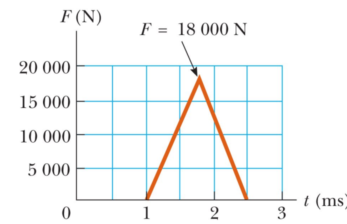
# Homework 2 Kinetic Theory

1. How fast can you set the Earth moving? In particular, when you jump straight up as high as you can, what is the order of magnitude of the maximum recoil speed that you give to the Earth? Model the Earth as a perfectly solid object. In your solution, state the physical quantities you take as data and the values you measure or estimate for them.

2. (a) A particle of mass m moves with momentum $p$. Show that the kinetic energy of the particle is given by $K = p^2/2m$. (b) Express the magnitude of the particle's momentum in terms of its kinetic energy and mass.

3. A friend claims that as long as he has his seat belt on, he can hold on to a 12.0-kg child in a 60.0 mi/h head-on collision with a brick wall in which the car passenger compartment comes to a stop in 0.050 0 s. Show that the violent force during the collision will tear the child from his arms. (A child should always be in a toddler seat secured with a seat belt in the back seat of a car.)

4. An estimated force – time curve for a baseball struck by a bat is shown in the figure. From this curve, determine (a) the impulse delivered to the ball, (b) the average force exerted on the ball, and (c) the peak force exerted on the ball.

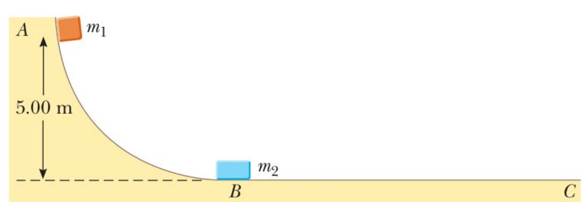



5. A tennis player receives a shot with the ball (0.06 kg) traveling horizontally at 50.0 m/s and returns the shot with the ball traveling horizontally at 40.0 m/s in the opposite direction. (a) What is the impulse delivered to the ball by the racquet? (b) What work does the racquet do on the ball?

6. In a slow-pitch softball game, a 0.200-kg softball crosses the plate at 15.0 m/s at an angle of 45.0° below the horizontal. The batter hits the ball toward center field, giving it a velocity of 40.0 m/s at 30.0° above the horizontal. (a) What is the impulse delivered to the ball? (b) If the force on the ball increases linearly for 4.00 ms, holds constant for 20.0 ms, and then decreases linearly to zero in another 4.00 ms, what is the maximum force on the ball?

7. A railroad car of mass $2.50 \times 10^4\ kg$ is moving with a speed of 4.00 m/s. It collides and couples with three other coupled railroad cars, each of the same mass as the single car and moving in the same direction with an initial speed of 2.00 m/s. (a) What is the speed of the four cars after the collision? (b) How much mechanical energy is lost in the collision?

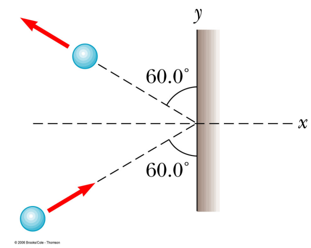
8. Two blocks are free to slide along the frictionless wooden track ABC shown in the figure. A block of mass $m_1 = 5.00\ kg$ is released from A. Protruding from its front end is the north pole of a strong magnet, repelling the north pole of an identical magnet embedded in the back end of the block of mass $m_2 = 10.0\ kg$, initially at rest. The two blocks never touch. Calculate the maximum height to which $m_1$ rises after the elastic collision.




9. A neutron of mass $m_n$ and velocity $\vec{v}_0$ makes an elastic, head-on collision with a carbon nucleus of mass $m_C$ at rest. (a) What fraction of the neutron's kinetic energy is transferred to the carbon nucleus? Get your answer entirely in terms of the masses - no velocities. (b) Assume that the initial kinetic energy of the neutron is $1.60 \times 10^{-13}\ J$. Find its final kinetic energy and the kinetic energy of the carbon nucleus after the collision. The carbon nucleus is about 12 times the neutron mass. (c) What is the final neutron velocity in terms of the masses, $v_0$, and any other constants?

10. In a time interval $\Delta t = t - 0 = t$, $N$ hailstones strike a glass window of area $A$ at an angle $\theta$ to the window surface. Each hailstone has a mass $m$ and a speed $v$. If the collisions are elastic, what are the average force and pressure on the window?

11. A 3.00-kg steel ball strikes a wall with a speed of 10.0 m/s at an angle of 60.0 deg with the surface. It bounces off with the same speed and angle (see figure). If the ball is in contact with the wall for 0.200 s, what is the average force exerted on the ball by the wall?




12. In a period $\Delta t = 2.0\ s$, $N = 4.0 \times 10^{23}$ molecules of nitrogen gas strike a wall of area $A = 10.0\ cm^2$. If the molecules move with a speed $v = 350\ m/s$ and strike the wall head on in elastic collisions, then what is the average pressure exerted on the wall? Get an equation for the pressure $P$ before you start inserting numbers. The mass of a nitrogen molecule is $m = 4.68 \times 10^{-26}\ kg$.

13. A vessel of volume $V = 4 \times 10^{-3}\ m^3$ contains $N = 10^{24}$ molecules of nitrogen gas at a pressure $P = 8 \times 10^5\ N/m^2$. What is the average translational kinetic energy of each molecule?

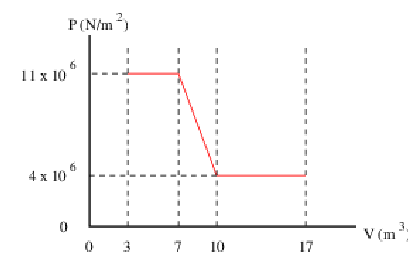
14. A $5.00 - L$ vessel contains nitrogen gas at 27.0°C and 3.00 $atm$. Find (a) the total translational kinetic energy of the gas molecules and (b) the average kinetic energy per molecule.

15. *Brownian motion.* Molecular motion is invisible in itself. When a small particle is suspended in a fluid, bombardment by molecules makes the particle jitter about at random. Robert Brown discovered this motion in 1827 while studying plant fertilization. Albert Einstein analyzed it in 1905 and Jean Perrin used it for an early measurement of Avogadro's number. The visible particle's average kinetic energy can be taken as $\frac{3}{2}k_BT$, the same as that of a molecule in an ideal gas. Consider a spherical particle of density 1000 $kg/m^3$ in water at 20°C. (a) For a particle of diameter 3.00 $\mu m$, evaluate the rms speed. (b) The particle's actual motion is a random walk, but imagine that it moves with constant velocity equal in magnitude to its rms speed. In what time interval would it move by a distance equal to its own diameter? (c) Repeat parts (a) and (b) for a particle of mass 70.0 $kg$, modeling your own body. (d) Find the diameter of a particle whose rms speed is equal to its own diameter divided by 1 $s$. (Note: You can solve all parts of this problem most efficiently by first finding a symbolic relationship between the particle size and its rms speed.)

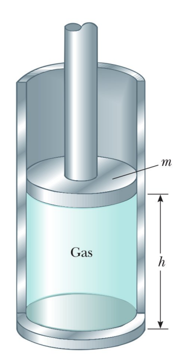
16. A spherical balloon of volume $V$ contains helium at a pressure $P$. How many atoms of helium are in the balloon if the average kinetic energy of the helium atoms is $\langle E_{kin} \rangle$? Your answer should be in terms of $P$, $V$, and $\langle E_{kin} \rangle$.

17. Long-term space missions require reclamation of the oxygen in the carbon dioxide exhaled by the crew. In one method of reclamation, 1.00 mol of carbon dioxide produces 1.00 mol of oxygen and 1.00 mol of methane as a byproduct. The methane is stored in a tank under pressure and is available to control the attitude of the spacecraft by controlled venting. A single astronaut exhales 1.09 kg of carbon dioxide each day. If the methane generated in the respiration recycling of three astronauts during one week of flight is stored in an originally empty 150-L tank at 45.0°C, what is the final pressure in the tank?

18. What is the work done by a fluid that expands from $A$ to $B$ as shown in the plot to the right.

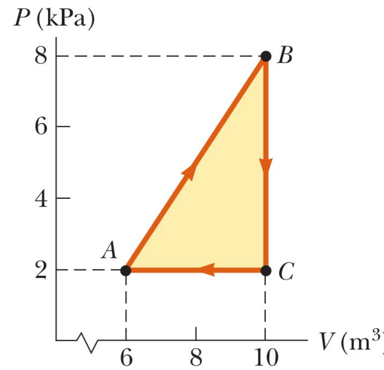



19. A vertical cylinder of cross-sectional area $A$ is fitted with a tight-fitting, frictionless piston of mass $m$ (see figure). (a) If $n$ moles of an ideal gas are in the cylinder at a temperature $T$, what is the height $h$ at which the piston is in equilibrium under its own weight? (b) What is the value for $h$ if $n = 0.200\ mol$, $T = 400\ K$, $A = 0.00800\ m^2$, and $m 20.0\ kg$?



20. (a) Show that the density of an ideal gas occupying a volume $V$ is given by $\rho = PM/RT$, where M is the molar mass. (b) Determine the density of oxygen gas at atmospheric pressure and 20.0°C.

21. A gas is taken through the cyclic process described in the figure. (a) Find the net energy transferred to the system by heat during one complete cycle. (b) If the cycle is reversed — that is, the process follows the path $ACBA$ — what is the net energy input per cycle by heat?




22. Consider the cyclic process depicted in the same figure as the previous problem. If $Q$ is negative for the process $BC$, and $\Delta E_{int}$ is negative for the process $CA$, what are the signs of $Q$ , $W$, and $\Delta E_{int}$ that are associated with each process?

23. One mole of an ideal gas does 3000 $J$ of work on its surroundings as it expands isothermally to a final pressure of 1.00 $atm$ and volume of 25.0 $L$. Determine (a) the initial volume and (b) the temperature of the gas.

24. A 1.00-mol sample of hydrogen gas is heated at constant pressure from 300 K to 420 K. Calculate (a) the energy transferred to the gas by heat, (b) the increase in its internal energy, and (c) the work done on the gas.

25. What is the change in internal energy of 3 mol of helium gas if its temperature is increased by 2.0 K.

26. In a constant-volume process, 209 $J$ of energy is transferred by heat to 1.00 mol of an ideal monatomic gas initially at 300 K. Find (a) the increase in internal energy of the gas, (b) the work done on it, and (c) its final temperature.

27. A certain molecule has $f$ degrees of freedom. Show that an ideal gas consisting of such molecules has the following properties: (1) its total internal energy is $fnRT/2$, (2) its molar specific heat at constant volume is $fR/2$, (3) its molar specific heat at constant pressure is $(f + 2)R/2$, (4) its specific heat ratio is $\gamma = C_P/C_V = (f + 2)/f$.

28. The heat capacity of a sample of a substance is the product of the mass of the sample and the specific heat of the substance. Consider 2.00 mol of an ideal diatomic gas. (a) Find the total heat capacity of the gas at constant volume assuming that the molecules rotate but do not vibrate. (b) Repeat the problem, assuming that the molecules both rotate and vibrate.

29. The Universe was created about twelve billion years ago in a cataclysmic explosion known as the Big Bang. As the Universe expanded after the explosion it cooled and its current temperature has been measured to be $T_b = 2.7\ K$. Its number density is $N/V = 10^6\ particles/m^3$ which is primarily due to protons (H nuclei of mass $m_p = 1.67 \times 10^{-27}\ kg$) and its radius is $r = 10^{26}\ m$. Let's treat the Universe as a spherical, ideal gas of protons. What is the root-mean-square speed of the protons in this 'gas'? What is the total thermal energy in the gas? How does this compare with the energy output of all the stars in the universe $E_{stars} = 10^{65}\ J$? What is the pressure? How does this compare with the lowest laboratory pressure ever reached of $10^{-14}\ N/m^2$?

30. Suppose we wanted to terraform the Moon, *i.e.* change it's climate to an Earth-like one. We would have to release lots of oxygen $O_2$ and nitrogen $N_2$ gases (from the interior of the Moon maybe?). In sunlight temperatures on the Moon's surface can reach $T_{moon} = 500\ K$. The escape velocity on the Moon (minimum speed needed for an object to escape the Moon's gravity) is $v_e = 2.4 \times 10^3\ m/s$. When we start releasing gases will the average $O_2$ or $N_2$ gas molecules in sunlight stay on the Moon or can they escape into outer space?

31. A room with a volume $V_0 = 100\ m^3$ is filled with an ideal diatomic gas (air) at a temperature $T_0 = 283\ K$ and pressure $P_0 = 1.0 \times 10^5\ N/m^2$. The air in the room is heated to a new temperature $T_1 = 297\ K$ with the pressure remaining at $P_0$ since the room is not airtight.

(a) What is the initial internal energy of the air in the room?

(b) What is the change in the internal energy of the air in the room? Does this result make sense? Explain. Notice the room is not airtight so air can move freely in and out of it.

32. An incandescent light bulb contains a fixed volume $V$ of argon at pressure $P_0$. The bulb is switched on and a constant power $\mathcal{P}_l$ is transferred to the argon for a time interval $\Delta t$. (a) Show the final pressure $P_1$ in the bulb at the end of this process is

$$P_1 = P_0 \left( 1 + \frac{\mathcal{P}_l \Delta t R}{P_0 V C_V} \right) \quad .$$

(b) Find the pressure in the spherical bulb of diameter $d = 0.08\ m$ after a time $\Delta t = 8.0\ s$. The power of the light bulb is $\mathcal{P}_l = 3.0\ J/s$ and the initial pressure is $P_0 = 1.01 \times 10^5\ N/m^2$.

33. A helium atom collides elastically head-on with an iron atom. Both particles are free to move in space. The initial velocity of the helium is $|\vec{v}_0| = 100\ m/s$ and the iron atom is at rest. What is the final velocity $\vec{v}_1$ of the helium atom? Ignore the effects of potential energy. If the mass of the target (the iron atom here) increases to infinity how are $v_0$ and $v_1$ related?

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| Helium mass: | $m_h = 6.7 \times 10^{-27}\ kg$ | $v_0 = 100\ m/s$ |
| Iron mass: | $m_i = 9.4 \times 10^{-26}\ kg$ | $v_i = 0\ m/s$ |