## Physics 132-2 Test 2

I pledge that I have neither given nor received unauthorized assistance during the completion of this work.

Name $\qquad$ Signature $\qquad$
Questions (5 for 8 pts . apiece) Answer in complete, well-written sentences WITHIN the spaces provided.

1. Use Newton's law of universal gravitation to show that the magnitude of the acceleration due to gravity on an object of mass $m$ at a height $h$ above the surface of the earth is given by the following expression

$$
\frac{G M_{e}}{(R+h)^{2}}
$$

where $G$ is the gravitational constant, $M_{e}$ is the mass of the Earth, and $R$ is the radius of the Earth. Hint: Because of the spherical symmetry of the Earth you can treat the mass of the Earth as if it were all concentrated at a point at the Earth's center. Explain.
2. For non-uniform forces (that is, forces that change with position), the work done by a force is given by

$$
W=-\int \vec{F} \cdot d \vec{s}
$$

which reduces to $W=\int F d s$ in one dimension. The diagram below shows the resulting general relationship between force $\vec{F}$ and potential energy $U$. Complete the equation on the right showing how to find the force if you know the potential energy $U$. Explain you reasoning.

3. Consider the circuit shown below. How would you connect an ammeter to measure the total current in the circuit? A sketch might be helpful. Explain.

4. In calculating the magnetic field due to a nearby line of current of length $L$ like the one shown in the figure, you encounter the integral shown below

$$
|\vec{B}|=\oint \frac{\mu_{o} \mathcal{I}}{4 \pi} \frac{y}{\left(x^{2}+y^{2}\right)^{(3 / 2)}} d x
$$

where $\mathcal{I}$ is the current and the point $P$ is over the midpoint of the line of current. What is the direction of the field at point $P$ ? Explain.

5. Consider the following idea for a novel propulsion for a ship or submarine. In this 'magnetohydrodynamic drive' seawater flows between the poles of a magnet as shown below and an electric field drives a current through the seawater. The magnetic force on this current propels the water towards the rear of the ship pushing the ship forward. What should be the direction of the applied electric field in the region between the poles of the magnet? Explain.


Problems (3). Clearly show all reasoning for full credit. Use a separate sheet to show your work.

1. 15 pts. The $x$ axis is the symmetry axis of a stationary, uniformly charged ring of radius $R$ and charge $Q$ (see figure). A particle with charge $Q$ and mass $M$ is located on this axis. Starting from the electric potential of a point charge, what is the electric potential along the $x$-axis in terms of $x, Q, R, M$, and any other
 constants?

Problems (3). Clearly show all reasoning for full credit. Use a separate sheet to show your work.
2. 20 pts. Many mass spectrometers have an additional device to insure the velocity of the particles entering the magnetic field of the spectrometer is the desired value. This 'velocity selector' consists of an electric field and a second magnetic field oriented perpendicular to the $\vec{E}$ field (see figure below). These two fields are set so that only particles with the desired velocity are undeflected as they move through the device. Negative ${ }^{14} \mathrm{C}$ ions with charge $q=-e$ are accelerated across the electric potential $V=10^{6} V$ and enter the velocity selector. The magnetic field of the velocity selector is 0.5 T . To pick out the ${ }^{14} \mathrm{C}$ ions, what should be the value of the electric field in the velocity selector and how should it be oriented?

3. 25 pts. In a fusion experiment a magnetic 'bottle' holds a gas of hydrogen atoms with an initial density $\rho=2.8 \times 10^{21}$ atoms $/ \mathrm{m}^{3}$. After reaching a high temperature the hydrogen gas within this volume becomes partially ionized meaning that each proton spends part of its time bound to an electron and part of the time alone. An applied electric field causes the bare protons to drift in a cylindrical region of cross sectional area $A=1.2 \times 10^{-3} \mathrm{~m}^{2}$ at $v_{p}=25 \mathrm{~m} / \mathrm{s}$ while the electrons drift in the opposite direction at $v_{e}=88 \mathrm{~m} / \mathrm{s}$ (see figure). The net current has magnitude $|\mathcal{I}|=21 A$. What fraction of the gas is ionized?


DO NOT WRITE BELOW THIS LINE.

## Physics 132-02 Equations Test 2

$$
\begin{gathered}
\vec{F}_{G}=-G \frac{m_{1} m_{2}}{r_{12}^{2}} \hat{r} \quad \vec{F}_{C}=k_{e} \frac{q_{1} q_{2}}{r_{12}^{2}} \hat{r} \quad \vec{E} \equiv \frac{\vec{F}}{q_{0}} \quad \vec{E}=k_{e} \sum_{i} \frac{q_{i}}{r_{i}^{2}} \hat{r}_{i} \quad \vec{E}=k_{e} \int \frac{d q}{r^{2}} \hat{r} \quad k_{e}=\frac{1}{4 \pi \epsilon_{0}} \\
\vec{E}_{\text {dipole }}=k_{e} \frac{q(2 a)}{\left(x^{2}+a^{2}\right)^{3 / 2}} \hat{j} \quad \vec{E}_{\text {ring }}=k_{e} \frac{q x}{\left(x^{2}+R^{2}\right)^{3 / 2}} \hat{i} \quad \vec{E}_{\text {plane }}=2 \pi k_{e} \eta \hat{k}=\frac{\eta}{2 \epsilon_{0}} \hat{k} \\
\vec{E}_{\text {disk }}=2 \pi k_{e} \eta\left[1-\frac{z}{\left.\sqrt{z^{2}+R^{2}}\right]}\right] \hat{k}=\frac{\eta}{2 \epsilon_{0}}\left[1-\frac{z}{\sqrt{z^{2}+R^{2}}}\right] \hat{k} \\
W \equiv \int \vec{F} \cdot d \vec{s} \quad \Delta V \equiv \frac{\Delta P E}{q_{0}}=-\int_{A}^{B} \vec{E} \cdot d \vec{s} \quad V=k_{e} \frac{q}{r} \quad V=k_{e} \sum_{i} \frac{q_{i}}{r_{i}} \\
V=k_{e} \int \frac{d q}{r} \quad V=E d \quad I=\frac{d Q}{d t} \quad Q=\int I d t \quad V=I R \quad P=I V \quad R_{\text {equiv }}=\sum R_{i} \\
\text { The algebraic sum of the potential changes } \quad I=n q v_{d} A \\
\text { across all the elements of a closed loop is zero. }
\end{gathered}
$$

$$
\begin{gathered}
\vec{F}_{B}=q \vec{v} \times \vec{B} \quad\left|\vec{F}_{B}\right|=|q v B \sin \alpha| \quad \vec{B}=k_{m} \int \frac{I d \vec{s} \times \hat{r}}{r^{2}} \quad k_{m}=\frac{\mu_{0}}{4 \pi} \quad \vec{B}_{\text {ring }}=\frac{\mu_{o} I R^{2}}{2} \frac{1}{\left(x^{2}+R^{2}\right)^{3 / 2}} \hat{i} \\
K E_{0}+P E_{0}=K E_{1}+P E_{1} \quad K E=\frac{1}{2} m v^{2} \quad P E=q V \\
\vec{F}=m \vec{a} \quad\left|\vec{F}_{\text {cent }}\right|=m \frac{v^{2}}{r} \quad x=\frac{a}{2} t^{2}+v_{0} t+x_{0} \quad v=a t+v_{0} \\
\frac{d x^{n}}{d x}=n x^{n-1} \quad \frac{d f(u)}{d x}=\frac{d f}{d u} \frac{d u}{d x} \quad \frac{d}{d x} f(x) \cdot g(x)=f \frac{d g}{d x}+g \frac{d f}{d x} \\
\langle x\rangle=\frac{1}{N} \sum_{i} x_{i} \quad \sigma=\sqrt{\frac{\sum_{i}\left(x_{i}-\langle x\rangle\right)^{2}}{N-1}} \quad A=4 \pi r^{2} \quad V=A h \quad V=\frac{4}{3} \pi r^{3} \\
\vec{A} \cdot \vec{B}=A_{x} B_{x}+A_{y} B_{y}+A_{z} B_{z} \quad \vec{A} \times \vec{B}=|\vec{A}||\vec{B}||\sin \alpha| \rightarrow r h r \\
\ln (a b)=\ln a+\ln b \quad \ln \left(\frac{a}{b}\right)=\ln a-\ln b \quad \ln x^{n}=n \ln x \quad x=e^{\ln x}=\ln \left(e^{x}\right) \\
\frac{d f(x)}{d x}=\lim _{\Delta x \rightarrow 0} \frac{f(x+\Delta x)-f(x)}{\Delta x} \quad \int_{a}^{b} f(x) d x=\lim _{\Delta x \rightarrow 0} \sum_{n=1}^{N} f(x) \Delta x \quad \frac{d \ln x}{d x}=\frac{1}{x} \\
\int \frac{1}{x} d x=\ln x \quad \int x^{n} d x=\frac{x^{n+1}}{n+1} \quad \int e^{a x} d x=\frac{e^{a x}}{a} \quad \int \frac{1}{\sqrt{x^{2}+a^{2}}} d x=\ln \left[x+\sqrt{x^{2}+a^{2}}\right]
\end{gathered}
$$

$$
\begin{gathered}
\int \frac{x}{\sqrt{x^{2}+a^{2}}} d x=\sqrt{x^{2}+a^{2}} \quad \int \frac{x^{2}}{\sqrt{x^{2}+a^{2}}} d x=\frac{1}{2} x \sqrt{x^{2}+a^{2}}-\frac{1}{2} a^{2} \ln \left[x+\sqrt{x^{2}+a^{2}}\right] \\
\int \frac{x^{3}}{\sqrt{x^{2}+a^{2}}} d x=\frac{1}{3}\left(-2 a^{2}+x^{2}\right) \sqrt{x^{2}+a^{2}} \quad \int \frac{1}{\left(x^{2}+a^{2}\right)^{3 / 2}} d x=\frac{x}{a^{2} \sqrt{x^{2}+a^{2}}} \\
\int \frac{x}{\left(x^{2}+a^{2}\right)^{3 / 2}} d x=\frac{-1}{\sqrt{x^{2}+a^{2}}} \int \frac{1}{x^{2}+a^{2}} d x=\frac{1}{a} \tan ^{-1}\left(\frac{x}{a}\right)
\end{gathered}
$$

## Physics 132-3 Constants

| $T_{\text {boiling }}\left(\mathrm{N}_{2}\right)$ | 77 K | $T_{\text {freezing }}\left(\mathrm{N}_{2}\right)$ | 63 K |
| :--- | :--- | :--- | :--- |
| $T_{\text {boiling }}$ (water) | 373 K or $100^{\circ} \mathrm{C}$ | $T_{\text {freezing }}($ water $)$ | $273 \mathrm{~K} \mathrm{or} 0^{\circ} \mathrm{C}$ |
| $L_{v}($ water $)$ | $2.26 \times 10^{6} \mathrm{~J} / \mathrm{kg}$ | $L_{f}$ (water) | $3.33 \times 10^{5} \mathrm{~J} / \mathrm{kg}$ |
| $L_{v}\left(\mathrm{~N}_{2}\right)$ | $2.01 \times 10^{5} \mathrm{~J} / \mathrm{kg}$ | $c$ (copper) | $3.87 \times 10^{2} \mathrm{~J} / \mathrm{kg}-{ }^{\circ} \mathrm{C}$ |
| $c($ water $)$ | $4.19 \times 10^{3} \mathrm{~J} / \mathrm{kg}-\mathrm{K}$ | $c$ (steam) | $0.69 \mathrm{~J} / \mathrm{kg}-\mathrm{K}$ |
| $c$ (iron) | $4.5 \times 10^{2} \mathrm{~J} / \mathrm{kg}-\mathrm{k}$ | $c$ (aluminum) | $9.0 \times 10^{2} \mathrm{~J} / \mathrm{kg}-\mathrm{K}$ |
| $\rho$ (water) | $1.0 \times 10^{3} \mathrm{~kg} / \mathrm{m}^{3}$ | $P_{\text {atm }}$ | $1.01 \times 10^{5} \mathrm{~N} / \mathrm{m}^{2}$ |
| $R$ | $8.31 \mathrm{~J} / \mathrm{K}-\mathrm{mole}$ | $g$ | $9.8 \mathrm{~m} / \mathrm{s}^{2}$ |
| $0 K$ | $-273^{\circ} \mathrm{C}$ | Speed of light $(\mathrm{c})$ | $3.0 \times 10^{8} \mathrm{~m} / \mathrm{s}$ |
| proton/neutron mass | $1.67 \times 10^{-27} \mathrm{~kg}$ | $k_{B}$ | $1.38 \times 10^{-23} \mathrm{~J} / \mathrm{K}$ |
| Gravitation constant | $6.67 \times 10^{-11} \mathrm{~N}-\mathrm{m}^{2} / \mathrm{kg}^{2}$ | 1.0 eV | $1.6 \times 10^{-19} \mathrm{~J}$ |
| $e$ electronic charge | $1.6 \times 10^{-19} \mathrm{C}$ | Electron mass | $9.11 \times 10^{-31} \mathrm{~kg}$ |
| Permittivity constant $\left(\epsilon_{0}\right)$ | $8.85 \times 10^{-12} \frac{\mathrm{~kg}^{2}}{\mathrm{N-m}}$ | 1 u | $1.67 \times 10^{-27} \mathrm{~kg}$ |
| Permeability constant $\left(\mu_{0}\right)$ | $4 \pi \times 10^{-7} \mathrm{Tm} / \mathrm{A}$ | Earth-Sun distance | $1.5 \times 10^{11} \mathrm{~m}$ |
| $k_{e}=1 / 4 \pi \epsilon_{0}$ | $8.99 \times 10^{9} \mathrm{~N}-\mathrm{m}^{2} / \mathrm{C}^{2}$ | Earth's mass | $5.97 \times 10^{24} \mathrm{~kg}$ |
| $k_{m}=\mu_{o} / 4 \pi$ | Earth's radius | $6.37 \times 10^{6} \mathrm{~m}$ |  |


| $\begin{array}{\|c} \substack{\text { madoges } \\ \mathbf{1} \\ \mathbf{H}} \end{array}$ |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | $\begin{aligned} & \substack{\text { nilum } \\ \text { ne } \\ \mathrm{He}} \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| , | ${ }^{\text {bempum }}$ |  |  |  |  |  |  |  |  |  |  |  | ${ }_{5}^{\text {amo }}$ | ${ }_{6}^{\text {antom }}$ | Hasm | ${ }_{8}^{\text {puxsen }}$ | ${ }^{\text {wame }}$ | 10 |
| Li | Be |  |  |  |  |  |  |  |  |  |  |  | B | C | N | 0 | F | Ne |
| Soll |  |  |  |  |  |  |  |  |  |  |  |  |  | silmen | ${ }_{\substack{\text { chemen } \\ 15}}^{\text {chans }}$ | ${ }^{\text {sumber }}$ | 17 | ${ }^{\text {ajomen }}$ |
| Na | Mg |  |  |  |  |  |  |  |  |  |  |  | Al | Si | P | S | Cl | Ar |
|  |  |  | ${ }^{\text {sanamum }}$ | ${ }^{\text {manumm }}$ | ${ }^{\text {anamum }}$ | ${ }^{\text {ancomum }}$ | manemes | ${ }_{26}^{100}$ | ${ }^{\text {cobat }}$ | ${ }^{\text {notect }}$ |  | ${ }_{\text {zne }}^{\substack{\text { ne } \\ 30}}$ | cos | 32 | 33 | 34 | 35 | 36 |
| K | Ca |  | Sc | Ti | V | Cr | Mn | Fe | Co | Ni | Cu | Zn | Ga | Ge | As | Se | Br | Kr |
| 㐋 |  |  | 9, |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| ${ }^{\text {dram }}$ | 38 |  | ${ }^{39}$ | ${ }^{40}$ | 41 | 42 | 43 | ${ }^{44}$ | ${ }^{4}$ | 46 | 47 | ${ }^{4}$ | ${ }^{49}$ | 50 | 51 | 52 | ${ }^{53}$ | ${ }^{5}$ |
| Rb | Sr |  | Y | Zr | Nb | Mo | Tc | Ru | Rh | Pd | Ag | Cd | In | Sn | Sb | Te | 1 | Xe |
| cosm |  | 57.70 |  |  | ${ }^{\substack{\text { chanem } \\ \text { tuam }}}$ | 74 | ${ }^{\text {chemm }}$ | come | 仿 |  | - | 80 |  | 82 | ${ }^{83}$ | ${ }^{4}$ | 85 | 86 |
| Cs | Ba | * | Lu | Hf | Ta | W | Re | Os | Ir | Pt | Au | Hg | TI | Pb | Bi | Po | At | Rn |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| ${ }^{87}$ | ${ }^{88}$ | 89-102 | 103 | Rf | dos | ${ }^{10}$ | -107 | 108 | 109 | 110 | mind | $1{ }^{\text {n }}$ |  | ${ }^{\text {momand }} 14$ |  |  |  |  |
| Fr | Ra | * | Lr | Rf | Db | Sg | Bh | Hs | Mt | Uun | Uuu | Uub |  | Uuq |  |  |  |  |


| *Lanthanide series | $\begin{array}{\|l\|} \hline \text { anamaum } \\ \text { La } \\ \mathrm{La} \end{array}$ | Ce | $\stackrel{5}{59}^{59}$ | $\mathrm{No}^{60}$ | Pm | Sm | Eu | $\mathrm{Gd}^{64}$ | Tb | Dy | Ho | Er | Tm | Yb |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| **Actinide series |  |  |  | $\begin{gathered} \substack{14,42 \\ \text { undm } \\ 92 \\ \hline} \end{gathered}$ | $\begin{gathered} \substack { \text { nits } \\ \begin{subarray}{c}{\text { nefum } \\ 93 \\ \mathrm{Nb}{ \text { nits } \\ \begin{subarray} { c } { \text { nefum } \\ 9 3 \\ \mathrm { Nb } } } \\ {\hline} \end{gathered}$ | $\begin{aligned} & 94 \\ & \mathrm{Pu} \end{aligned}$ |  |  |  | Cf | Es | Fm | Md | +102 |
|  | Ac |  | Pa | U | Np | P |  |  | Bk |  | Es | Fm | ${ }_{\text {c }}^{1289}$ | No |

The Periodic Chart.

