Consider two twins. One sets out at the age of 25 on a spaceship from Earth at a speed of $0.99c$ where c is the speed of light. The Earthbound twin goes on about her/his business accumulating the normal accouterments of advancing age (gray hair, drooping body parts, *etc.*). After twenty years have passed for the Earthbound twin, the spacefaring one returns. When they finally meet the voyager is NOT twenty years older!

She/He looks only a few years older than when she/he left and shows few signs of age. How much has she/he aged during the journey?

1. Physics is the same in all inertial reference frames (hopefully).

The Postulates

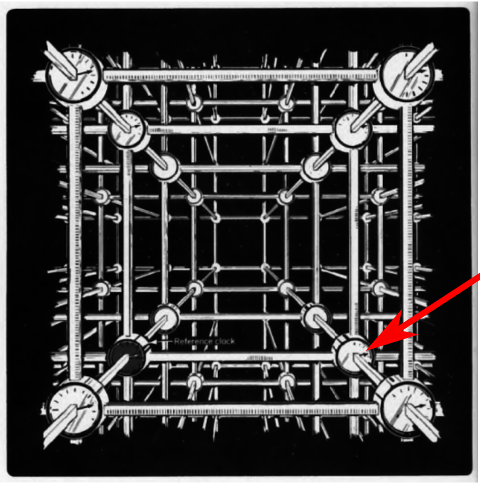
1. Physics is the same in all inertial reference frames (hopefully).
2. The speed of light is the same in all inertial reference frames.

1. In 1971 Hafele and Keating at the old National Bureau of Standards (now National Institute for Standards and Technology) took four cesium-beam atomic clocks aboard commercial airliners and flew twice around the world, first eastward, then westward, and compared the clocks against those of the United States Naval Observatory.

	nanoseconds gained			
	predicted			measured
	gravitational (general relativity)	kinematic (special relativity)	total	
eastward	144 ± 14	-184 ± 18	-40 ± 23	-59 ± 10
westward	179 ± 18	96 ± 10	275 ± 21	273 ± 7

2. Mountaintop muon decay measurements.
3. GPS and many others.

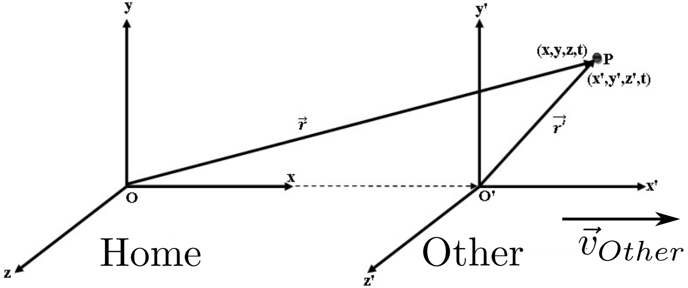
Inertial Frames - 1



$$(t, x, y, z)$$

$$\vec{v}_{Home} = 0$$
$$\vec{v}_{Other} = \text{constant}$$

The coordinates (t, x, y, z) describe the spacetime position P of an event in the *Home* frame. The *Other* frame is aligned with the x axis of the *Home* frame and is moving at a velocity $\vec{v}_{Other}$. The coordinates (t', x', y', z') describe the spacetime position P of the same event in the *Other* frame. Assume the clocks both start at the same moment and the origins coincide at that moment. How are (t, x, y, z) and $\vec{v}$ in *Home* related to (t', x', y', z') and $\vec{v}'$ in the *Other* frame in Galilean Relativity?



Galilean
$x' = x - vt$
$y' = y$
$z' = z$
$t' = t$
$v'_x = v_x - v_O$
$v'_y = v_y$
$v'_z = v_z$

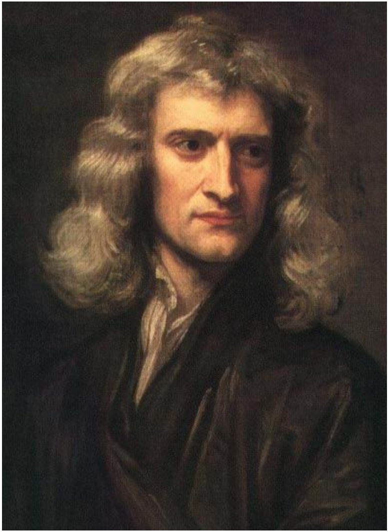
primes refer to the frame moving with velocity v_O.

v_O - velocity of moving/Other/B frame.

v_i - i^{th} component of the velocity in the stationary/Home/A frame.

v'_i - i^{th} component of the velocity in the moving/Other/B frame.

Newtonian or absolute time (1) exists independently of any perceiver, (2) progresses at a consistent pace throughout the universe, (3) is measurable but imperceptible, and (4) can only be truly understood mathematically. Absolute time and space were independent and separate aspects of objective reality, and not dependent on physical events or on each other. It is universal, *i.e.* the same for everyone everywhere in the universe.

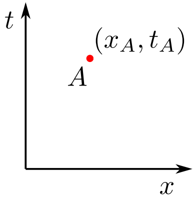


Defining Coordinate Time - 1

1. Inertial frame of reference - coordinates moving at a constant $\vec{v}_{Other}$.
2. Synchronized clocks in an inertial frame - A light flash is emitted at clock A at time t_A and received at clock B (in the same frame) at a later time t_B.
3. The distance between the clocks is $c(T_B - T_A)$.

Spacetime Diagrams

1D spacetime



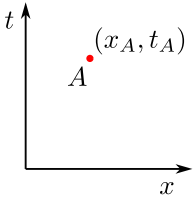
What is the worldline of a stationary point?

Describe the motion shown here.

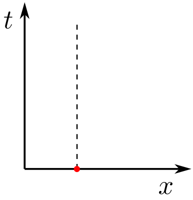
What is the worldline of a point with constant $\vec{v}$?

Spacetime Diagrams

1D spacetime



What is the world-line of a stationary point?



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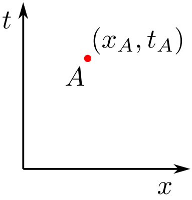

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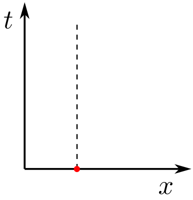

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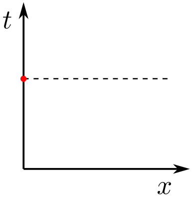

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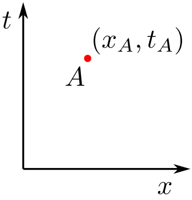

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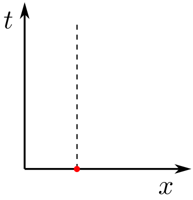

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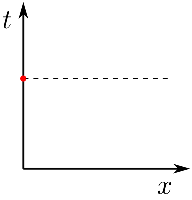

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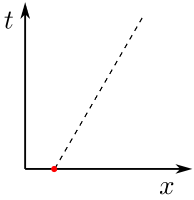
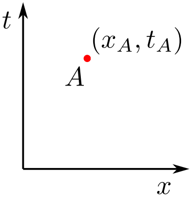


What is the world-line for a photon in the $+x$ direction?

Spacetime Diagrams

1D spacetime



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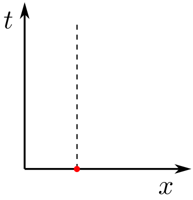

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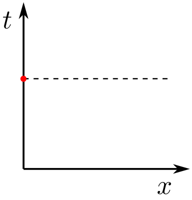

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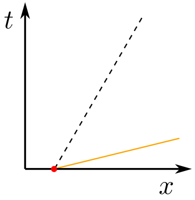

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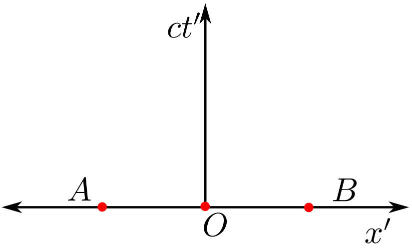
The coordinate time Δt is the time difference between two events A and B with two clocks at rest in an inertial frame located at the positions of the two events.



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What is $\Delta t'$ in this frame?

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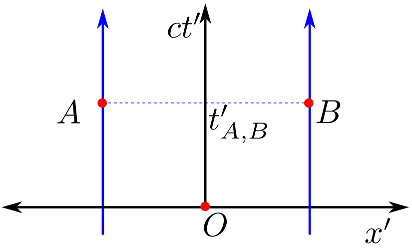

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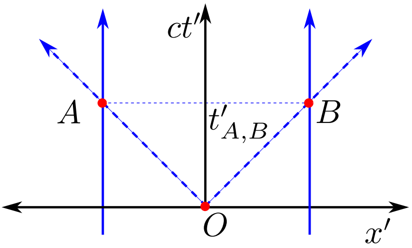

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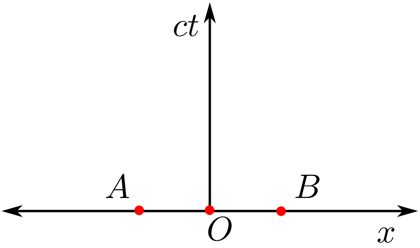

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You're now in the Home frame and the Other frame is moving in the positive x direction.

What will the worldlines of the clocks look like in the Home frame?
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Where do we put the clocks in the Home frame when they reach $\pm x'$?
What is Δt in this frame? Is $\Delta t = \Delta t'$?

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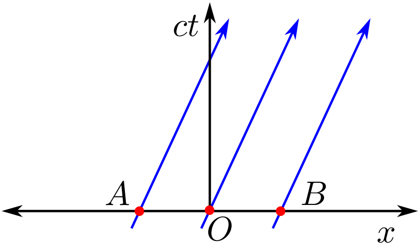

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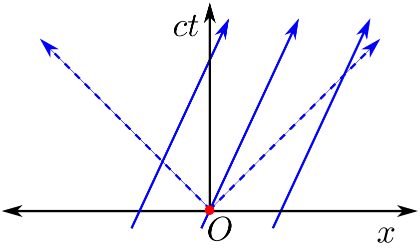

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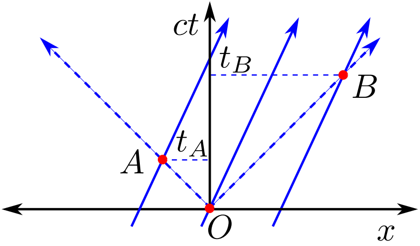

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The time interval Δt is messed up, what about the space interval Δx?

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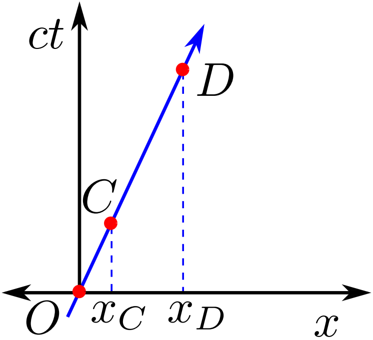

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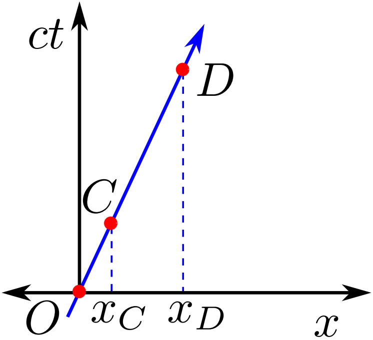
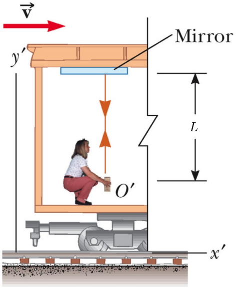
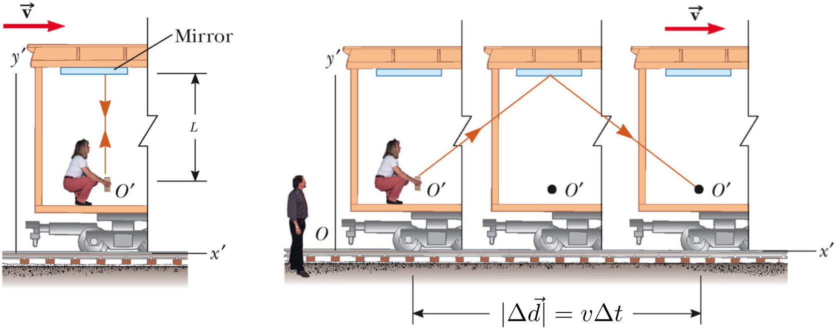


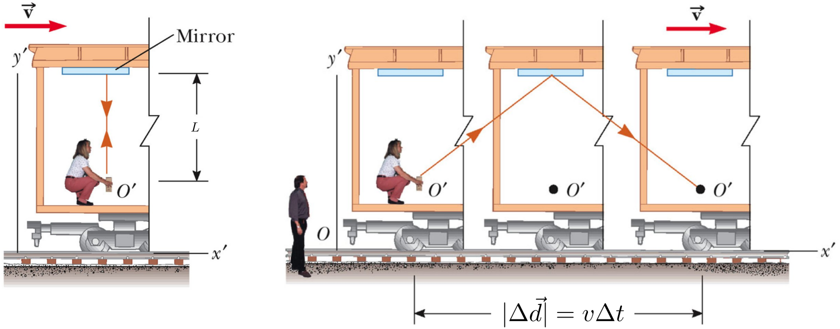
Consider two events that occur at the origin in the Other frame at different times. Both events occur at $x' = 0$ in the Other frame, but they will in different locations in the Home frame. The Home frame is shown here. What is Δx in this frame? Is $\Delta x = \Delta x'$?



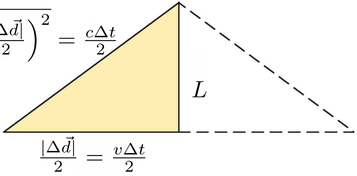
$\vec{v}$
Mirror
y'
L
O'
x'


$\vec{v}$
Mirror
y'
L
O'
x'
O
$|\Delta\vec{d}| = v\Delta t$

Spacetime Interval - 1



$$L_{Home} = \sqrt{L^2 + \left(\frac{|\Delta\vec{d}|}{2}\right)^2} = \frac{c\Delta t}{2}$$



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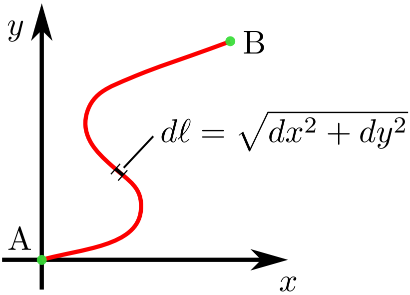
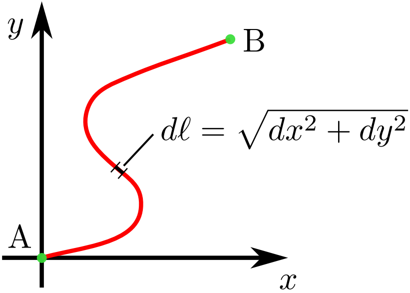
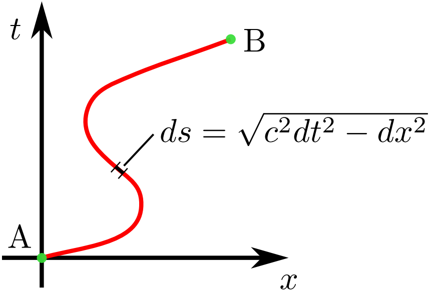
Spacetime Interval - 2

In the Home frame two events are observed to occur with a spatial separation $\Delta d = 360\ cm = 12\ ns$ and a coordinate time separation of $\Delta t = 24\ ns$. (a) An inertial clock travels between these events so it is present at both events. What time interval does this clock measure? (2) What is the speed of the clock in the Home Frame?



$$\Delta\ell_{AB} = \int_{path} \sqrt{dx^2 + dy^2}$$





$$\Delta \ell_{AB} = \int_{path} \sqrt{dx^2 + dy^2}$$

$$\Delta s_{AB} = \int_{path} \sqrt{c^2 dt^2 - dx^2}$$

The proper time between event A and event B is measured in the frame moving with the clock and in SR and SI units it is

$$\Delta s_{AB} = \int_{t_A}^{t_B} \sqrt{1 - \frac{v^2}{c^2}} c dt \; (SI\ units)$$

$$\Delta \tau_{AB} = \int_{t_A}^{t_B} \sqrt{1 - v^2} dt \; (SR\ units)$$

This equation describes how to use measurements in an inertial frame to determine the proper time between any two events A and B along an arbitrary worldline. The limits t_A and t_B are the times of the events A and B respectively, dt is the coordinate time differential, and $\vec{v}(t)$ is the clock speed all measured in the same inertial frame.

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Summary of Time Measurements

	Coordinate Time	Proper Time	Spacetime Interval
Definition	Time between two events in an inertial frame measured with synchronized clocks	Time between two events measured by the same clock at both events.	Time between two events measured by the same, inertial clock at both events.
Equation	Δt	$\Delta s_{AB} = \int_{t_A}^{t_B} \sqrt{1 - \frac{v^2}{c^2}}\, c dt$	$\Delta s^2 = c^2 \Delta t^2 - \Delta d^2$
Frame independent?	No	Yes	Yes
Geometric analog	coordinate difference	pathlength	distance

The proper time between event A and event B is measured in the frame moving with the clock and in SR and SI units is

$$\Delta s_{AB} = \int_{t_A}^{t_B} \sqrt{1 - \frac{v^2}{c^2}} cdt \ (SI\ units)$$

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Suppose the speed of a spaceship relative to an inertial frame fixed to the Sun is given by $|\vec{v}| = v = |\vec{a}|t = at$ where $a = 9.8 \ m/s$. Where would the proper time be measured? How long does it take to accelerate from rest to $v_f = 0.5c$ in the frame of the Sun? How long does it take to accelerate from rest in the spaceship frame?

Consider

The Paradox of the Twins Paradox - 1

Consider

1. In the problem we found the Earthbound twin (EB) aged 20 years.

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2. The Spacefaring twin (SF) aged just 2.8 years.

The Paradox of the Twins Paradox - 1 41

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3. EB took a relativity course in college and expects SF be younger upon arrival.

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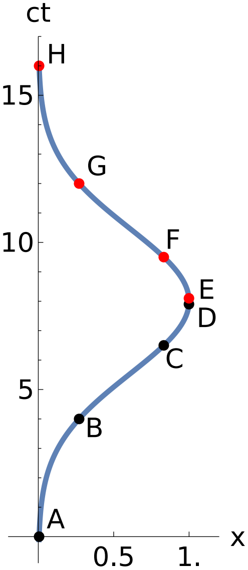
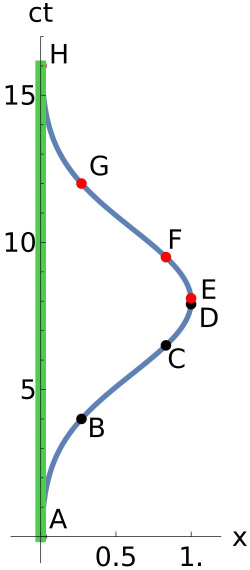
SF was not in an inertial frame during the entire tip. She/he had to accelerate to turn around and return.



A	spaceship leaves
B	reaches cruising speed
C	starts decelerating
D	reaches turn-around point
E	starts return trip
F	reaches cruising speed
G	starts decelerating
H	return

Blue	Spacefaring
Green	Earthbound



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B	reaches cruising speed
C	starts decelerating
D	reaches turn-around point
E	starts return trip
F	reaches cruising speed
G	starts decelerating
H	return

Blue	Spacefaring
Green	Earthbound