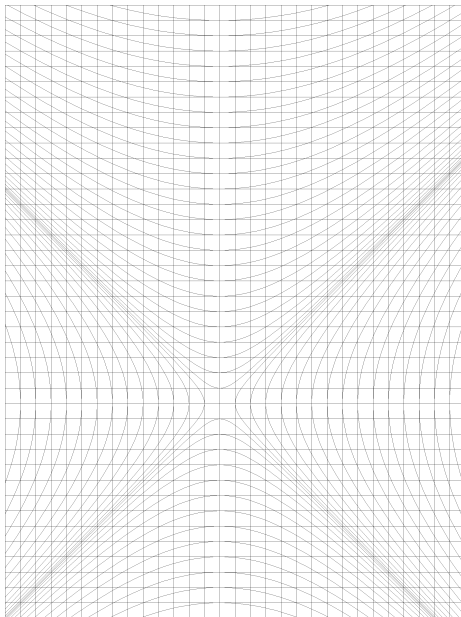
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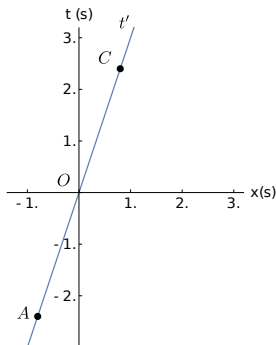


Consider 3 events:

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Blue line - world line

Dashed line - light pulse

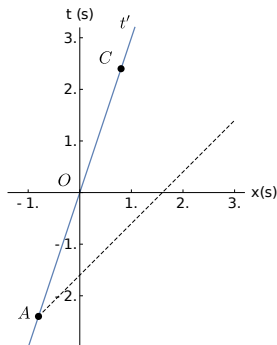
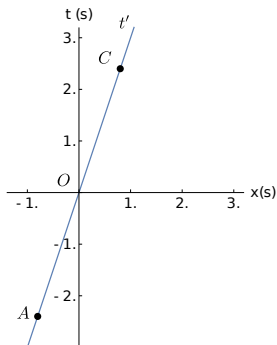


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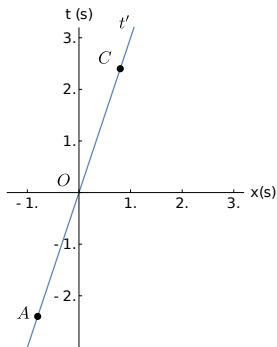
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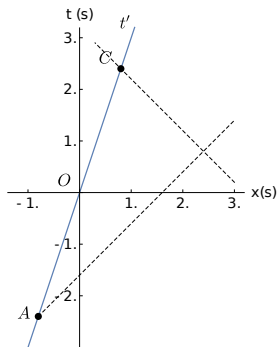
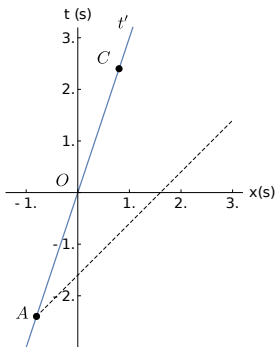
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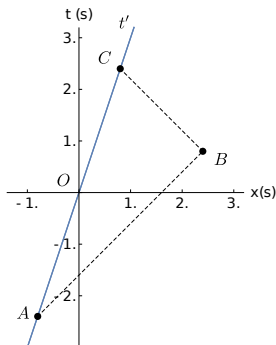


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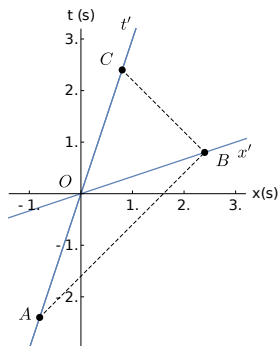
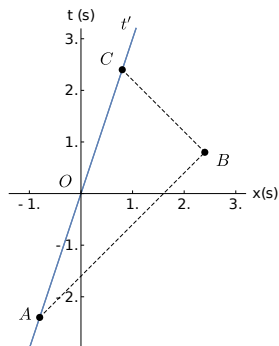


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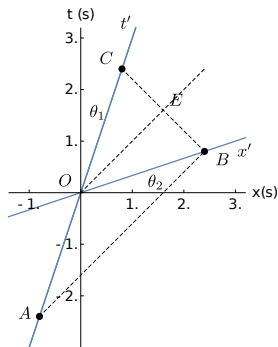
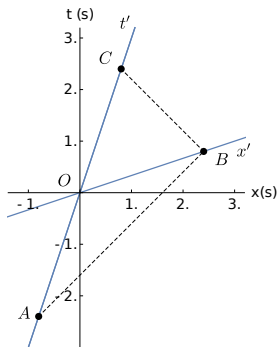
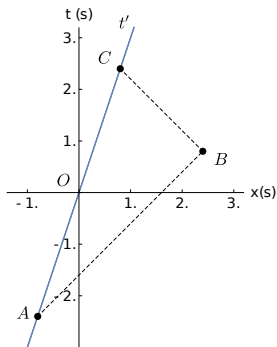


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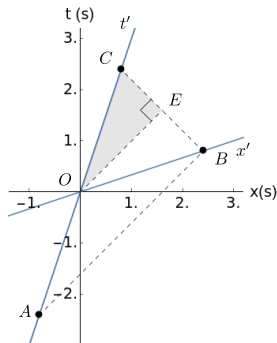


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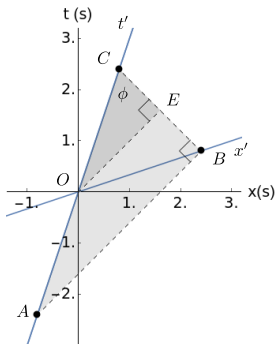
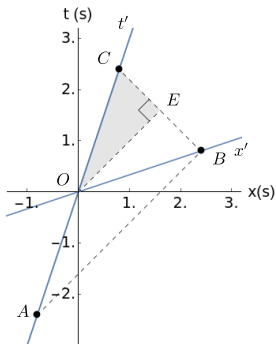


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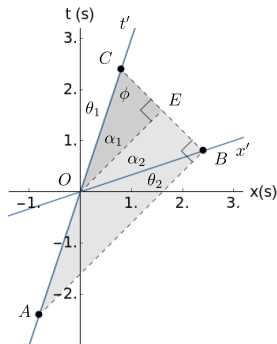
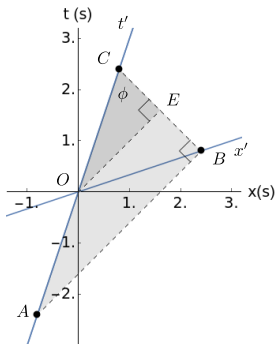
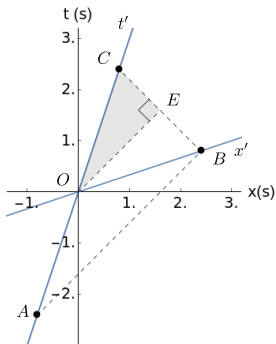


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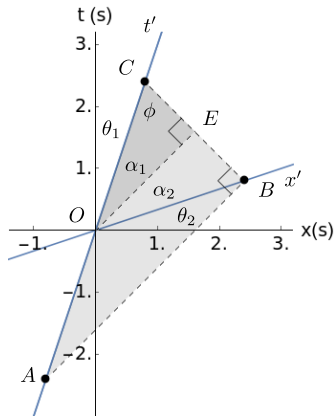
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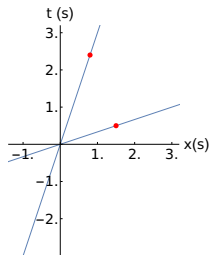
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- Events A , O , and C are at the origin so they define the t' axis.
- Events A and C must be symmetric to O so $\overline{AO} = \overline{OC}$.
- Events O and B occur at $t' = 0$ so they define the x' axis.
- Let a light pulse be emitted from the origin going in the positive x' direction.
- $\triangle ABC$ and $\triangle OEC$ have the same internal angles so they are similar.
- The hypotenuse of $\triangle ABC$ is twice as long as $\triangle OEC$ because $\overline{AO} = \overline{OC}$.
- $\therefore \overline{BC} = 2 \times \overline{EC}$.
- $\therefore \triangle OEC$ similar to $\triangle OEB$
- $\triangle OEC$ and $\triangle OEB$ share \overline{OE} .
- $\therefore \overline{CE} = \overline{BE}$.
- $\therefore \alpha_1 = \alpha_2$
- \overline{OE} has a slope of 1 so $\theta_1 = \theta_2$



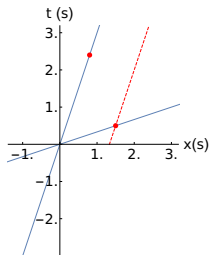
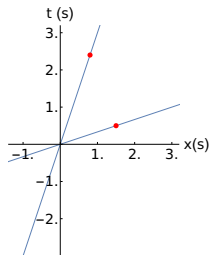
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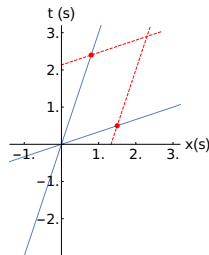
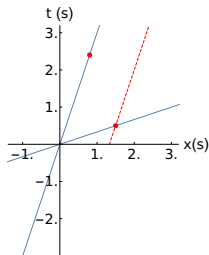
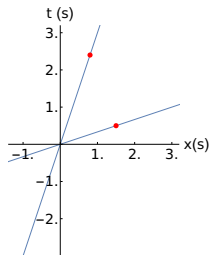
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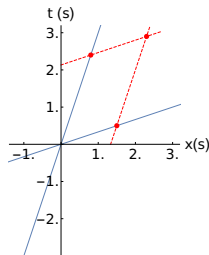
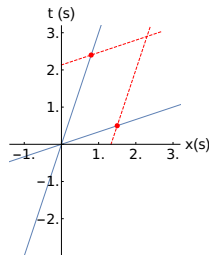
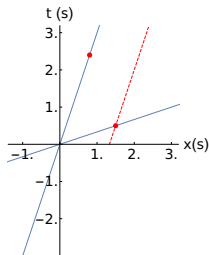
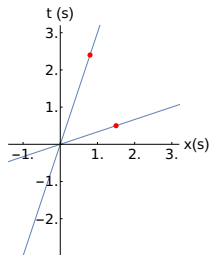
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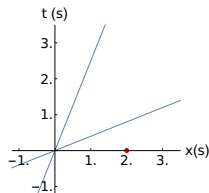
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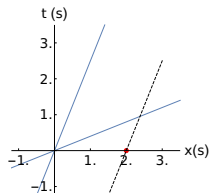
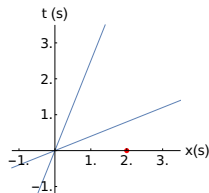
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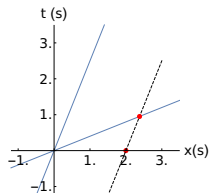
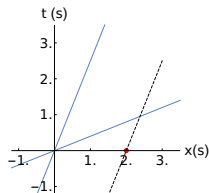
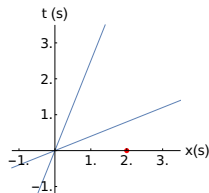
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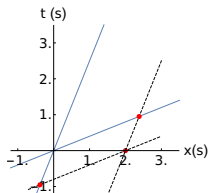
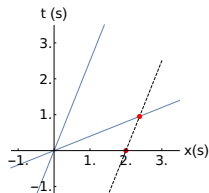
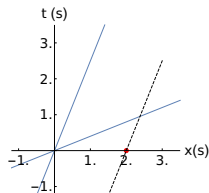
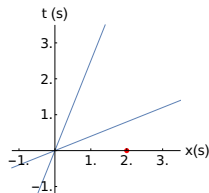
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$x' = x - vt$ $y' = y$ $z' = z$ $t' = t$ $u'_x = u_x - v$ $u'_y = u_y$ $u'_z = u_z$	

primes refer to the frame moving with velocity v .

v - velocity of moving frame.

u_i - i^{th} component of the velocity of an object in the stationary frame.

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$\gamma = \frac{1}{\sqrt{1-v^2/c^2}}$ where c is the speed of light.

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$y' = y$	$y' = y$
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$t' = t$	$t' = \gamma(t - vx/c^2)$
$u'_x = u_x - v$	$u'_x = \frac{u_x - v}{1 - u_x v/c^2}$
$u'_y = u_y$	$u'_y = u_y$
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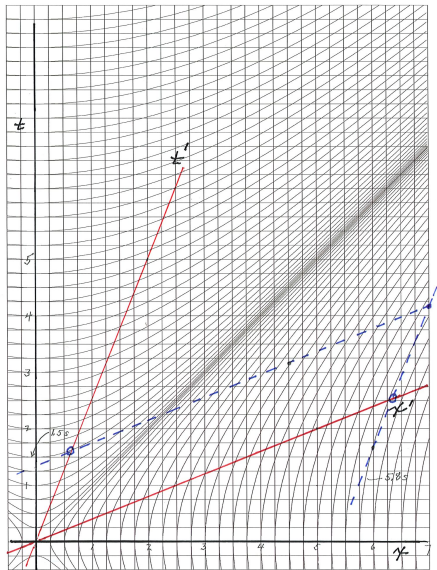
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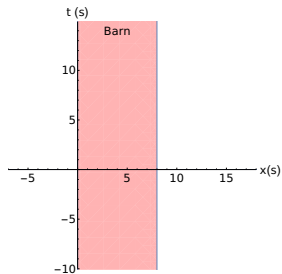


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- Now consider the point of view of the runner. She sees the pole as stationary and the barn approaching at high speed. The pole is still 10 m long, and the barn is now less than $(8\text{ m})\sqrt{1 - 0.8^2} = 4.8\text{ m}$ long. Surely the runner is in trouble if the doors close while she is inside. Is she?



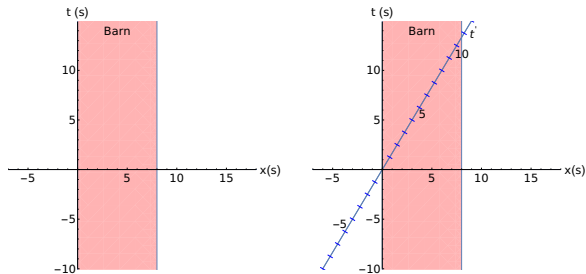
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$$L_{pole} = 10 \text{ m}$$

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The Barn and Pole Paradox

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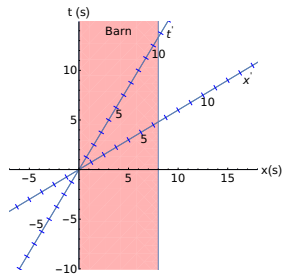
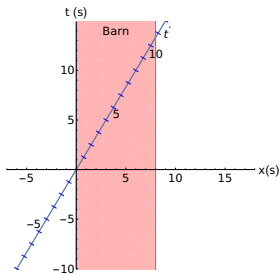
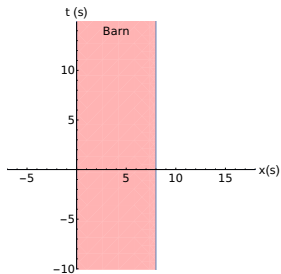
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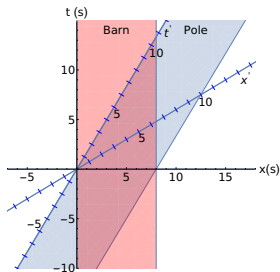
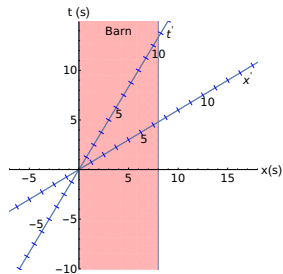
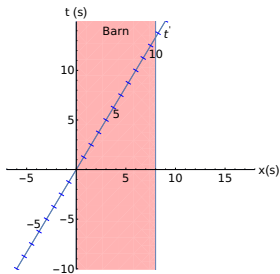
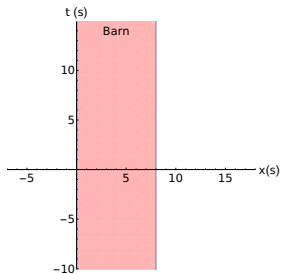
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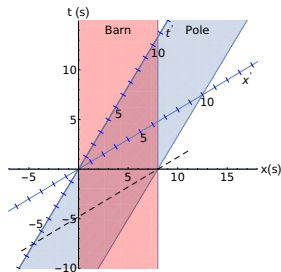
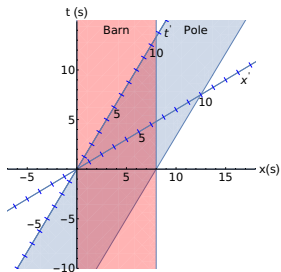
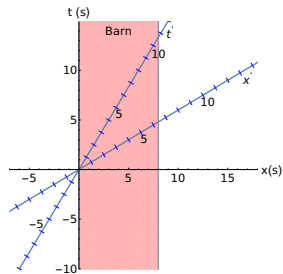
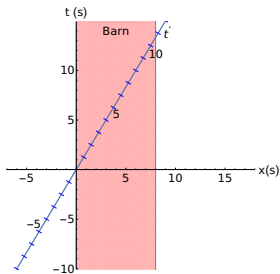
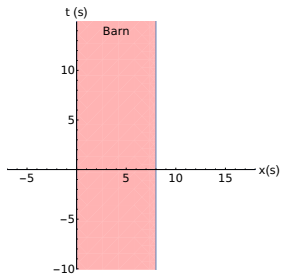
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$$L_{pole} = 10 \text{ m}$$

$$L_{barn} = 8 \text{ m}$$

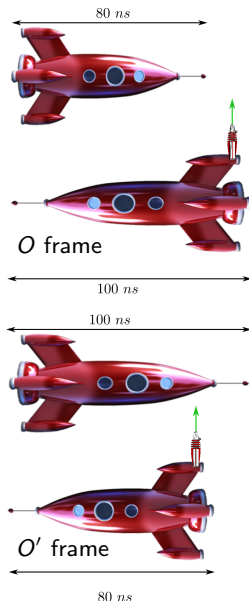
The Space Wars Paradox

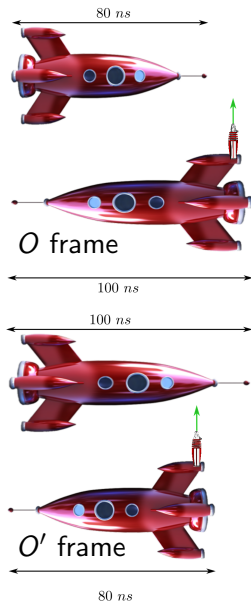
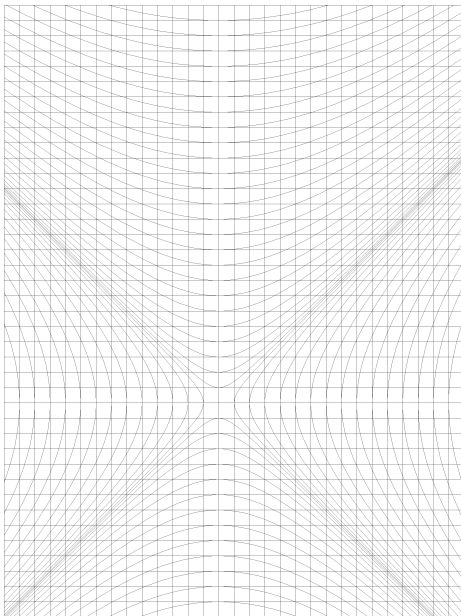
45

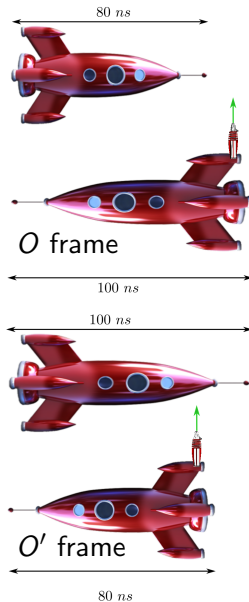
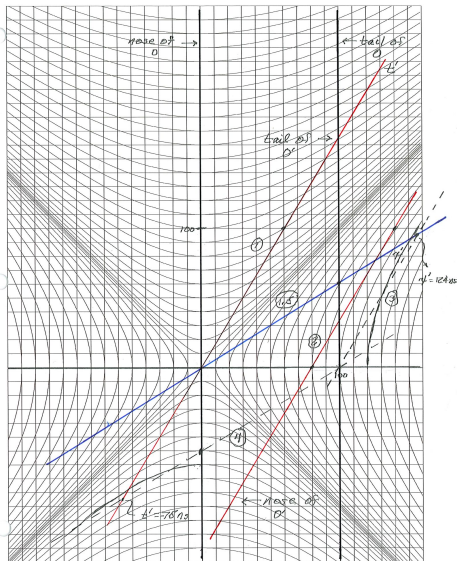
Two spacecraft of equal rest length $L_R = 100\text{ ns}$ pass very close to each other as they travel in opposite directions at a relative speed $|\beta| = \frac{3}{5}$. See figure. The captain of ship O intends to fire her laser cannon at the instant her bow is lined up with the tail of ship O' . Since O' is Lorentz-contracted to a length of 80 ns in the frame of ship O , she expects the laser burst to miss the other ship by 20 ns . However, to the captain on ship O' , it is ship O that has contracted to 80 ns so the laser burst will strike ship O' about 20 ns behind the bow

What really happens if O carries out her plan? Construct a two-observer spacetime diagram. Define event A to be coincidence of the bow of ship O and the tail of O' and event B to be the firing of the laser. Let A define the origin event in both frames. When and where does this event occur as measured in the O' frame? The travel time of the laser from one ship to another is negligible. Verify your results with the Lorentz transformations.

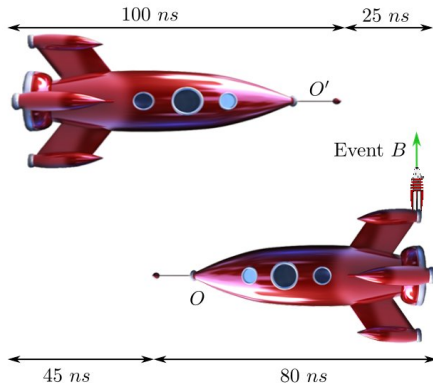
Adapted from *Spacetime Physics* by Taylor and Wheeler, 1966.







What O' sees at $t' = -75ns$.



What O' sees at $t' = 0ns$.

