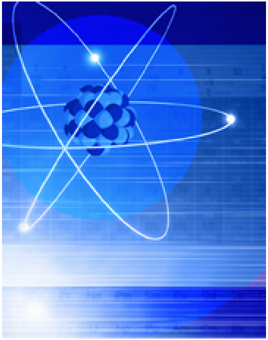
Where is the Electron?

The Coulomb force binds an electron and a proton into a hydrogen atom with a force that is mathematically identical to the gravitational force that binds the planets in our Solar System, the Moon to the Earth, *etc.* For an electron with energy E_e where can it be found?



What We Already Know.

- The Organizing Principle.

$$\begin{aligned} ME_0 &= ME_1 \\ KE_0 + PE_0 &= ME_1 + PE_1 \\ \frac{1}{2}mv_0^2 + PE_0 &= \frac{1}{2}mv_1^2 + PE_1 \end{aligned}$$

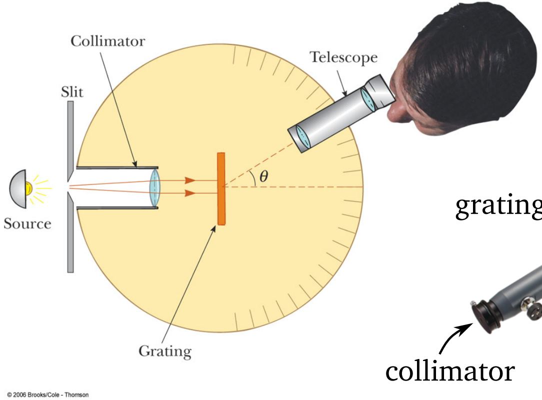
- The Forces

$$\vec{F}_{grav} = -\frac{Gm_1 m_2}{r_{12}^2}\hat{r}_{12} \qquad \vec{F}_{coul} = \frac{k_e q_1 q_2}{r_{12}^2}\hat{r}_{12}$$

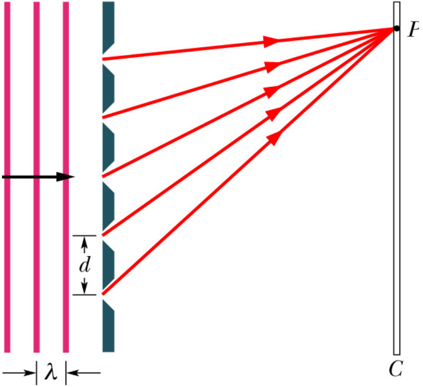
The simulation is here.

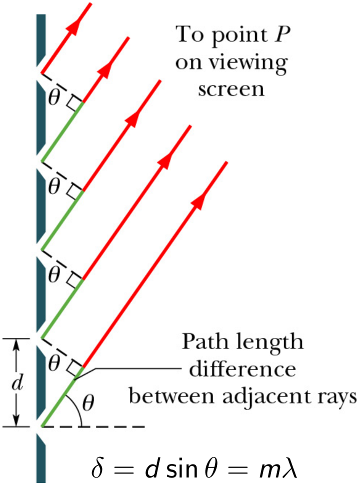
Atomic Spectroscopy -1

Light from a hydrogen spectrum tube is incident on a diffraction grating in a spectrometer. A narrow, red line appears at $\theta_1 = 20.50°$. The grating has a line density of 13,400 lines/inch. What is the wavelength of the light? What is the energy of the photons?



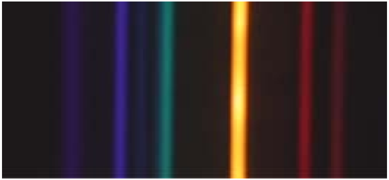
The Diffraction Grating





$$\delta = d \sin\theta = m\lambda$$
$$m = 0, \pm 1, \pm 2, \pm 3, \ldots$$

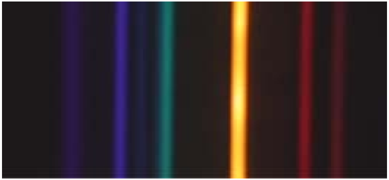
What You See.



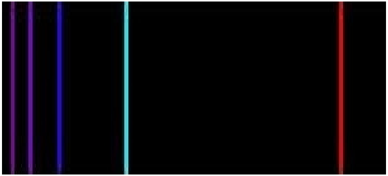
Visible emission spectrum of helium.

Video is here.

What You See.

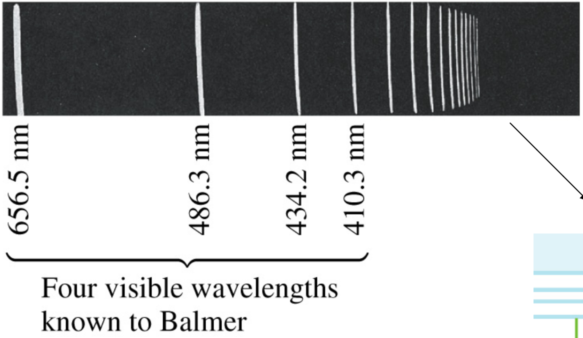


Visible emission spectrum of helium.

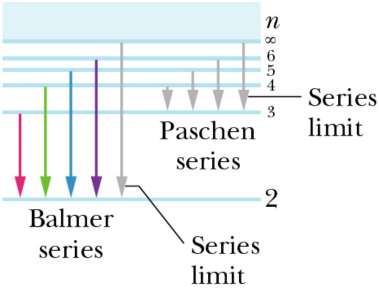


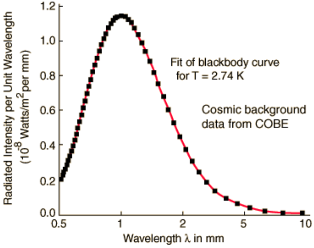
Visible emission spectrum of hydrogen.

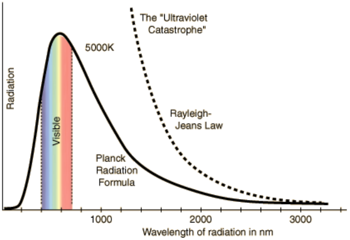
Video is here.



Transition	Energy (eV)
$5 \to 2$	2.856
$4 \to 2$	2.550
$3 \to 2$	1.889







Data adapted from J.C. Mather, *et al.*, Astro. Jour. 354, L37 (1990).

$$\langle E \rangle = \int_o^\infty E \cdot P(E) dE \rightarrow \langle E \rangle = \sum_0^\infty E \cdot P(E) \Delta E$$

where

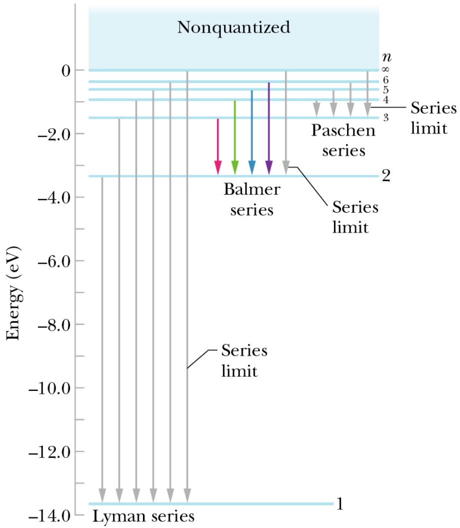
$$\Delta E = \epsilon = h\nu = h\frac{c}{\lambda}$$

Hydrogen in the Galaxy.

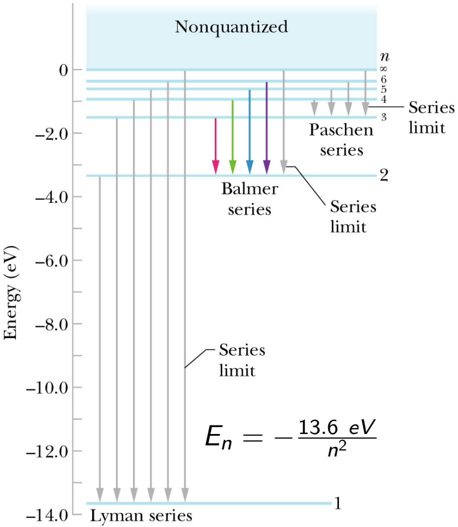
Our galaxy is filled with large gas clouds left over from its formation. The light emitted from these clouds can tell us about their composition and the nature of the processing going inside them. An astronomer has measured an emission line with a wavelength $\lambda = 1216$ Å (see figure). Does this line indicate the presence of hydrogen in the cloud?

Transition	Energy (eV)
$5 \rightarrow 2$	2.856
$4 \rightarrow 2$	2.550
$3 \rightarrow 2$	1.889

The Hydrogen Lines

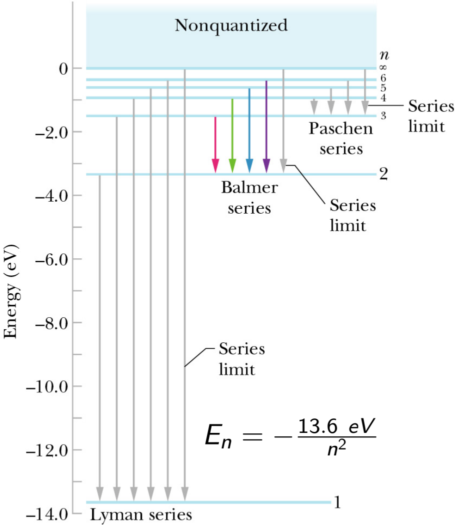


Hydrogen Quantum Numbers



$$E_n = -\frac{13.6\ eV}{n^2}$$

Hydrogen Quantum Numbers



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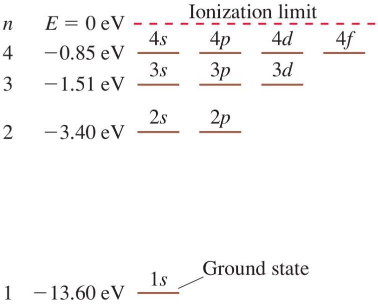
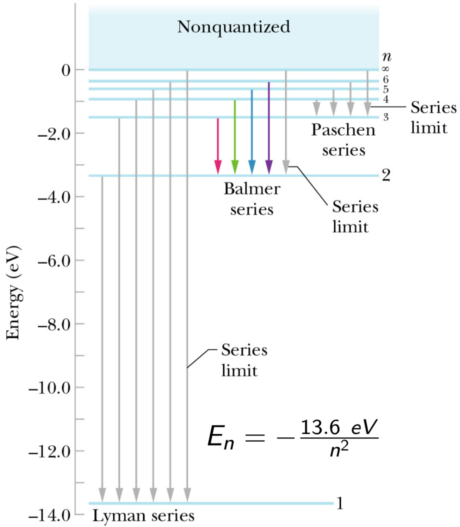
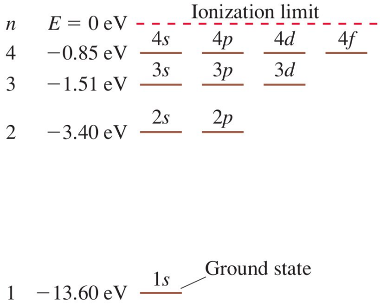


Hydrogen Quantum Numbers



$$E_n = -\frac{13.6\ eV}{n^2}$$



$n \quad \ell$

$3p$

n - principal quantum number

ℓ - angular momentum

Orbits

A Russian Artica satellite that monitors polar weather follows an elliptical orbit around the Earth at an altitude of $h = 300\ km$ above the surface (radius $r_s = 6.67 \times 10^6\ m$) at a velocity

$$\vec{v} = 4.1 \times 10^3\ m/s\ \hat{r} + 7.5 \times 10^3\ m/s\ \hat{\theta} \qquad .$$

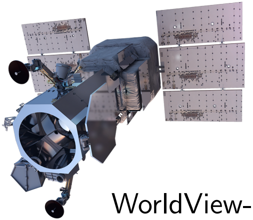
What is the angular momentum? What is the total energy? What is the distance of closest approach to the Earth? The satellite mass is $m_s = 600\ kg$.

Arktica-M

$$R_{earth} = 6.37 \times 10^6\ m$$
$$m_{earth} = 5.97 \times 10^{24}\ kg$$
$$G = 6.673 \times 10^{-11}\ Nm^2/kg^2$$

Changing Orbits

The WorldView-4 satellite is in a circular orbit around the Earth at an altitude of $h = 640\ km$ above the surface at a speed $v_0 = 7.52 \times 10^3\ m/s$. It quickly burns it rockets pointing in a direction away from the Earth and parallel to the direction of the gravitational force. It reaches a new mechanical energy $ME_1 = -1.14 \times 10^{10}\ J$. What will be its new angular momentum and new distance of closest approach to the Earth? Is this rocket burn a good idea? The satellite mass is $m_s = 726\ kg$.



WorldView-4

$$R_{earth} = 6.37 \times 10^6\ m$$
$$m_{earth} = 5.97 \times 10^{24}\ kg$$
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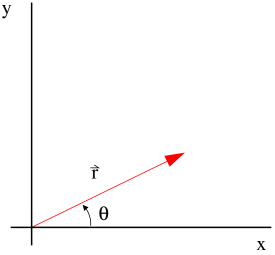
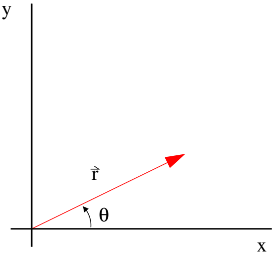
The Kinetic Energy in Polar Coordinates - 1

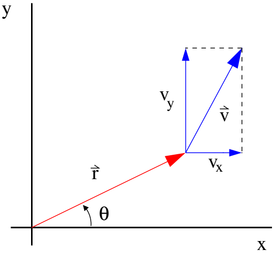


$$\vec{r} = x\hat{i} + y\hat{j}$$

The Kinetic Energy in Polar Coordinates - 1

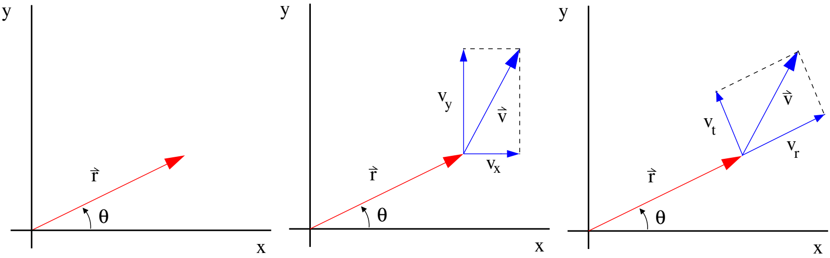


$$\vec{r} = x\hat{i} + y\hat{j}$$



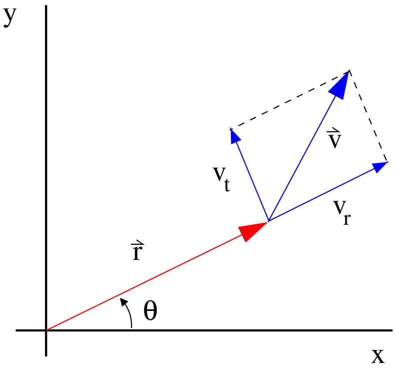
$$\vec{v} = v_x\hat{i} + v_y\hat{j}$$

The Kinetic Energy in Polar Coordinates - 1

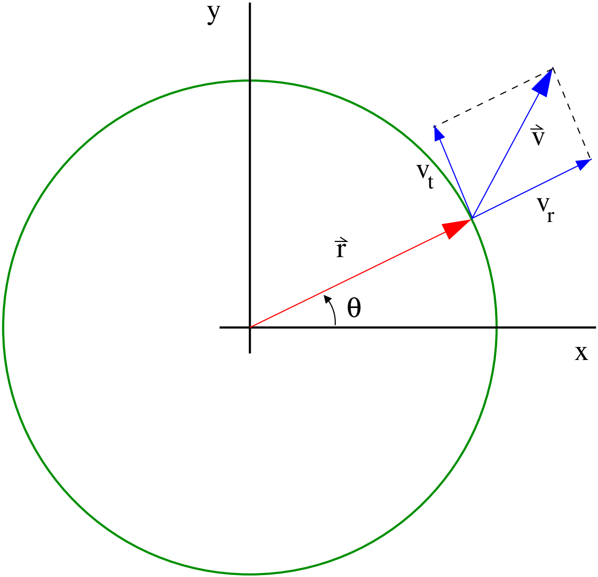


$\vec{r} = x\hat{i} + y\hat{j}$ $\qquad$ $\vec{v} = v_x\hat{i} + v_y\hat{j}$ $\qquad$ $\vec{v} = v_r\hat{r} + v_t\hat{\theta}$

The Kinetic Energy in Polar Coordinates - 2



The Kinetic Energy in Polar Coordinates - 2



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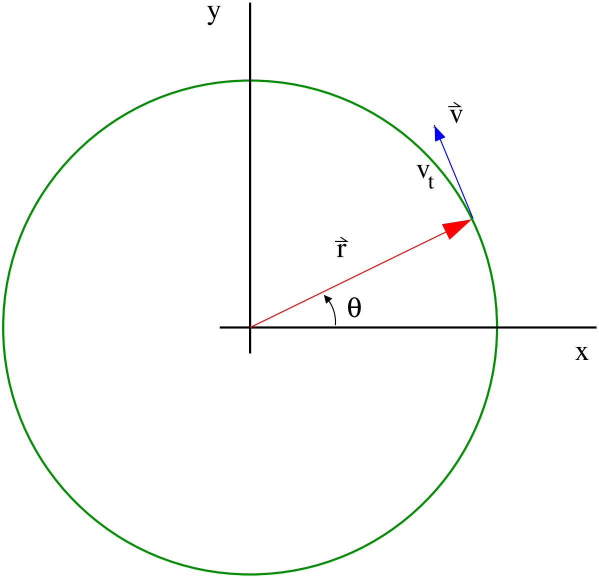
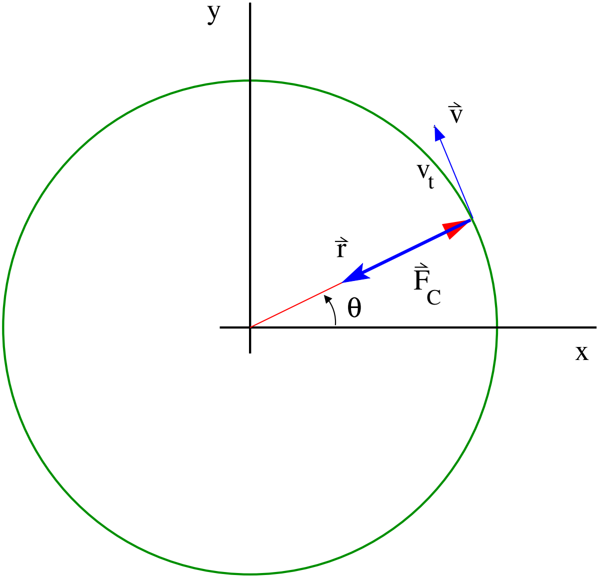
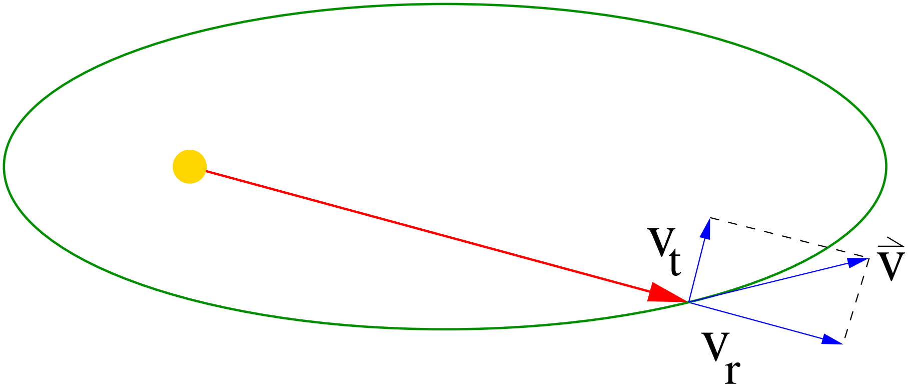


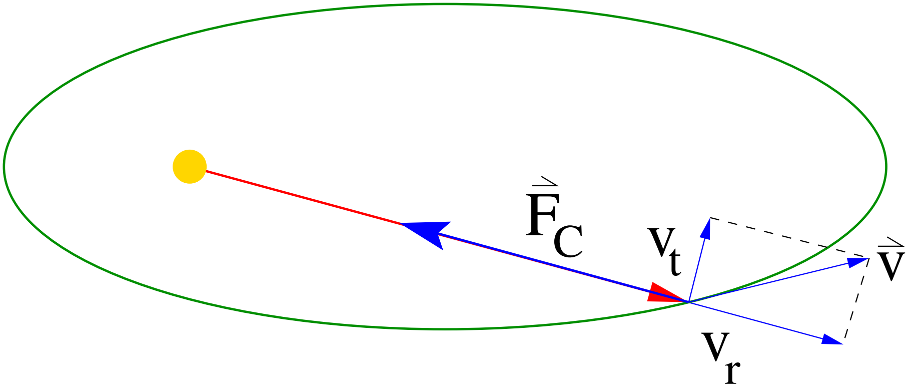
The Kinetic Energy in Polar Coordinates - 2



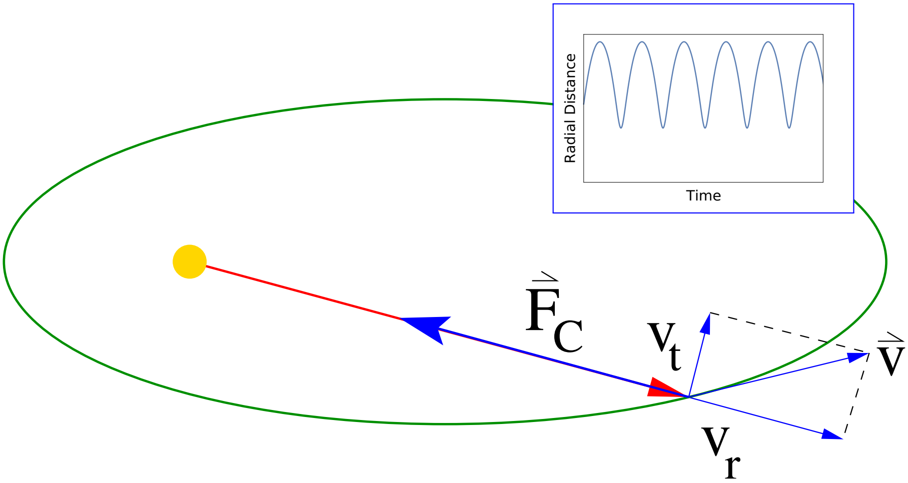
The Kinetic Energy in Polar Coordinates - 3



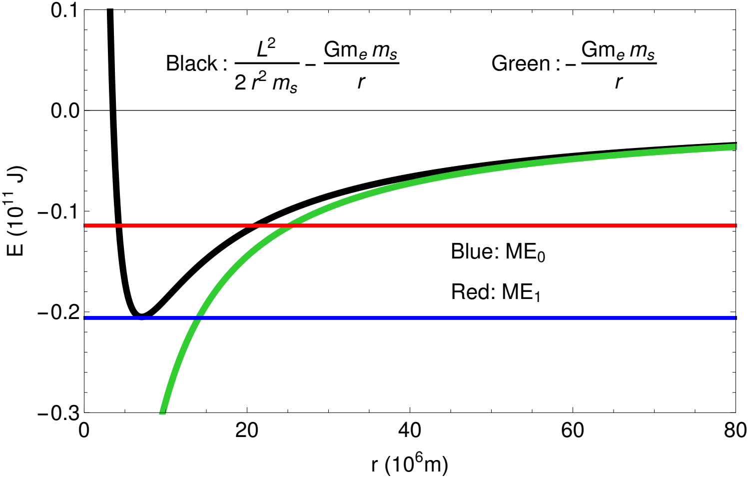
The Kinetic Energy in Polar Coordinates - 3



The Kinetic Energy in Polar Coordinates - 3



Changing Orbits



Classical Physics versus Quantum Mechanics

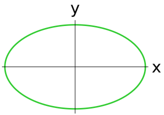
Classical Physics

1. Start with Newton's Laws.
2. Insert the force/potential.
3. Solve the differential equation with initial conditions

$$\vec{F} = m\frac{d^2\vec{r}}{dt^2}$$

where $\vec{r}$ is the position.

4. Get the position $\vec{r}(t)$ as a function of time.



Quantum Physics

Start with Schroedinger's equation.

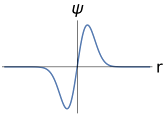
Insert the force/potential.

Solve the differential equation with initial conditions

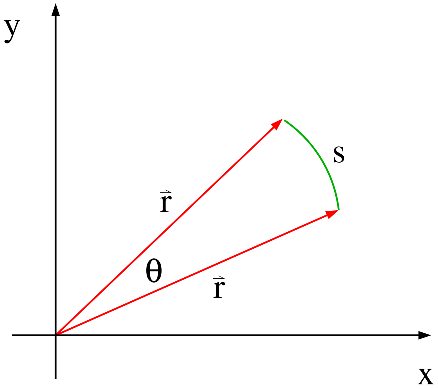
$$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dr^2} + \frac{L^2}{2mr^2}\psi + V\psi = E\psi(\vec{r})$$

where ψ is a wave function.

Get the probability $|\psi(\vec{r})|^2$ as a function of time.



Angles



$$\theta = \frac{s}{r}$$

$$\frac{d\theta}{dt} = \omega = \frac{d}{dt}(s/r)$$

$$\omega = \frac{1}{r}\frac{ds}{dt}$$

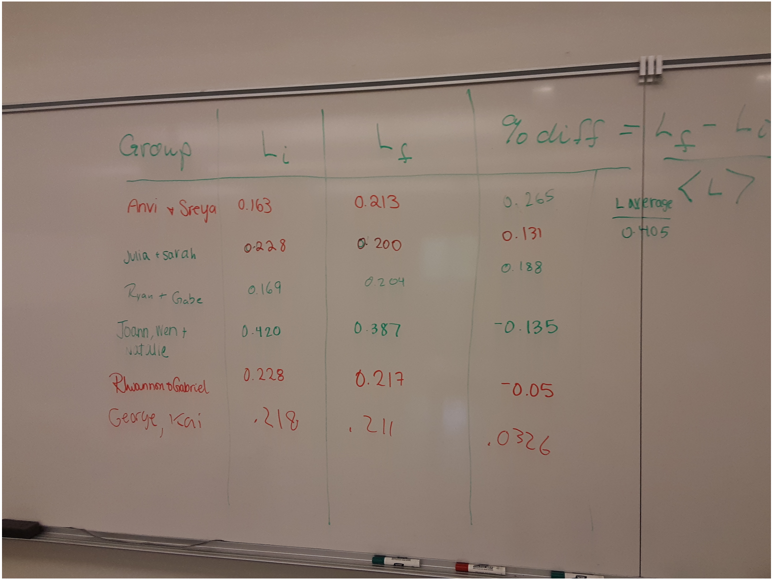
$$\omega = \frac{v_t}{r}$$

$$v_t = r\omega$$

Linear $\rightarrow$ Rotational Quantities

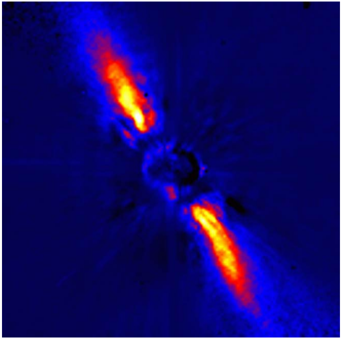
Linear Quantity	Connection	Rotational Quantity
s	$s = r\theta$	$\theta = \frac{s}{r}$
v_t	$v_t = r\omega$	$\omega = \frac{v_t}{r} = \frac{d\theta}{dt}$
$KE = \frac{1}{2}mv_t^2$		$KE = \frac{1}{2}I\omega^2$
$\vec{p}_t = m\vec{v}_t$	$\vec{L} = \vec{r} \times \vec{p}_t$	$\vec{L} = I\vec{\omega}$
$\vec{p}_i = \vec{p}_f$		$\vec{L}_i = \vec{L}_f$

Testing Conservation of Momentum

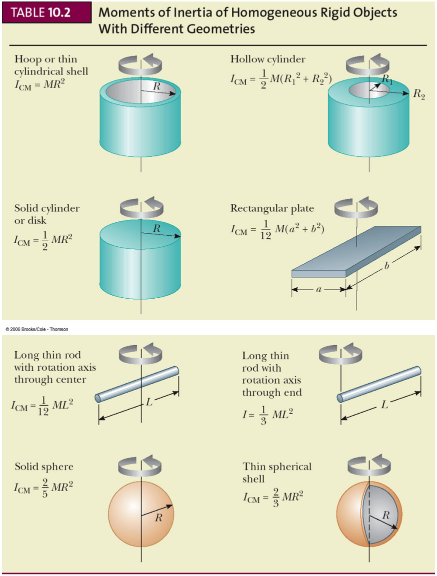


Angular Momentum - Conservation

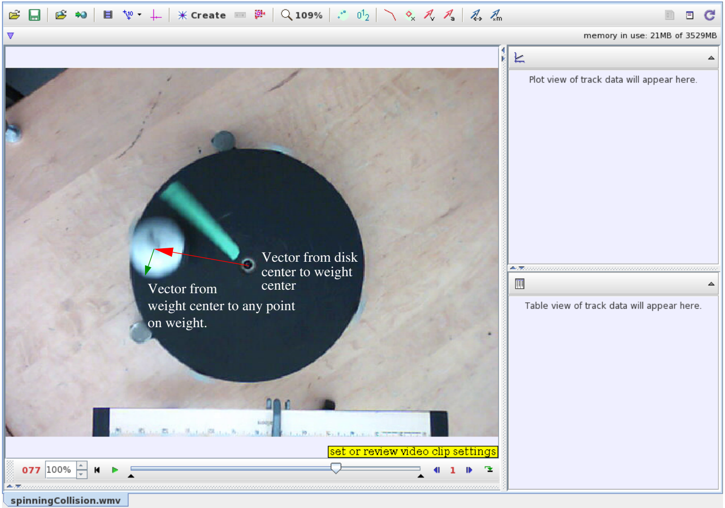
The picture here is an infrared image of a planet-forming disk of dust and gas in the constellation β Pictoris taken by a ground-based telescope. Our own Sun was formed by the gravitational collapse of such a cloud. If the cloud has a radius $R_1 = 10^{13}\ m$ and forms a star like the Sun, then what was the original angular speed and rotational period of the gas cloud? The mass of the Sun is $m_S = 1.99 \times 10^{30}\ kg$, it's rotational period is $T_S = 25.4\ d$, and its radius is $R_S = 6.96 \times 10^8\ m$.



Angular Momentum - Moments of Inertia

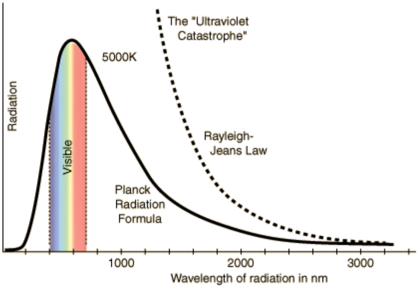


Angular Momentum - Parallel Axis Theorem

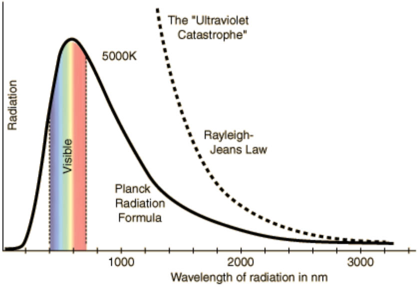


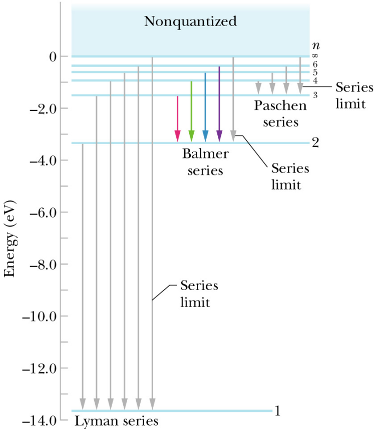
$$I_w = I_{cm} + MD^2$$

Problems for Classical Physics

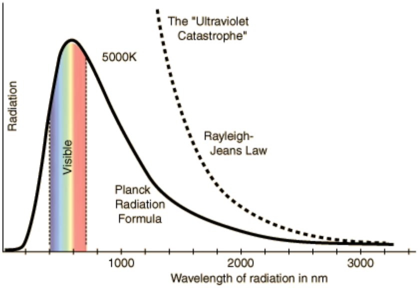


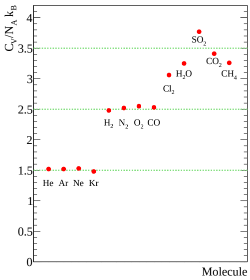
Problems for Classical Physics

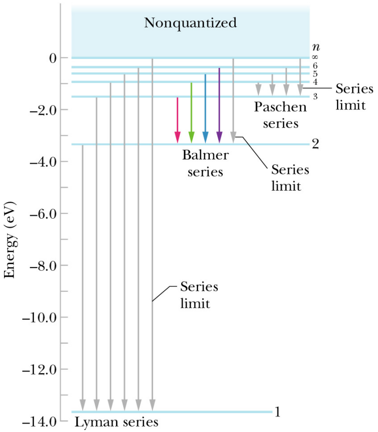




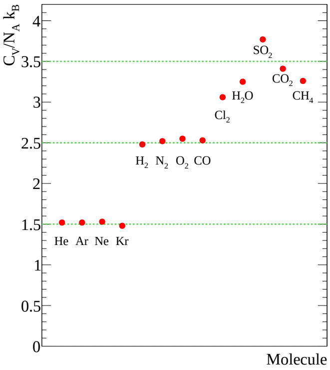
Problems for Classical Physics

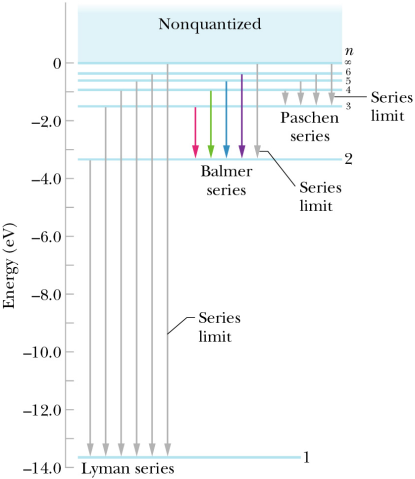






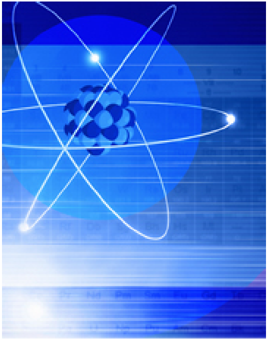
Problems for Classical Physics



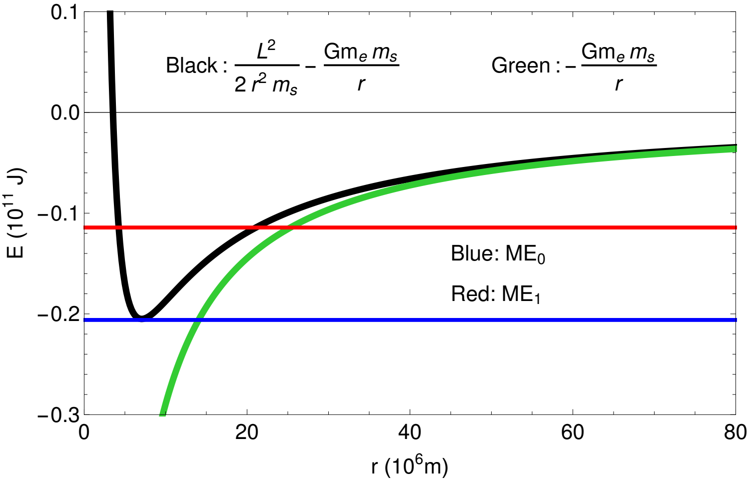


Where is the Electron?

The Coulomb force binds an electron and a proton into a hydrogen atom with a force that is mathematically identical to the gravitational force that binds the planets in our Solar System, the Moon to the Earth, *etc.* For an electron with energy E_e where can it be found as a function of r where r is the distance from the proton?

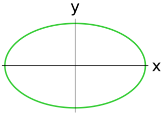


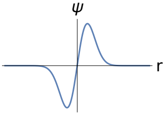
Changing Orbits



Classical Physics versus Quantum Mechanics

	Classical Physics	**Quantum Physics**
1	Start with Newton's Laws.	Start with Schroedinger's equation.
2	Insert the force/potential.	Insert the force/potential.
3	Solve the differential equation with initial conditions $$\vec{F} = m\frac{d^2\vec{r}}{dt^2}$$ where $\vec{r}$ is the position.	Solve the differential equation with initial conditions $$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} + V\psi = E\psi(\vec{r})$$ where ψ is a wave function.
4	Get the position $\vec{r}(t)$ as a function of time.	Get the probability $\|\psi(\vec{r})\|^2$ as a function of time.





The Postulates of Quantum Mechanics

1. The quantum state of a particle is characterized by a wave function $\Psi(\vec{r}, t)$, which contains all the information about the system an observer can possibly obtain. The square of the magnitude of the wave function $|\Psi(\vec{r}, t)|^2$ is interpreted as a probability or probability density for the particle's presence.
2. The things we measure (*e.g.* energy, momentum) are called observables. Each observable has a corresponding mathematical object called an operator that does 'something' to the wave function $\Psi(\vec{r}, t)$. The radial dependence of the wave function $\Psi(\vec{r}, t)$ is governed by the energy operator which generates a famous expression called the Schrödinger equation.

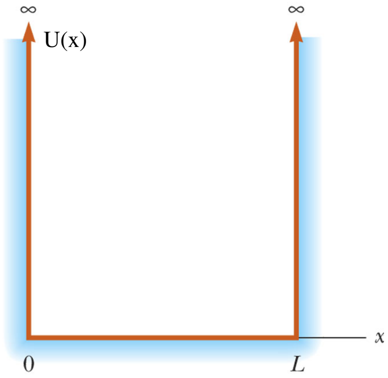
$$-\frac{\hbar^2}{2m}\left(\frac{d^2}{dr^2}\right)\Psi(r) + \frac{L^2}{2mr^2}\Psi(r) + V\Psi(r) = E\Psi(r)$$

Applying the Postulates

Consider a quantum particle trapped between two, infinitely strong walls (*i.e.*, their potential energy is infinite) as shown in the figure. The potential energy is

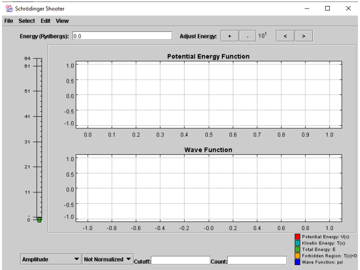
$$
\begin{aligned}
U(x) &= 0 \quad 0 < x < L \\
&= \infty \quad x \leq 0 \text{ and } x \geq L
\end{aligned}
$$

where L is the width of the well. What is the solution of the Schroedinger equation for this $U(x)$ for all x? What is the energy of the particle? Where is the particle located? What is the probability density for the particle?



Getting Schroedinger Shooter

1. Go to http://quantumconcepts.bu.edu/Test_Site/software/shooter/installer/install_shooter.html.
2. Download the installer for your machine which includes Java VM.
3. Double-click on `install_shooter_0040.exe`.
4. You may have to approve the installation of the software despite warnings from your computer. If you have troubles, ask me for help. Downloader the installer to your desktop.
5. You should get the installer after approving changes to your machine. Double-click on it to start. Your computer may continue to complain, but forge ahead. If you have troubles, ask me for help. I usually just take the defaults in the installer. This often means clicking `Next`.
6. When complete search for `shooter` or `Shooter_2.2.0036` to find the executable and double-ckick it. You should get a window like the one below.



A Theory for the Hydrogen Atom - Results

What did your ground-state wave function look like?

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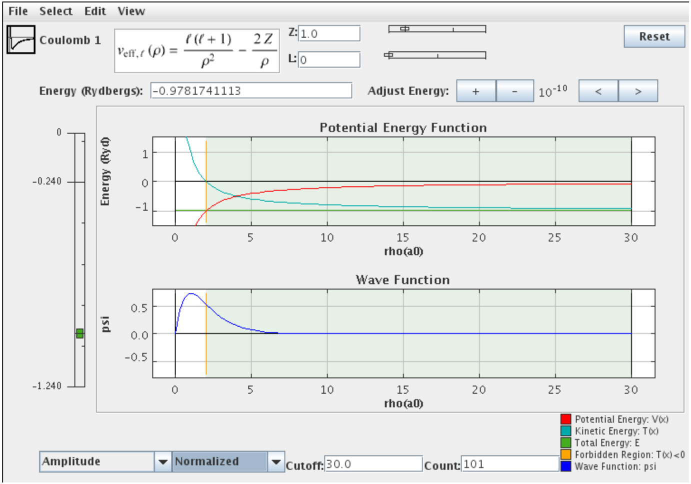
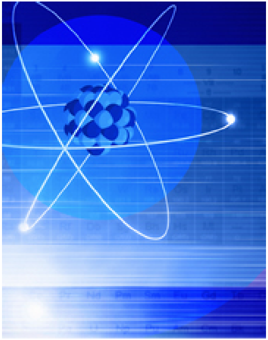


Where is the Electron?

The Coulomb force binds an electron and a proton into a hydrogen atom with a force that is mathematically identical to the gravitational force that binds the planets in our Solar System, the Moon to the Earth, *etc.* For an electron with energy E_e where can it be found as a function of r where r is the distance from the proton?

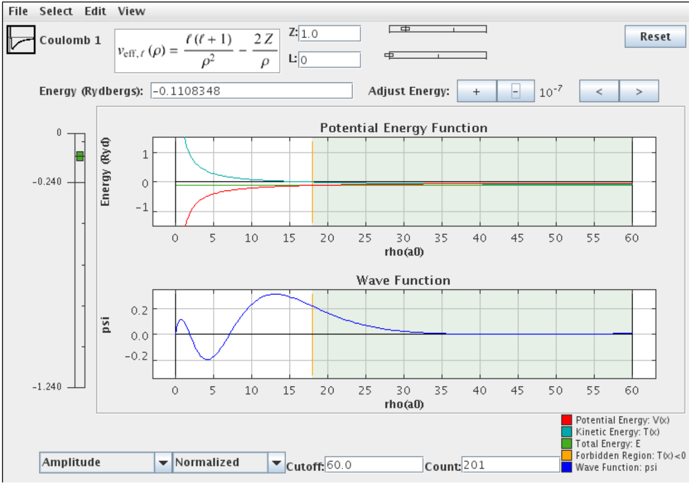


A Theory for the Hydrogen Atom - Results

What did your n=3 wave function look like?

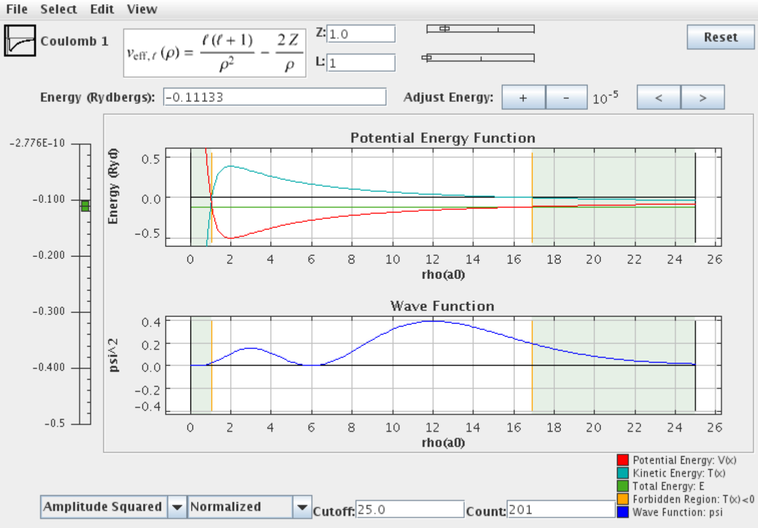
A Theory for the Hydrogen Atom - Results

What did your n=3 wave function look like?

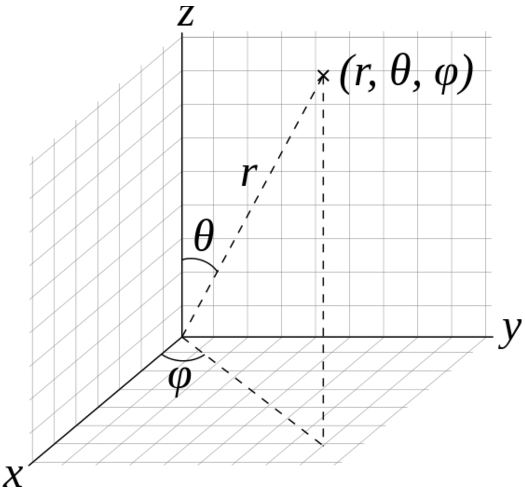


A Weird Result

For $n = 3$, $L = 1$.

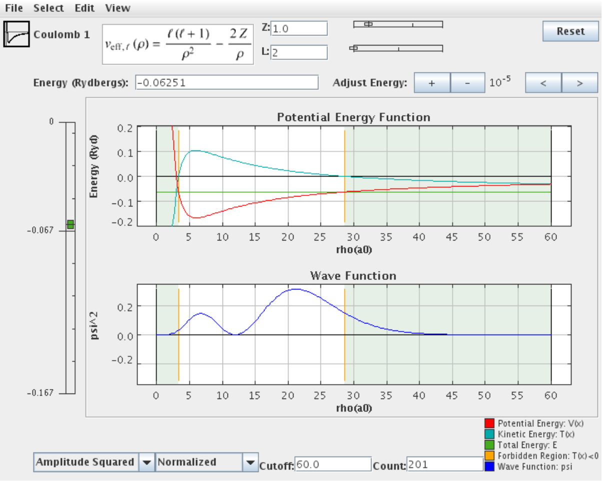


Spherical Coordinates



Comparing Pictures

$n = 4,\ L = 2,\ m = 0$



Comparing Pictures

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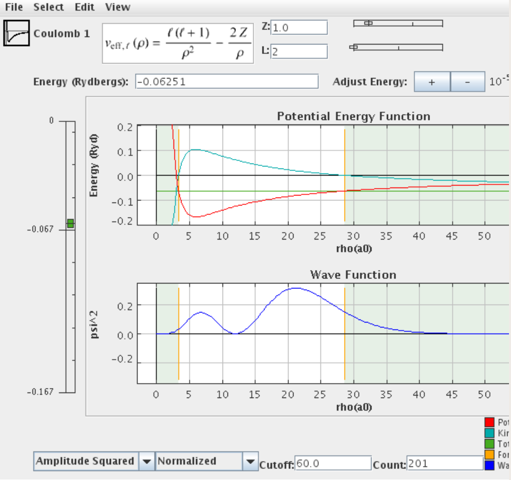

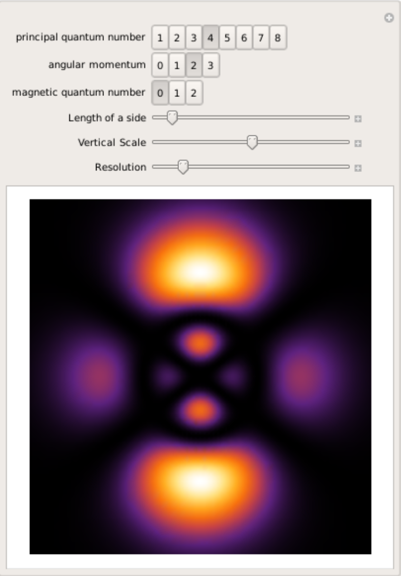

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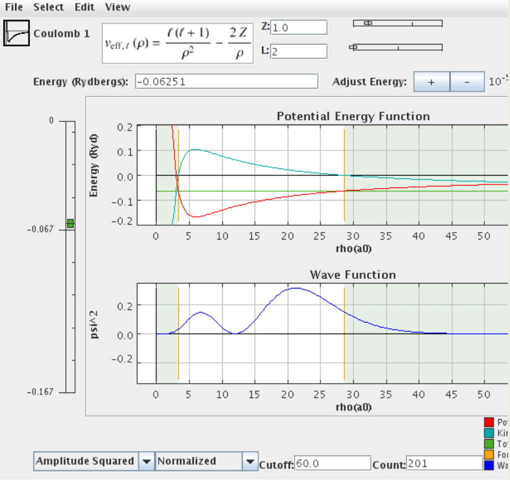

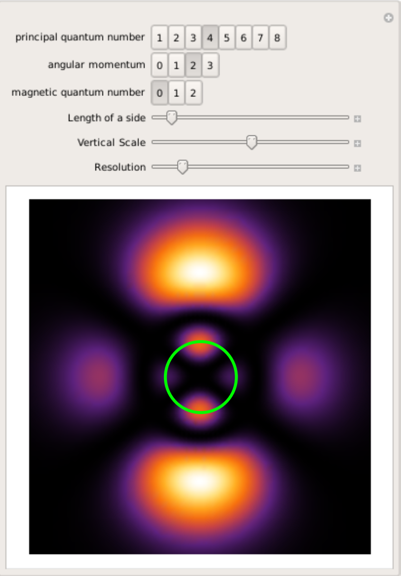

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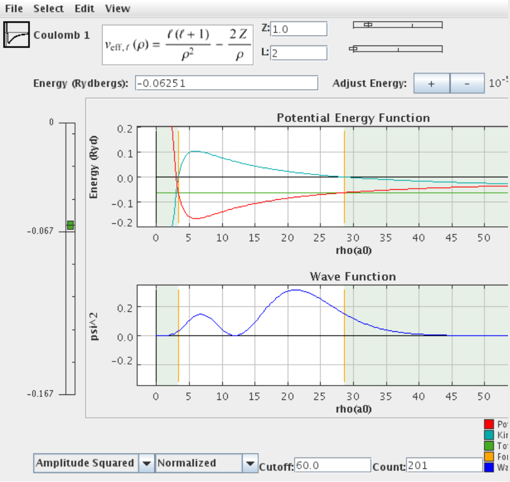

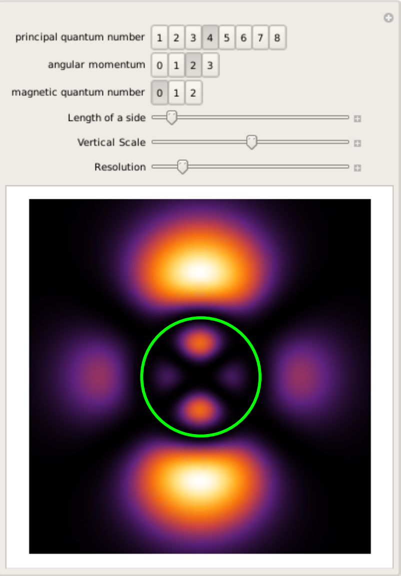

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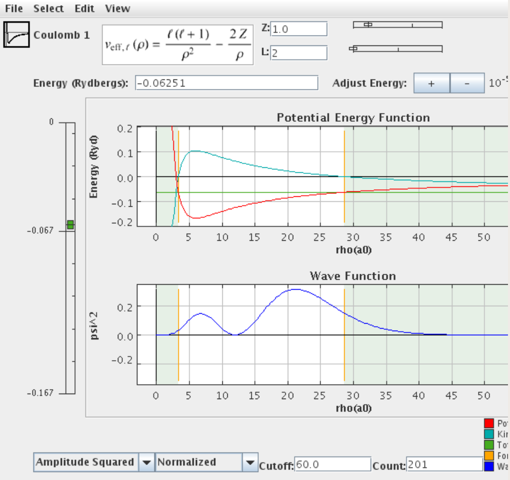

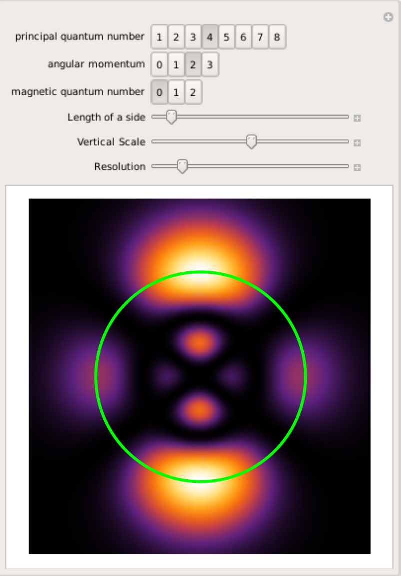

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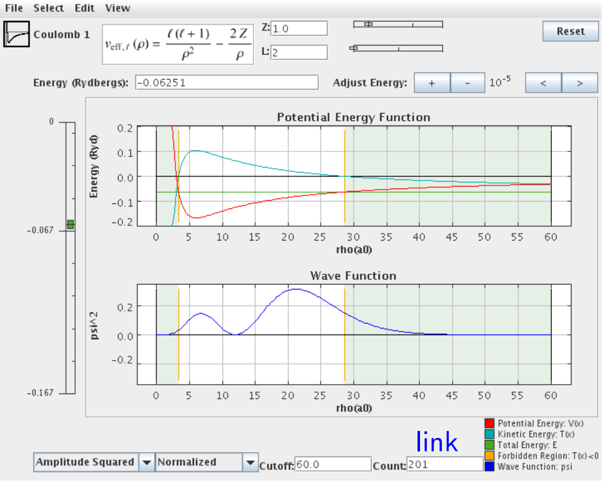

link

Comparing Pictures

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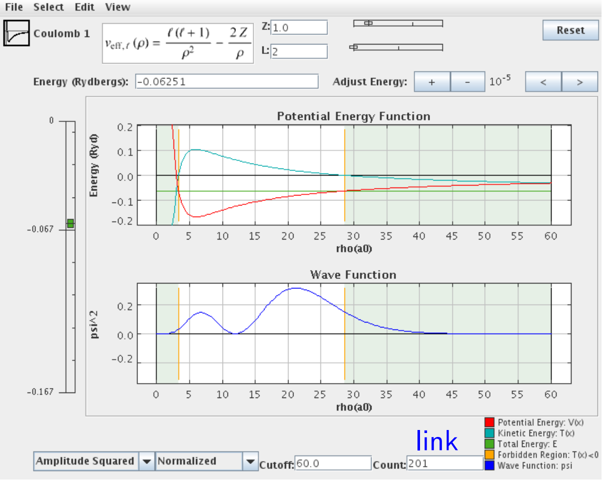

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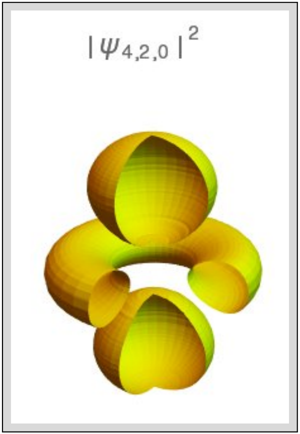

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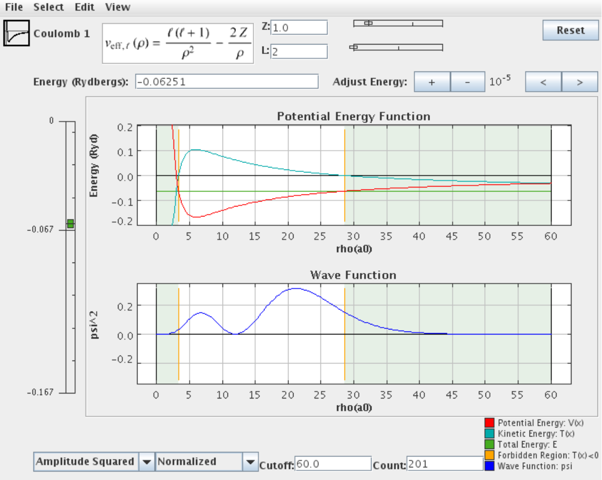

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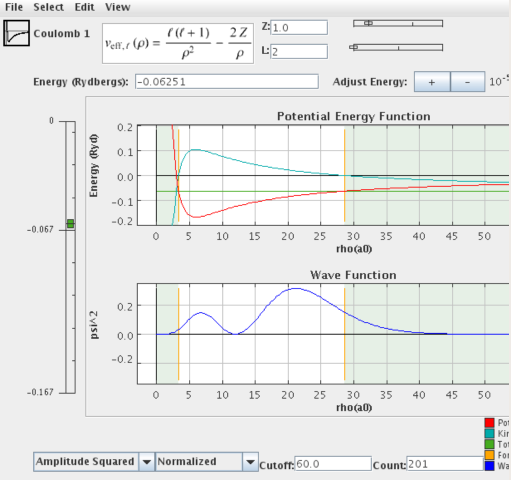
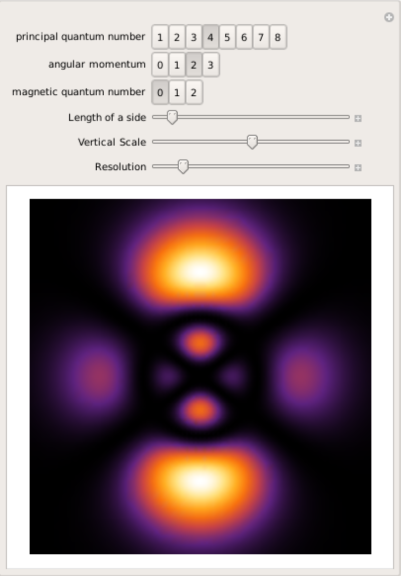
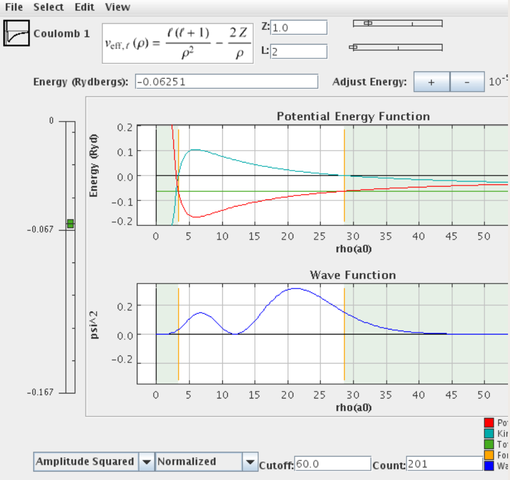
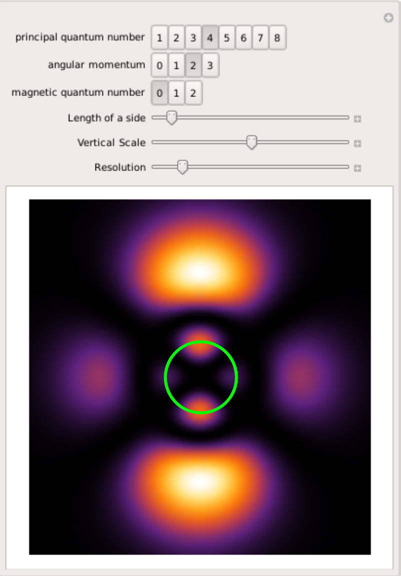




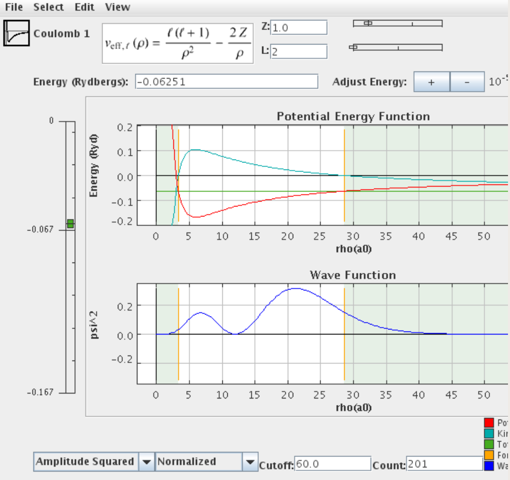
$n = 4, \ L = 2, \ m = 0$

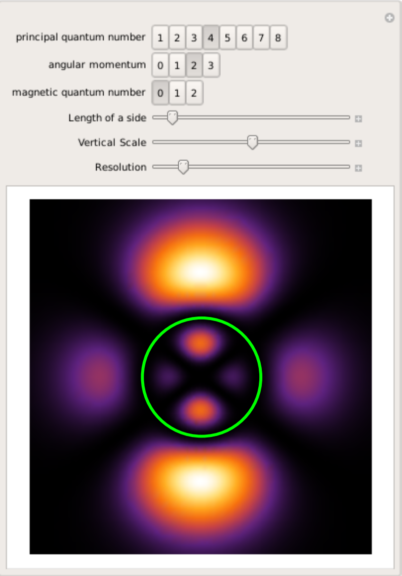

File Select Edit View
Coulomb 1
$v_{\mathrm{eff},\ell}(\rho) = \frac{\ell(\ell+1)}{\rho^2} - \frac{2Z}{\rho}$
Z: 1.0
L: 2
Energy (Rydbergs): -0.06251
Adjust Energy: + -
Potential Energy Function
Energy (Ryd)
rho(a0)
Wave Function
psi^2
rho(a0)
0
-0.067
-0.167
Amplitude Squared
Normalized
Cutoff: 60.0
Count: 201


principal quantum number 1 2 3 4 5 6 7 8
angular momentum 0 1 2 3
magnetic quantum number 0 1 2
Length of a side
Vertical Scale
Resolution

Comparing Pictures

$n = 4,\ L = 2,\ m = 0$





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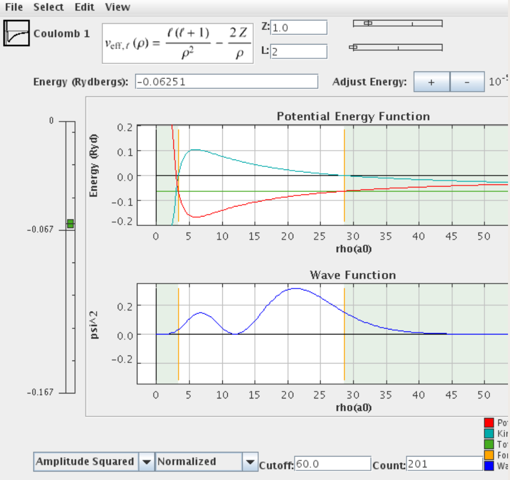

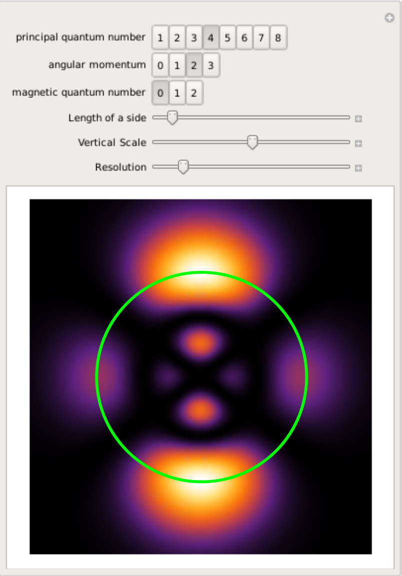

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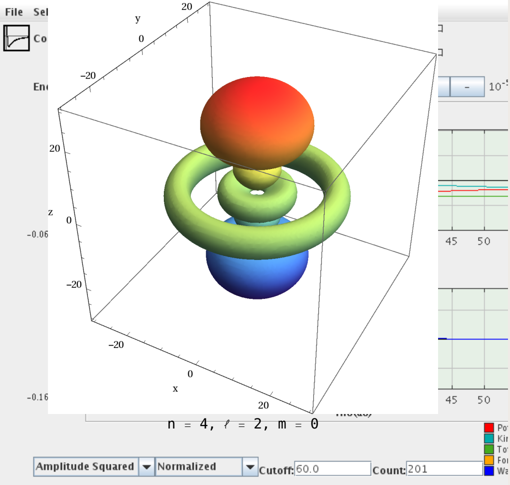

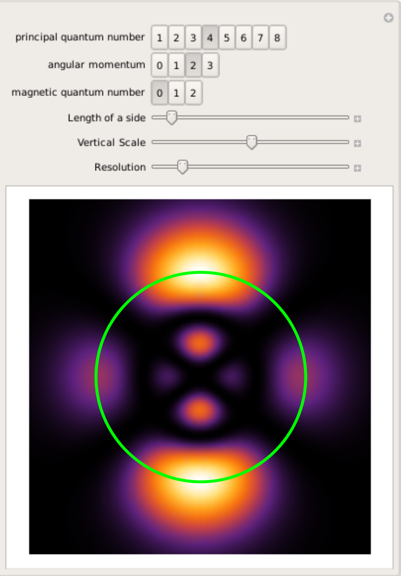

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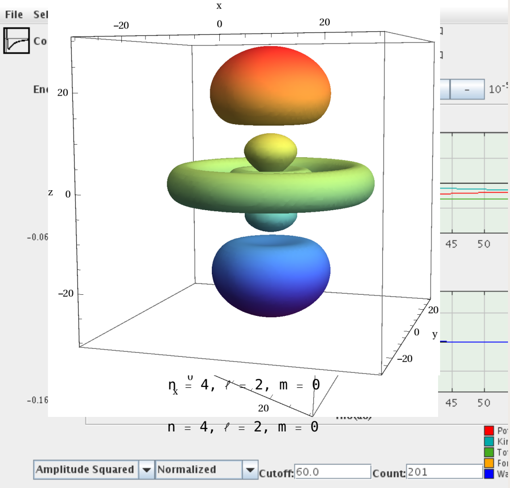

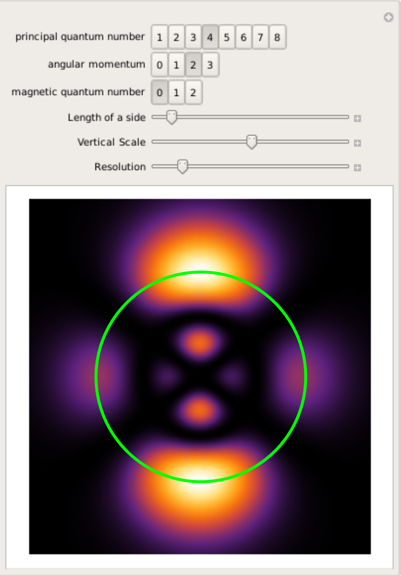

Atomic Orbitals