What is the Energy of an Electron? 1

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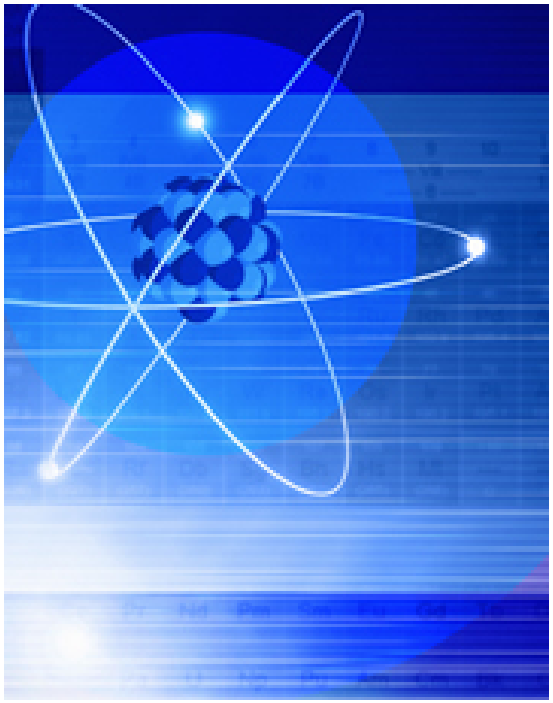


- The Organizing Principle.

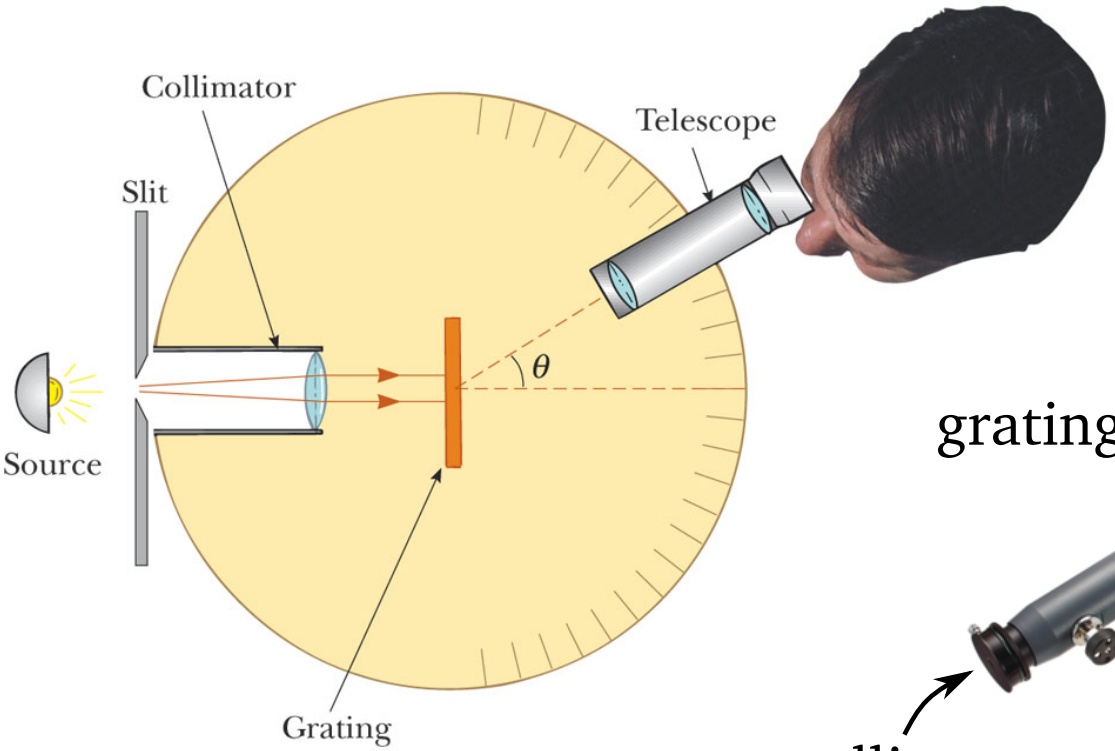
$$\begin{aligned} ME_0 &= ME_1 \\ KE_0 + PE_0 &= ME_1 + PE_1 \\ \frac{1}{2}mv_0^2 + PE_0 &= \frac{1}{2}mv_1^2 + PE_1 \end{aligned}$$

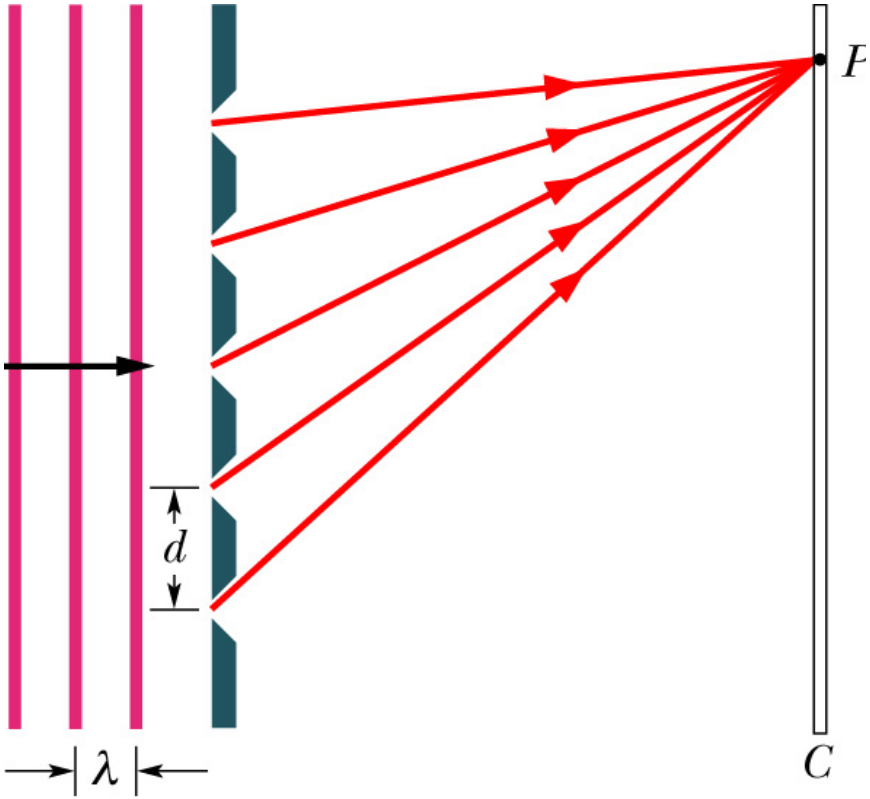
- The Forces

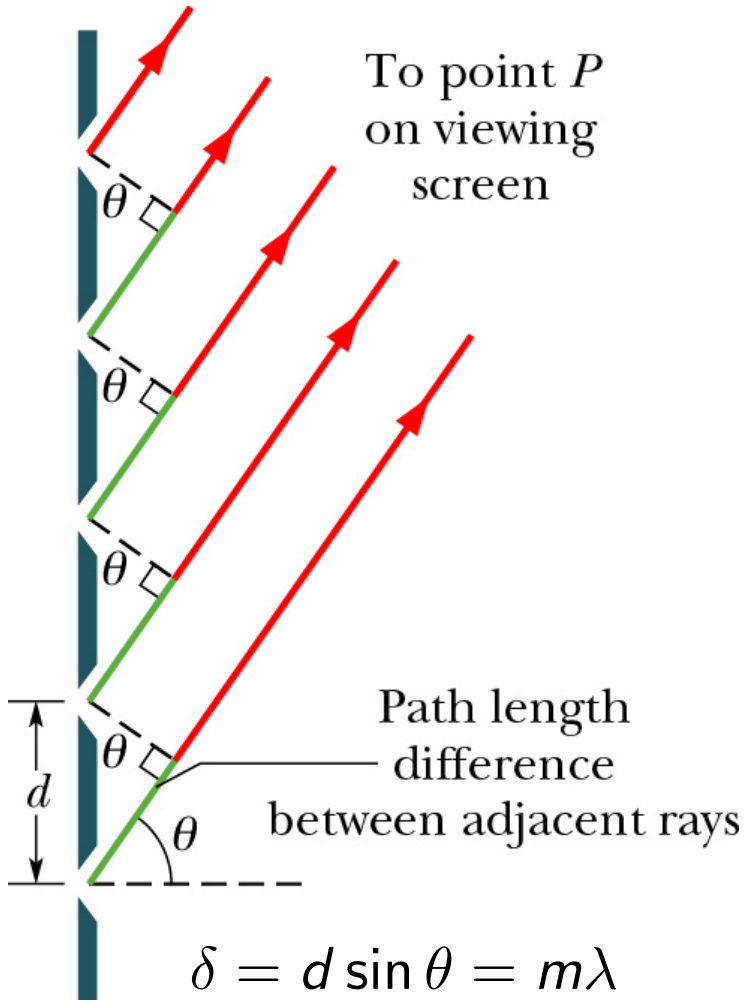
$$\vec{F}_{grav} = -\frac{Gm_1m_2}{r_{12}^2}\hat{r}_{12} \qquad \vec{F}_{coul} = \frac{k_e q_1 q_2}{r_{12}^2}\hat{r}_{12}$$

The simulation is here.

Light from a hydrogen spectrum tube is incident on a diffraction grating in a spectrometer. A narrow, red line appears at $\theta_1 = 20.50°$. The grating has a line density of 13,400 lines/inch. What is the wavelength of the light? What is the energy of the photons?

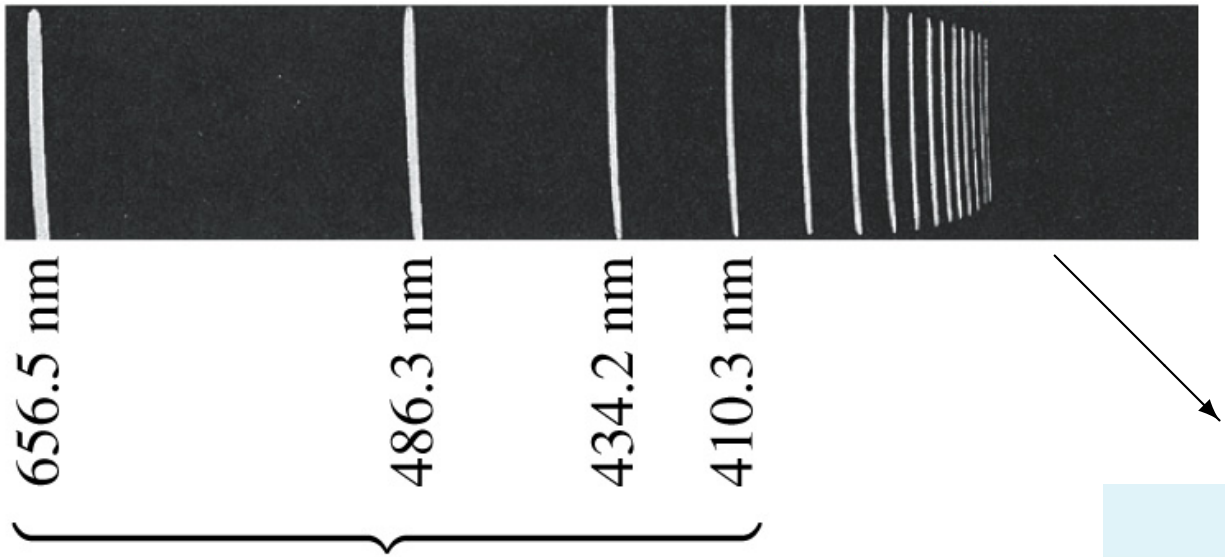






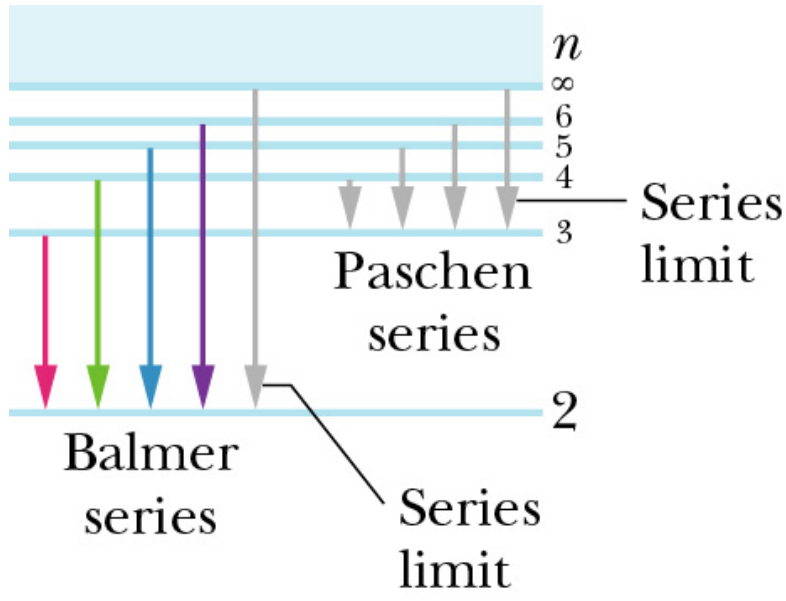
$$\delta = d \sin\theta = m\lambda$$
$$m = 0, \pm 1, \pm 2, \pm 3, \ldots$$

What We Measured In Lab.

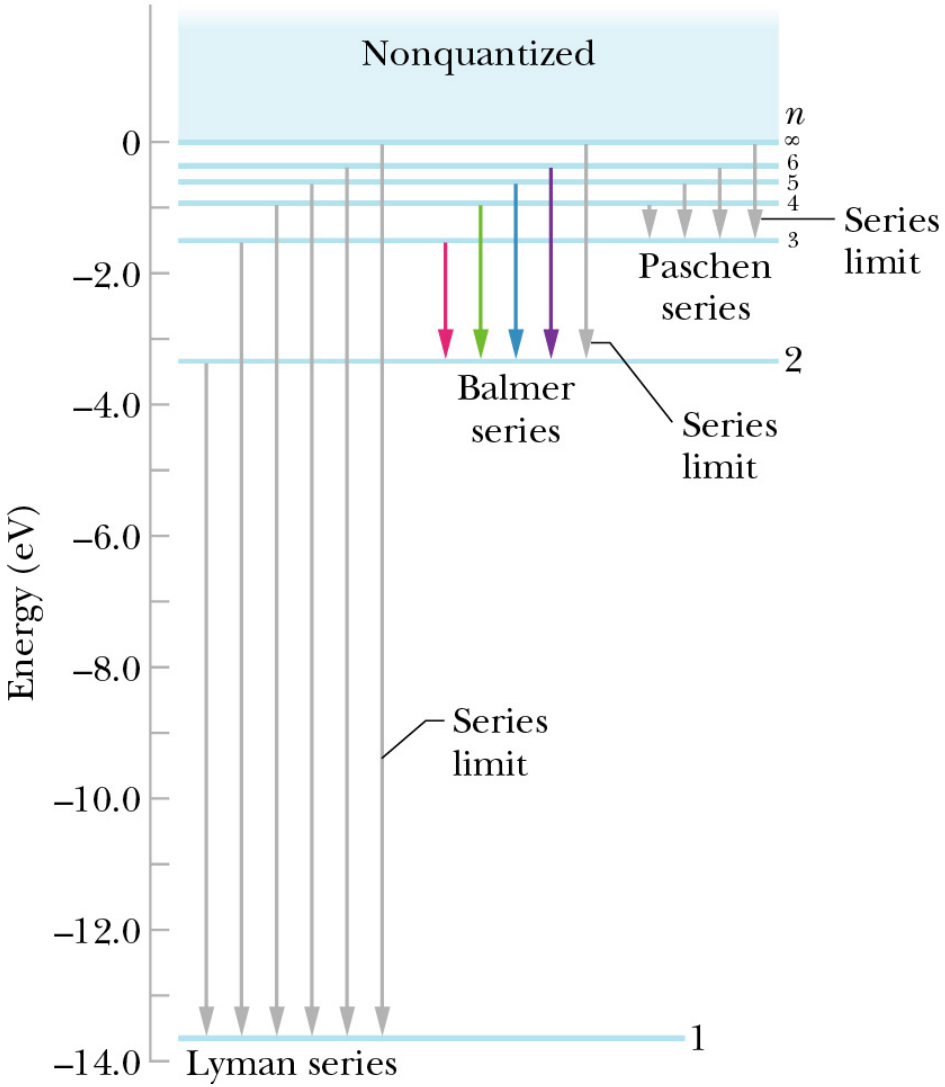


Four visible wavelengths known to Balmer

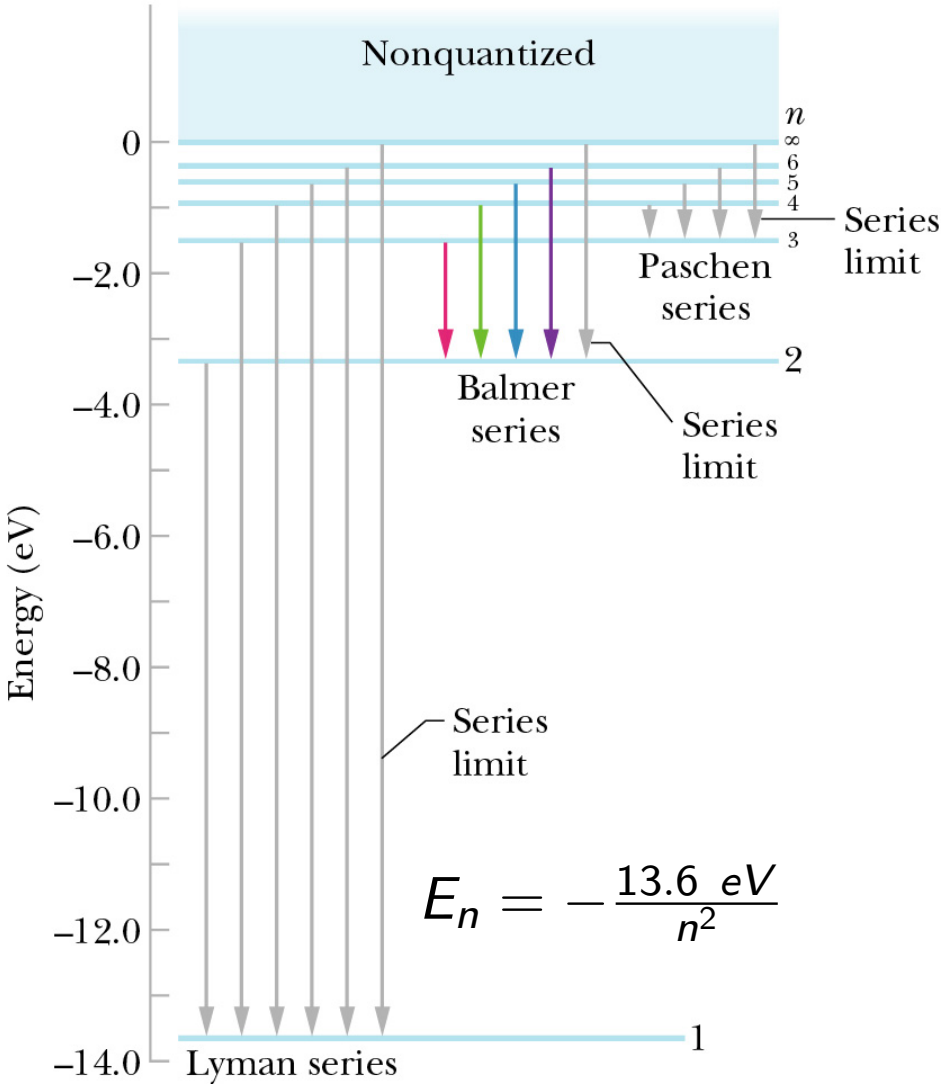
Transition	Energy (eV)
$5 \rightarrow 2$	2.856
$4 \rightarrow 2$	2.550
$3 \rightarrow 2$	1.889



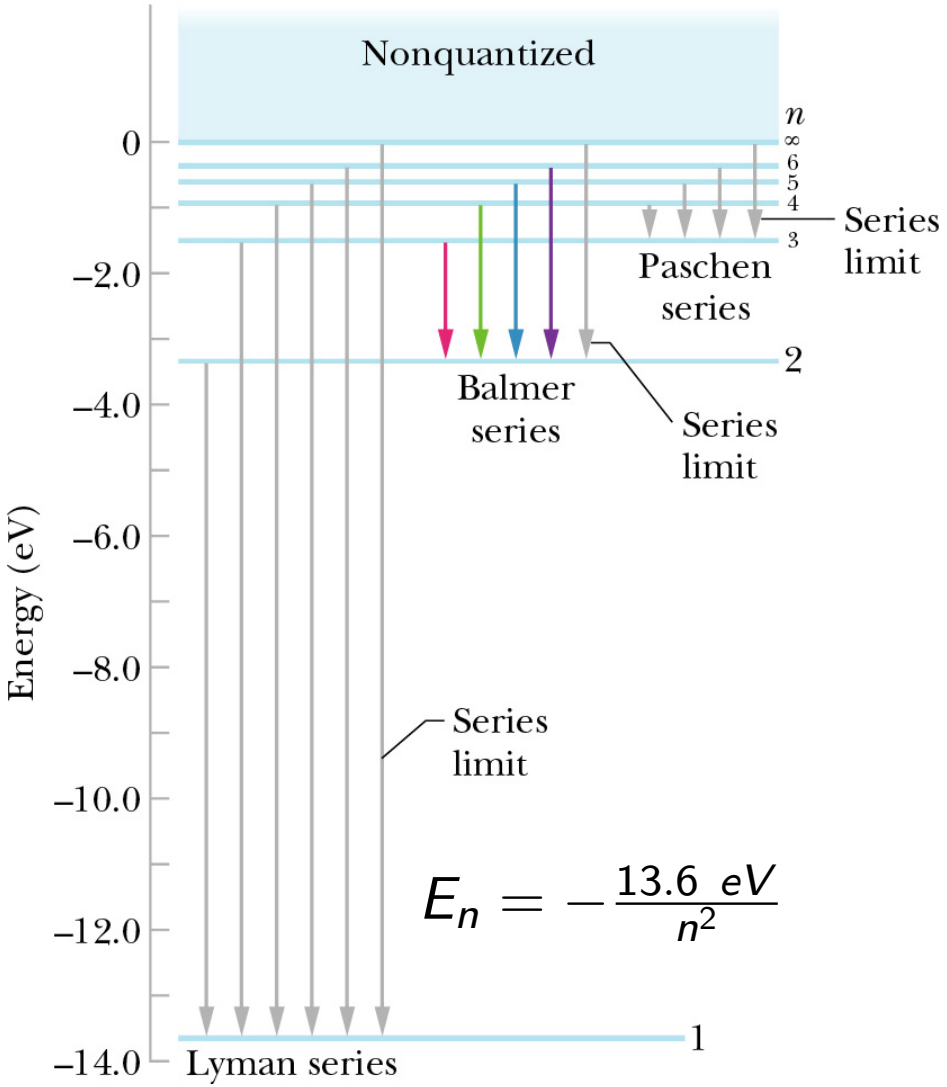
The Hydrogen Lines



Hydrogen Quantum Numbers



$$E_n = -\frac{13.6\ eV}{n^2}$$



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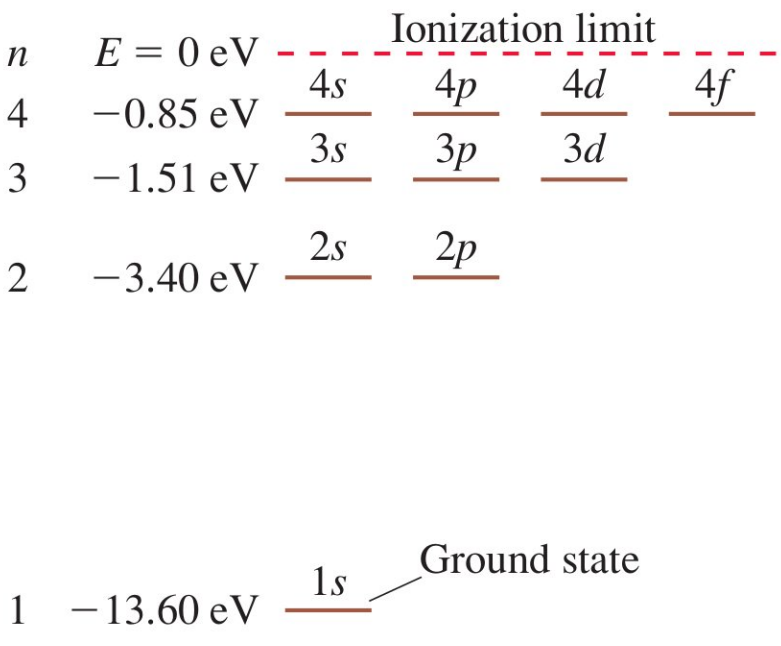

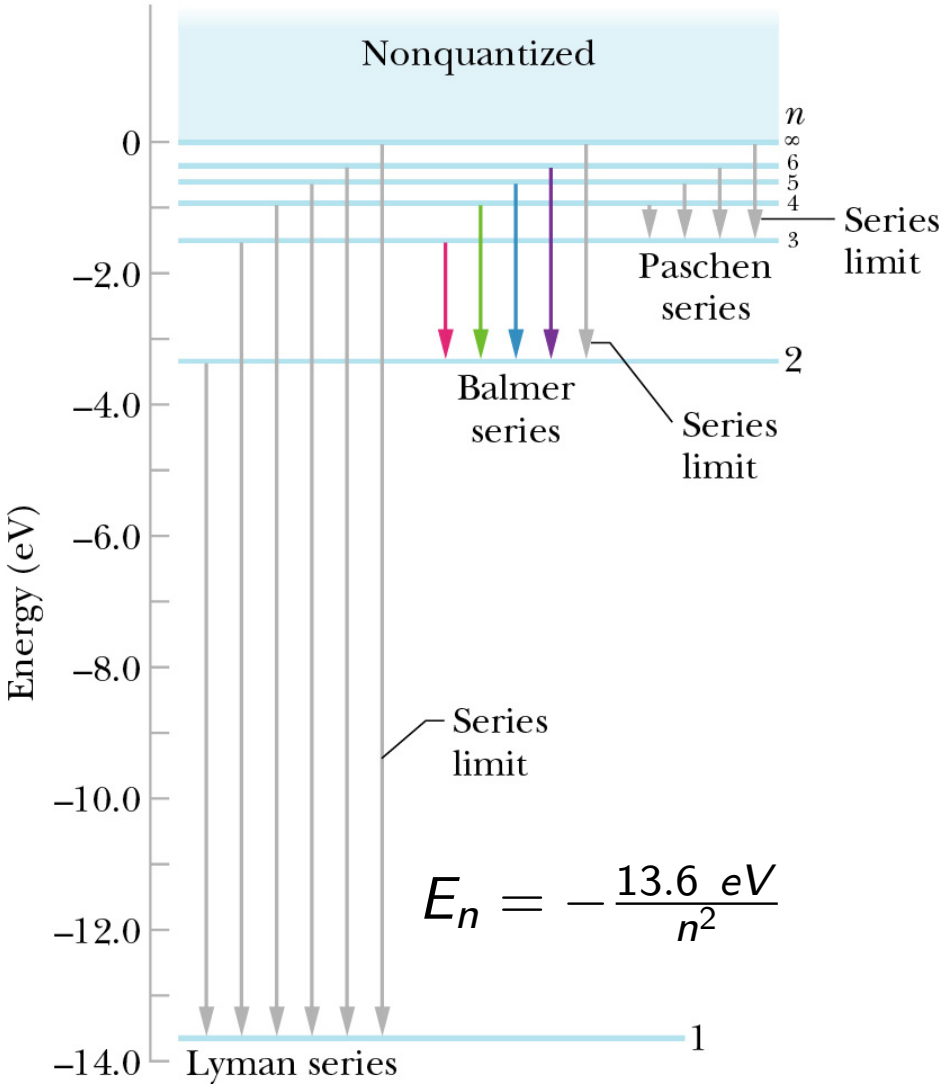

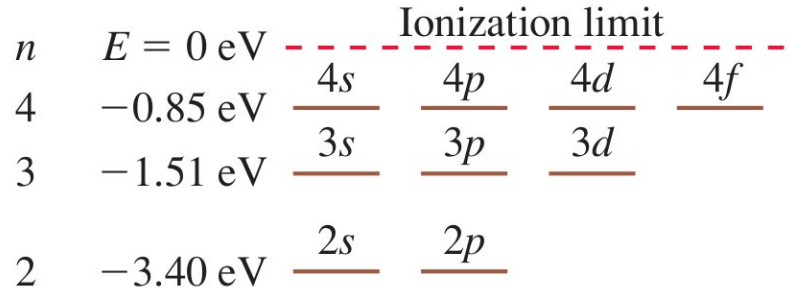

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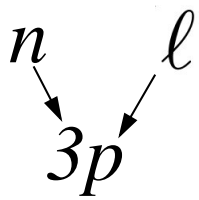

n - principal quantum number

ℓ - angular momentum

Electron Scattering on Hydrogen. 10

An electron beam strikes a gas of hydrogen atoms.

1. What is the minimum speed the electrons must have to cause the emission of $\lambda = 656\ nm$ light from the $3 \rightarrow 2$ transition of hydrogen?
2. What is the electric potential difference the electrons must fall through to be accelerated to this speed?

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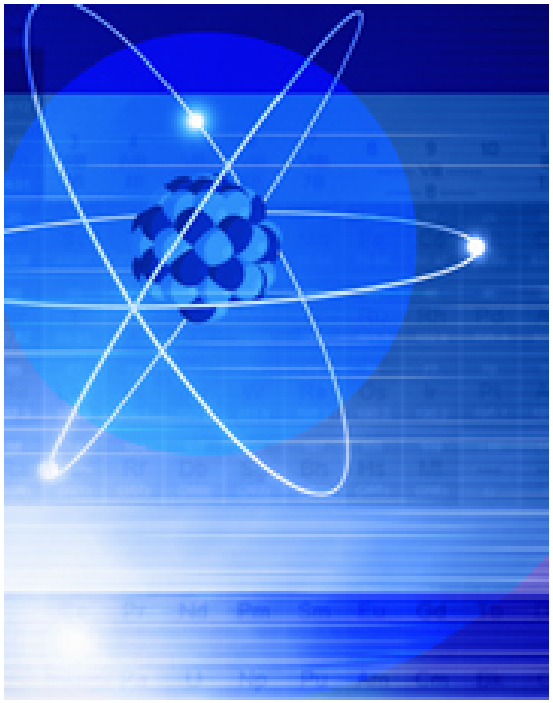

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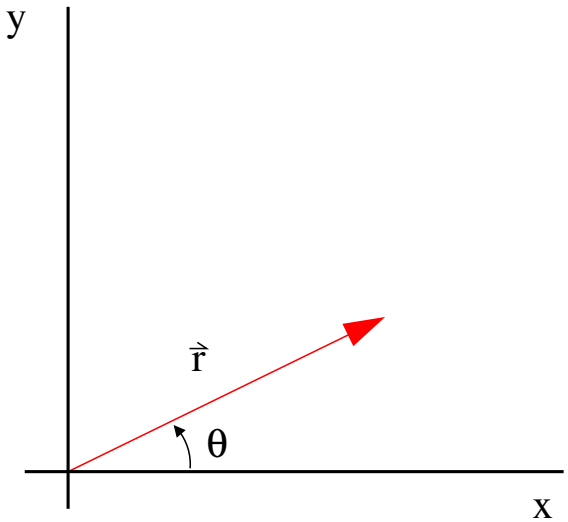
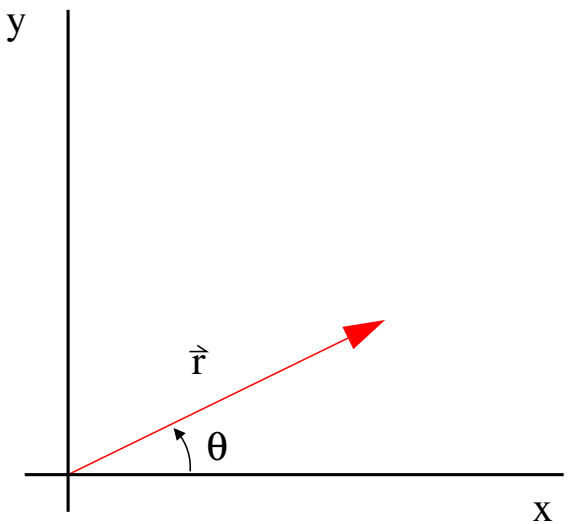


$$\vec{r} = x\hat{i} + y\hat{j}$$



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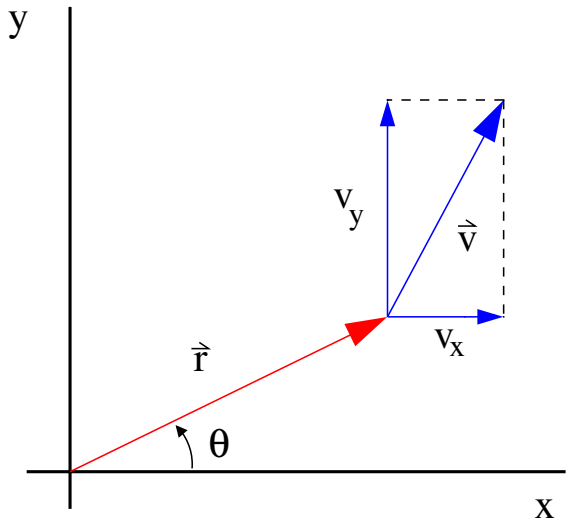
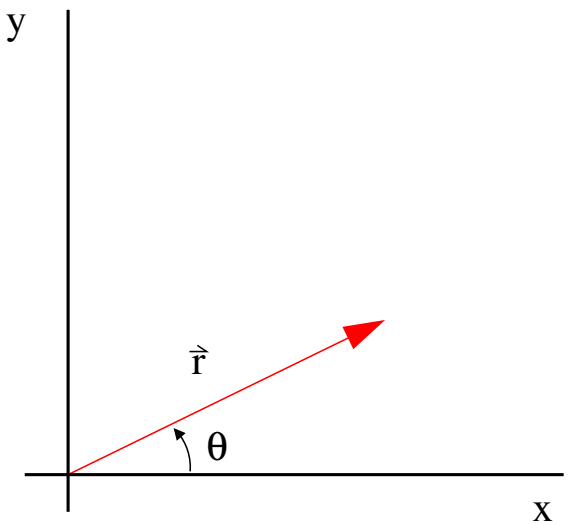


$$\vec{v} = v_x\hat{i} + v_y\hat{j}$$



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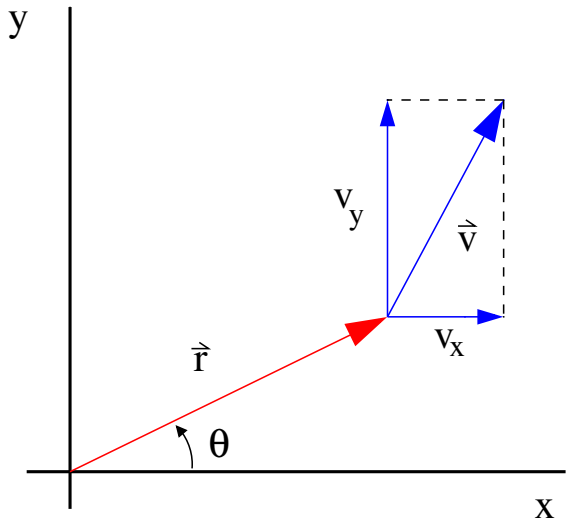

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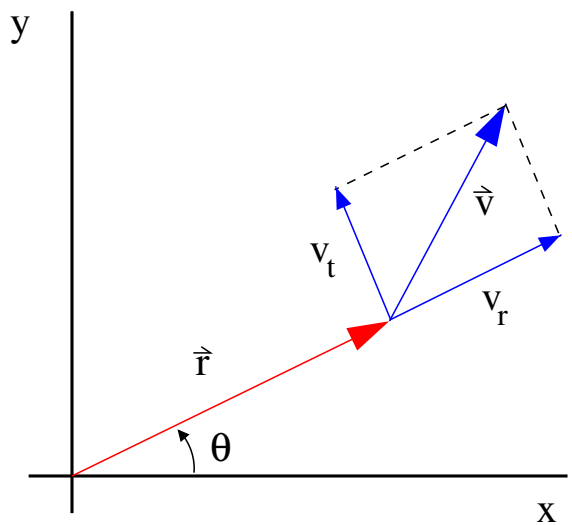
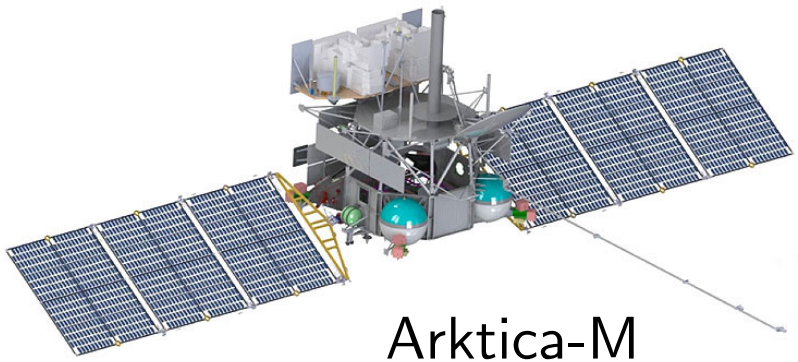


$$\vec{v} = v_r\hat{r} + v_t\hat{\theta}$$

A Russian Artica satellite that monitors polar weather follows an elliptical orbit around the Earth at an altitude of $h = 300\ km$ above the surface (radius $r_s = 6.67 \times 10^6\ m$). At one point in its orbit its velocity is measured to be

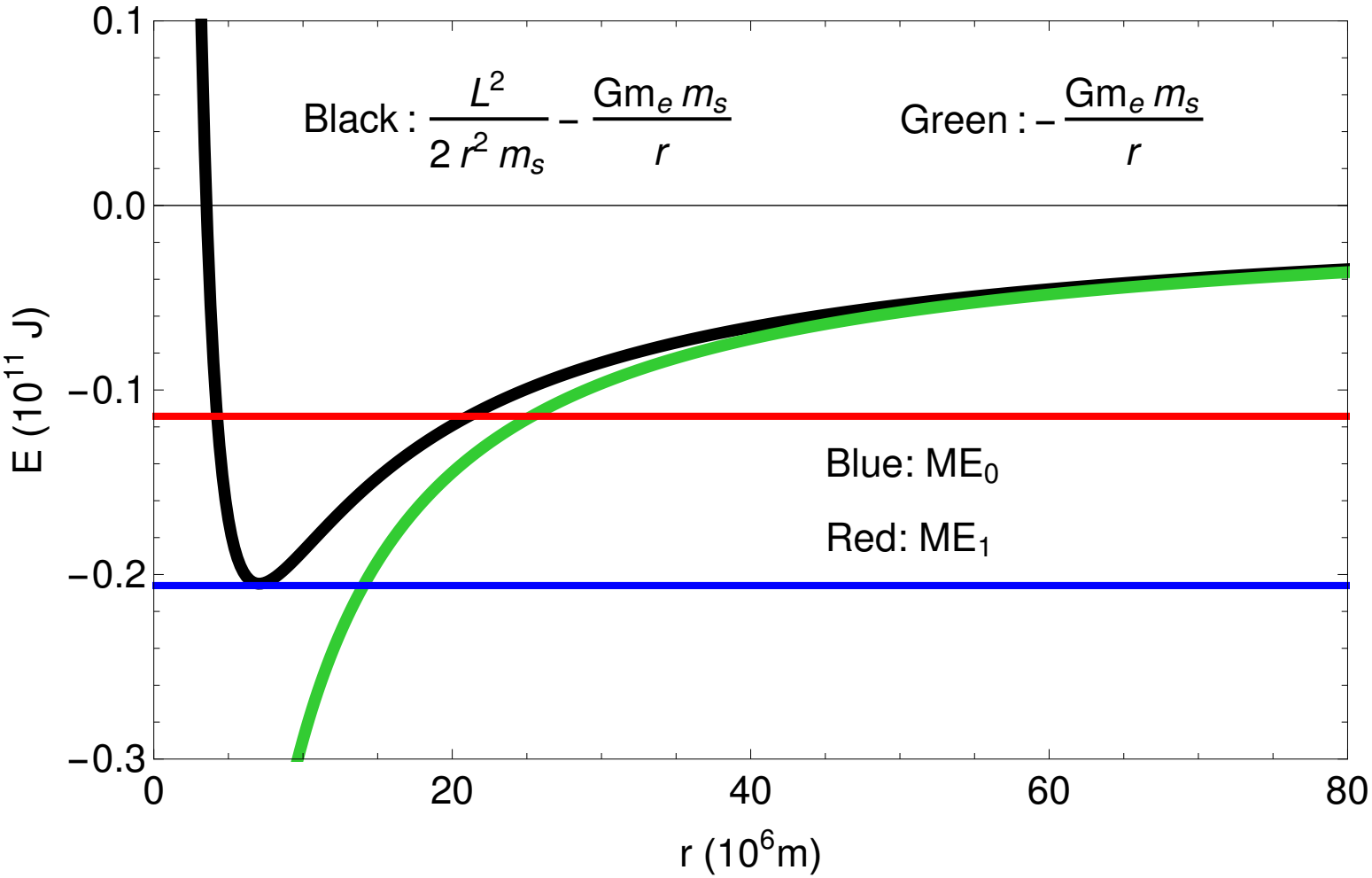
$$\vec{v} = 4.1 \times 10^3\ m/s\ \hat{r} + 7.5 \times 10^3\ m/s\ \hat{\theta} \quad .$$

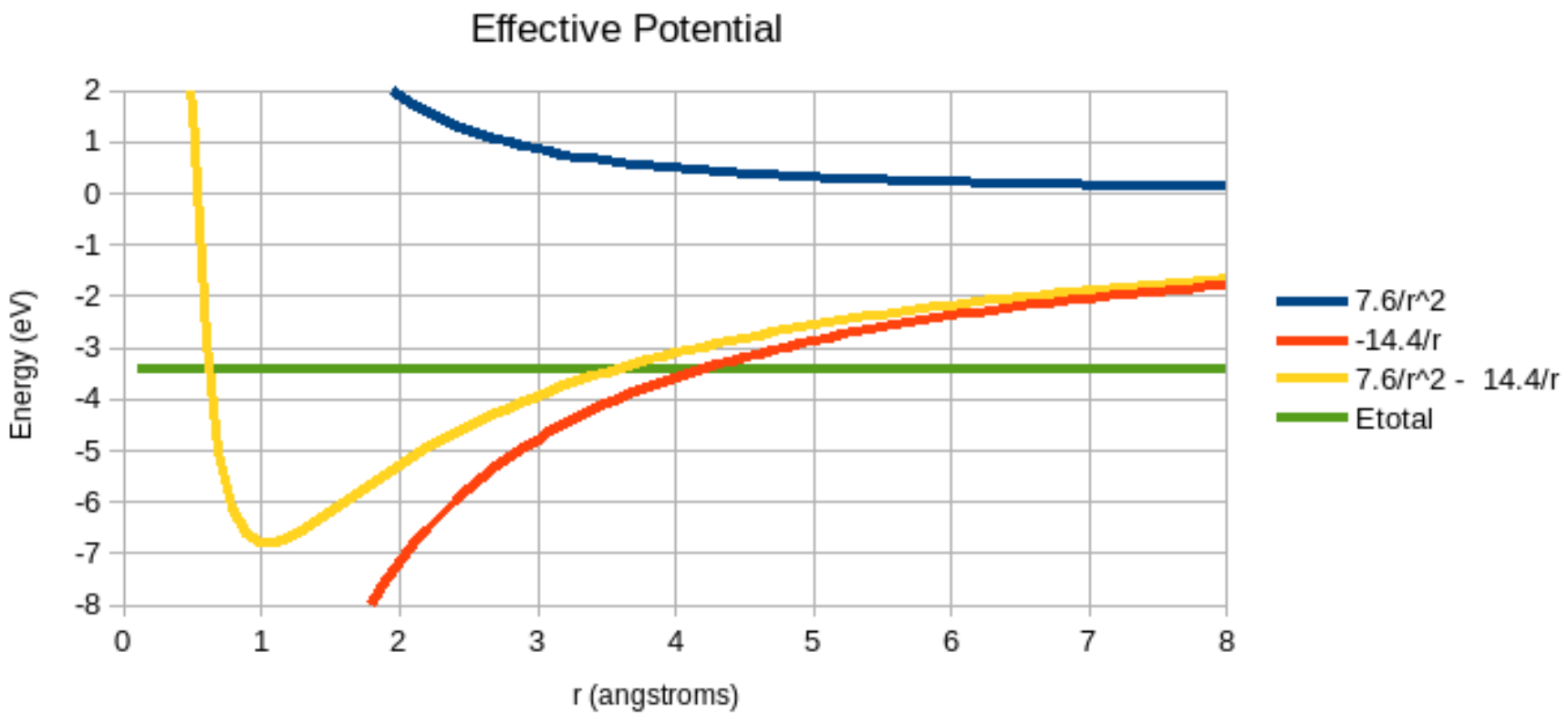
What is the angular momentum? What is the total energy? What is the distance of closest approach to the Earth? The satellite mass is $m_s = 600\ kg$.



Arktica-M

$R_{earth} = 6.37 \times 10^6\ m$
$m_{earth} = 5.97 \times 10^{24}\ kg$
$G = 6.673 \times 10^{-11}\ Nm^2/kg^2$


Black : $\frac{L^2}{2\,r^2\,m_s} - \frac{Gm_e\,m_s}{r}$
Green : $-\frac{Gm_e\,m_s}{r}$
Blue: ME_0
Red: ME_1
E (10^{11} J)
r (10^6m)
0.1
0.0
−0.1
−0.2
−0.3
0
20
40
60
80


Effective Potential
Energy (eV)
2
1
0
-1
-2
-3
-4
-5
-6
-7
-8
0
1
2
3
4
5
6
7
8
r (angstroms)
7.6/r^2
-14.4/r
7.6/r^2 - 14.4/r
Etotal