

Assume that a pure, ideal gas is made of tiny particles that bounce into each other and the walls of their cubic container of side ℓ . Show the average pressure P exerted by this gas is

$$P = \frac{1}{3} \frac{N}{V} \overline{mv_{total}^2}$$

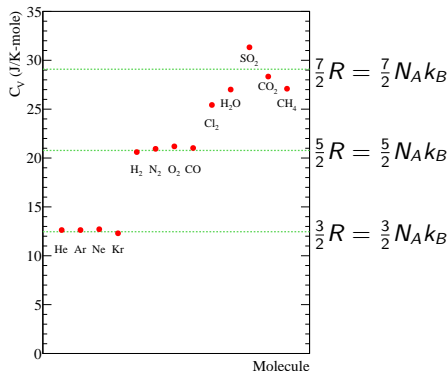
Use the ideal gas law ($PV = Nk_B T = nRT$) and the conservation of energy ($\Delta E_{int} = C_V \Delta T$) to calculate the specific heat of an ideal gas and show the following.

$$C_V = \frac{3}{2} N_A k_B = \frac{3}{2} R$$

Is this right?

N - number of particles
 k_B - Boltzmann constant
 N_A - Avogadro's number

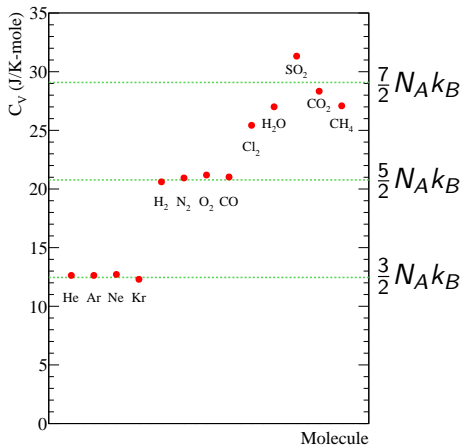
$V = \ell^3$
 m - atomic mass
 v_{total} - atom's speed



$$P = \frac{1}{3} \frac{N}{V} \overline{mv_{total}^2} = \frac{2}{3} \frac{N}{V} \langle E_{kin} \rangle$$

$$\langle E_{kin} \rangle = \frac{3}{2} N k_B T$$

$$C_V = \frac{3}{2} N_A k_B = \frac{3}{2} R$$



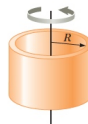
Classically

$$E_{\text{rot}} = \frac{\mathcal{L}^2}{2\mathcal{I}}$$

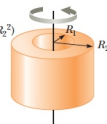
where

$$\mathcal{I} = \sum m r_i^2 = \int r^2 dm$$

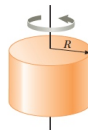
Hoop or thin
cylindrical shell
 $I_{\text{CM}} = MR^2$



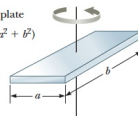
Hollow cylinder
 $I_{\text{CM}} = \frac{1}{2} M(R_1^2 + R_2^2)$



Solid cylinder
or disk
 $I_{\text{CM}} = \frac{1}{2} MR^2$



Rectangular plate
 $I_{\text{CM}} = \frac{1}{12} M(a^2 + b^2)$



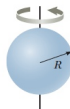
Long, thin rod
with rotation axis
through center
 $I_{\text{CM}} = \frac{1}{12} ML^2$



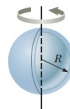
Long, thin
rod with
rotation axis
through end
 $I = \frac{1}{3} ML^2$



Solid sphere
 $I_{\text{CM}} = \frac{2}{5} MR^2$



Thin spherical
shell
 $I_{\text{CM}} = \frac{2}{3} MR^2$



Classically

$$E_{\text{rot}} = \frac{\mathcal{L}^2}{2\mathcal{I}}$$

where

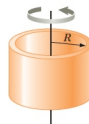
$$\mathcal{I} = \sum m r_i^2 = \int r^2 dm$$

Quantum mechanically

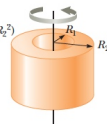
$$E_{\text{rot}}^{\text{qm}} = \ell(\ell + 1) \frac{\hbar^2}{2\mathcal{I}} \quad \ell = 0, 1, 2, \dots$$

where ℓ is the angular momentum quantum number.

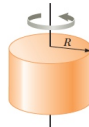
Hoop or thin
cylindrical shell
 $I_{\text{CM}} = MR^2$



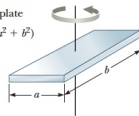
Hollow cylinder
 $I_{\text{CM}} = \frac{1}{2} M(R_1^2 + R_2^2)$



Solid cylinder
or disk
 $I_{\text{CM}} = \frac{1}{2} MR^2$



Rectangular plate
 $I_{\text{CM}} = \frac{1}{12} M(a^2 + b^2)$



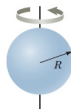
Long, thin rod
with rotation axis
through center
 $I_{\text{CM}} = \frac{1}{12} ML^2$



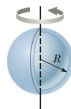
Long, thin
rod with
rotation axis
through end
 $I = \frac{1}{3} ML^2$



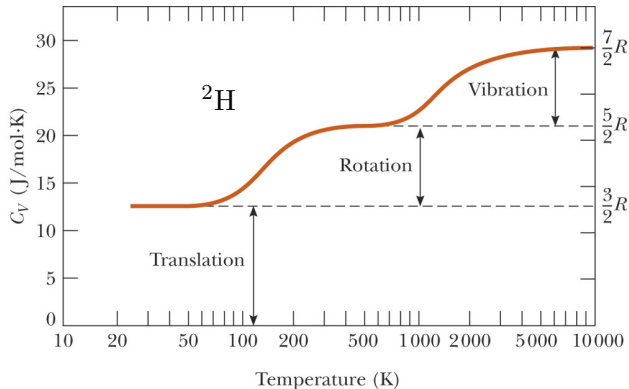
Solid sphere
 $I_{\text{CM}} = \frac{2}{5} MR^2$



Thin spherical
shell
 $I_{\text{CM}} = \frac{2}{3} MR^2$



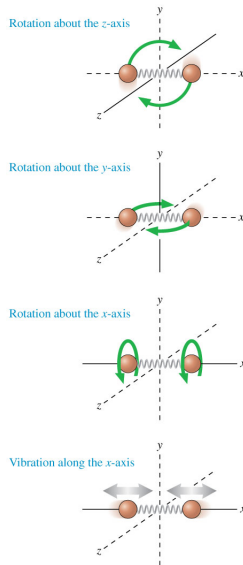
- ① The gas consists of a large number of small, **mobile** particles and their average separation is large.
- ② The particles obey Newton's Laws and the conservation laws, but their motion can be described statistically.
- ③ The particles' collisions are elastic.
- ④ The inter-particle forces are small until they collide.
- ⑤ The gas is pure.
- ⑥ The gas is in thermal equilibrium with the container walls.



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$$E_{rot}^{qm} = \frac{\ell(\ell + 1)\hbar^2}{2\mathcal{I}} \quad \ell = 0, 1, 2, 3, \dots$$

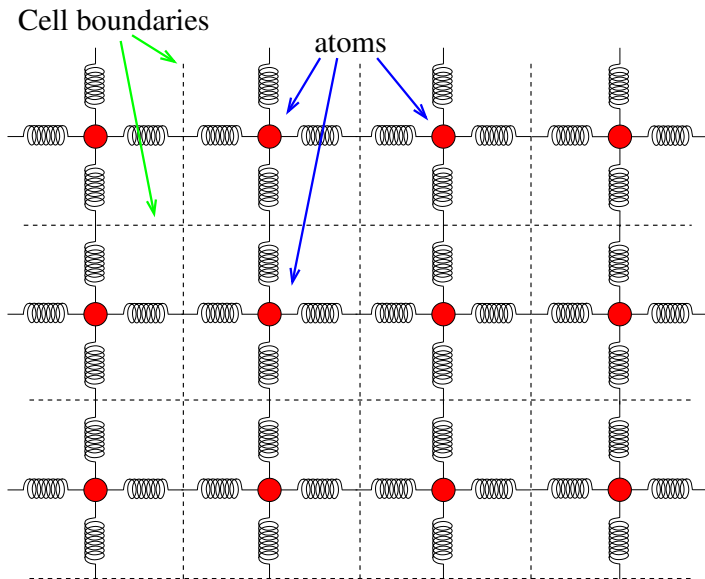
where ℓ is the angular momentum quantum number and \mathcal{I} is the moment of inertia.

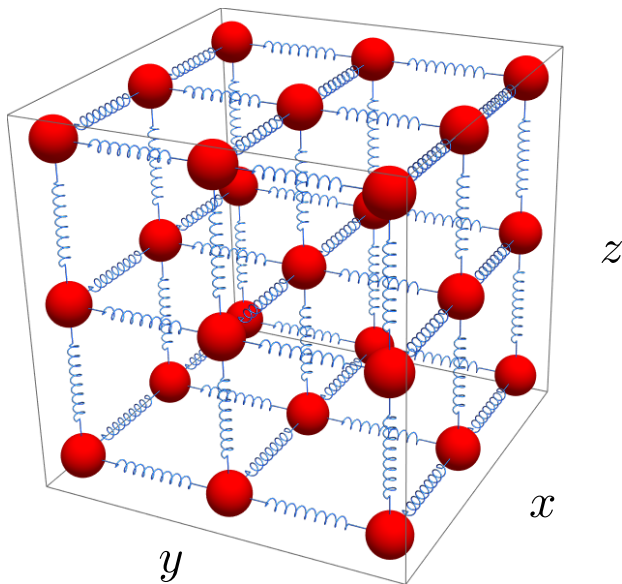


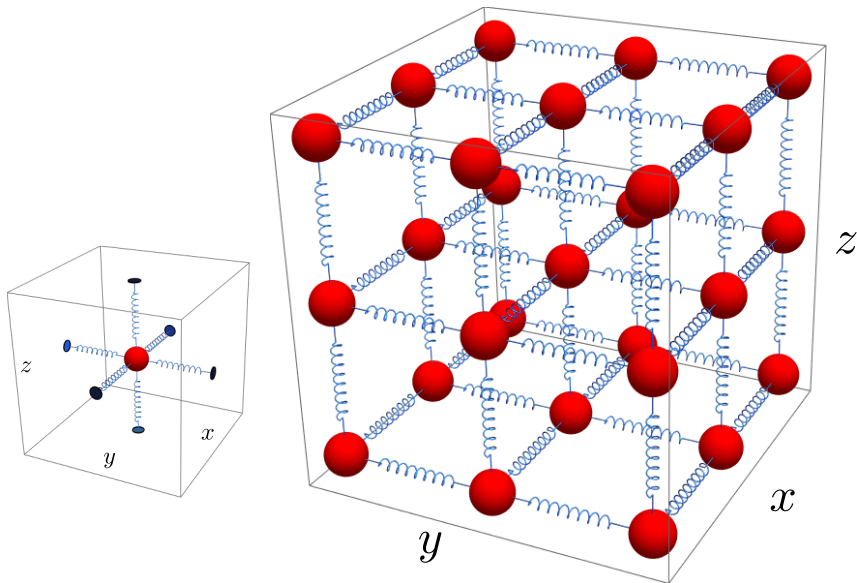
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Solid	Molar Specific Heat (J/K-mole)	Our Results (J/K-mole)
Lead	26.4	22 ± 6
Zinc	25.4	36 ± 14
Aluminum	26.4	24 ± 4
Copper	24.5	27 ± 8
Tin	27.0	43 ± 8
Gold	25.4	
Silver	25.4	
Iron	25.0	

Solid	Molar Specific Heat (J/K-mole)	Our Results (J/K-mole)	$3R$ (J/K-mole)
Lead	26.4	22 ± 6	24.9434
Zinc	25.4	36 ± 14	24.9434
Aluminum	26.4	24 ± 4	24.9434
Copper	24.5	27 ± 8	24.9434
Tin	27.0	43 ± 8	24.9434
Gold	25.4		24.9434
Silver	25.4		24.9434
Iron	25.0		24.9434







$$E_{atom} = n_x \hbar \omega + n_y \hbar \omega + n_z \hbar \omega$$

The n_i are integers and \hbar and ω are constants.

An Einstein solid is made of N , three-dimensional harmonic oscillators containing q quanta of energy.

- 1 What is the multiplicity of a single Einstein solid?
- 2 What is the multiplicity of two Einstein solids in thermal contact?
- 3 How would you determine the most likely microstate of the system?
- 4 How is entropy related to temperature?
- 5 How is the energy related to temperature?
- 6 What is the molar specific heat of an elemental solid?

Solid	Molar Specific Heat (J/K-mole)	Our Results (J/K-mole)
Lead	26.4 ± 0.7	22 ± 6
Zinc	25.4 ± 0.6	36 ± 14
Aluminum	26.4 ± 0.2	24 ± 4
Copper	24.5 ± 0.6	27 ± 8
Tin	27.0 ± 0.6	43 ± 8
Gold	25.4 ± 0.6	
Silver	25.4 ± 0.6	
Iron	25.0 ± 0.6	



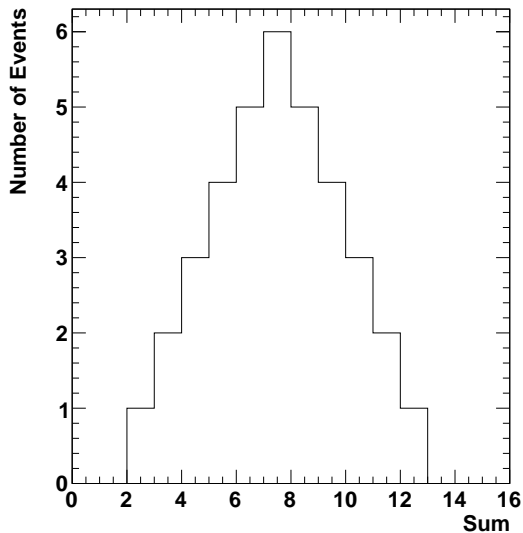
Total	Combinations	No. of combos
2	1-1	1
3	1-2,2-1	2
4	1-3,2-2,3-1	3
5	1-4,2-3,3-2,4-1	4
6	1-5,2-4,3-3,4-2,5-1	5
7	1-6,2-5,3-4,4-3,5-2,6-1	6
8	2-6,3-5,4-4,5-3,6-2	5
9	3-6,4-5,5-4,6-3	4
10	4-6,5-5,6-4	3
11	5-6,6-5	2
12	6-6	1

Total	Combinations	No. of combos
2	1-1	1
3	1-2,2-1	2
4	1-3,2-2,3-1	3
5	1-4,2-3,3-2,4-1	4
6	1-5,2-4,3-3,4-2,5-1	5
7	1-6,2-5,3-4,4-3,5-2,6-1	6
8	2-6,3-5,4-4,5-3,6-2	5
9	3-6,4-5,5-4,6-3	4
10	4-6,5-5,6-4	3
11	5-6,6-5	2
12	6-6	1

macrostate

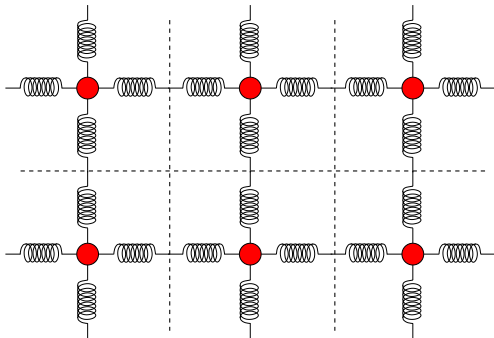
microstates

Throwing Dice



An Einstein solid is made of N , three-dimensional harmonic oscillators containing q quanta of energy as shown below.

- 1 What is the energy of a single oscillator? of N oscillators?
- 2 How many microstates Ω exist for a 'system' with $N = 1$ and $q = 3$? for $N = 2$, $q = 2$?
- 3 What is Ω for any N and q ?



Microstates for $N_A = 2, q_A = 2$

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[illegible][illegible]

Microstates for $N_A = 2, q_A = 2$ $\Omega = 21$

n_1	n_2	n_3	n_4	n_5	n_6
1	1				
1		1			
1			1		
1				1	
1					1
	1	1			
	1		1		
	1			1	
	1				1
		1	1		
		1		1	

n_1	n_2	n_3	n_4	n_5	n_6
		1			1
			1	1	
			1		1
				1	1
2					
	2				
		2			
			2		
				2	
					2

$$E_{int} = (n_x + n_y + n_z)\hbar\omega = \sum_{i=1}^{3N} n_i \hbar\omega \quad \text{for } N \text{ atoms}$$

multiplicity (Ω)	number of microstates
macrostate	configuration of a solid defined by bulk properties like N and E/U .
microstate	one of the configurations of quanta consistent with the macrostate.

$$\Omega(N_A, q_A) = \frac{(q_A + 3N_A - 1)!}{q_A!(3N_A - 1)!}$$

$$\Omega_{AB} = \Omega_A \Omega_B \quad \text{where} \quad \begin{array}{l} \Omega_{AB} - \text{multiplicity of combined state} \\ \Omega_{A,B} - \text{individual multiplicities.} \end{array}$$

E_{int}

oms

multiplicity

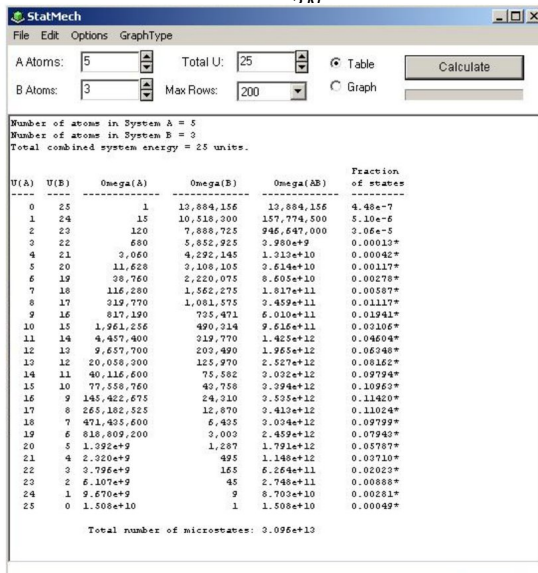
macrostate

microstate

 Ω_{AB}

ilk proper-

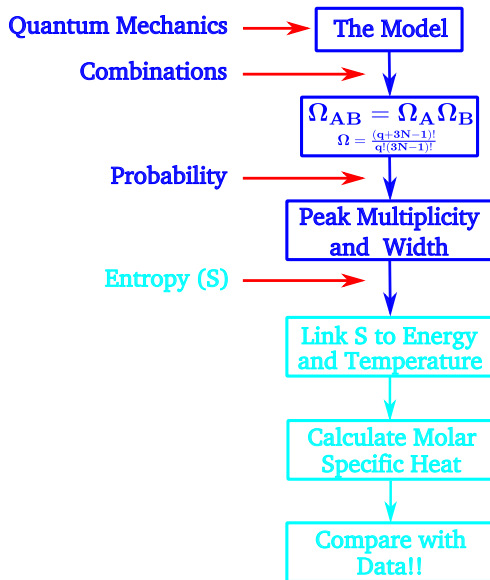
consistent

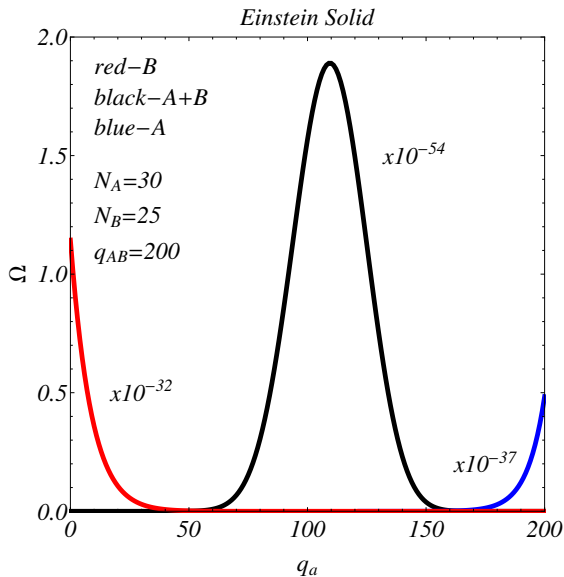
rbed state
icities.

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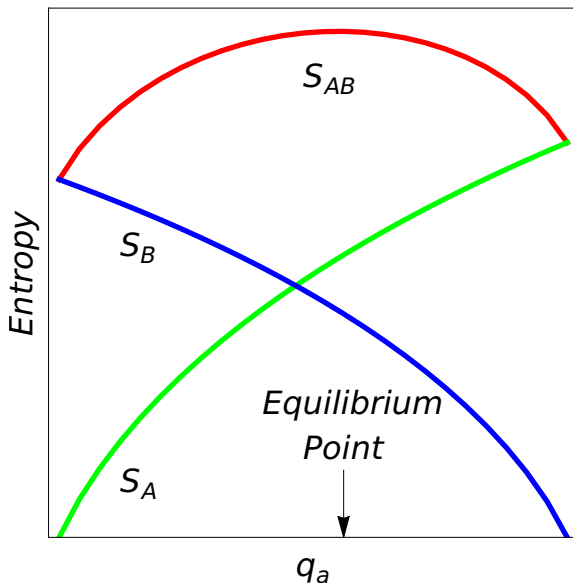


Product Rule $\log(xy) = \log x + \log y$

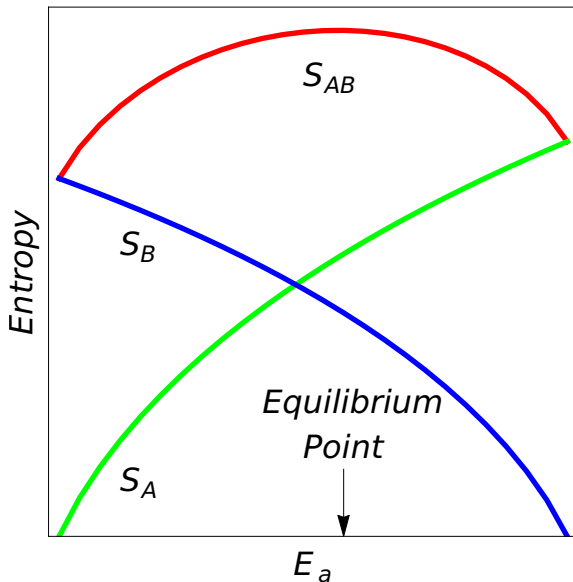
Quotient Rule $\log\left(\frac{x}{y}\right) = \log x - \log y$

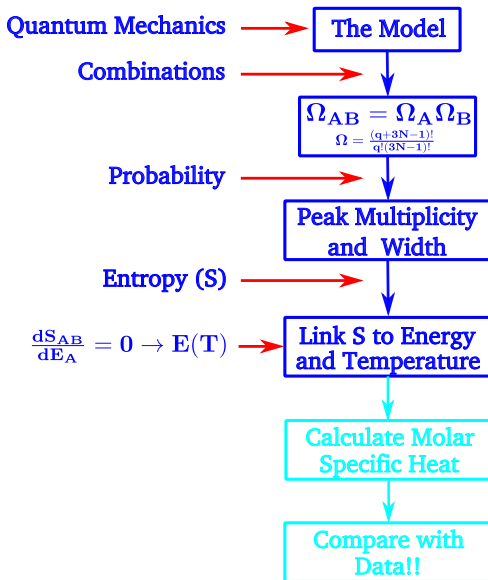
Power Rule $\log(x^p) = p \log x$

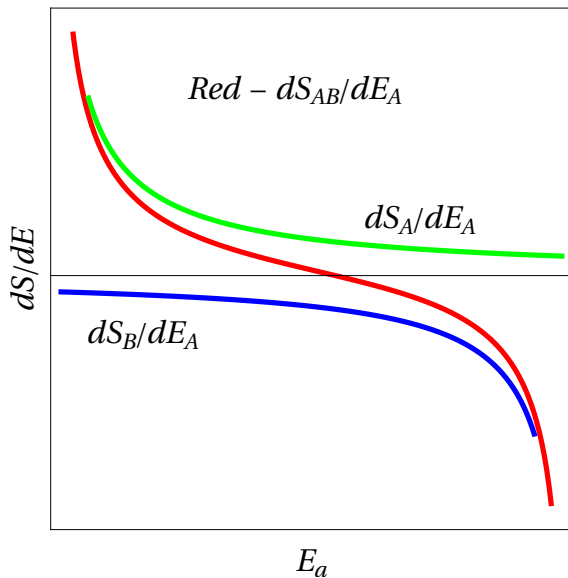
Equality Rule If $\log x = \log y$, then $x = y$

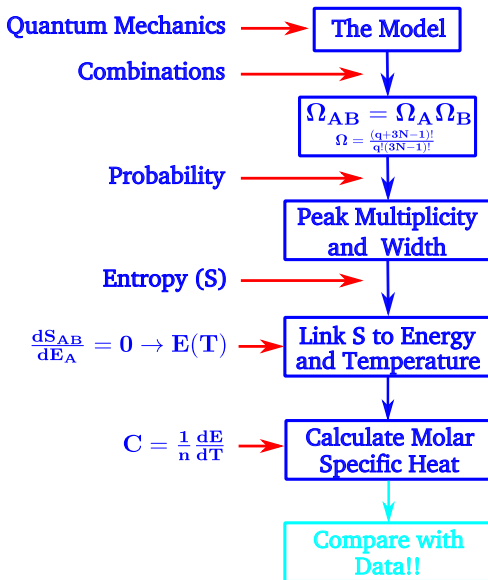


Entropy of Two Einstein Solids in Energy Terms 28



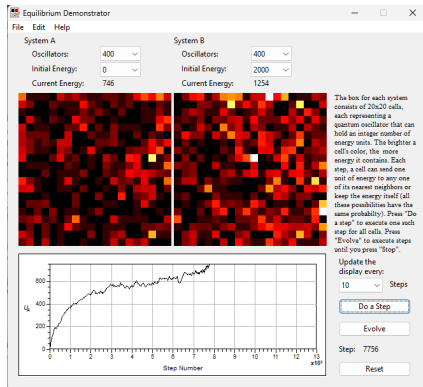
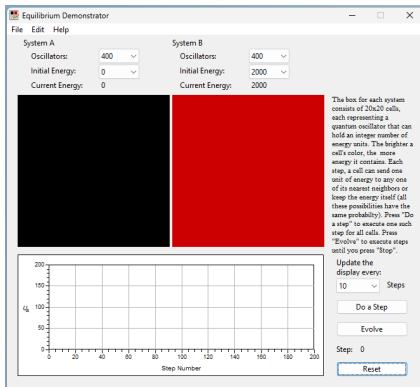


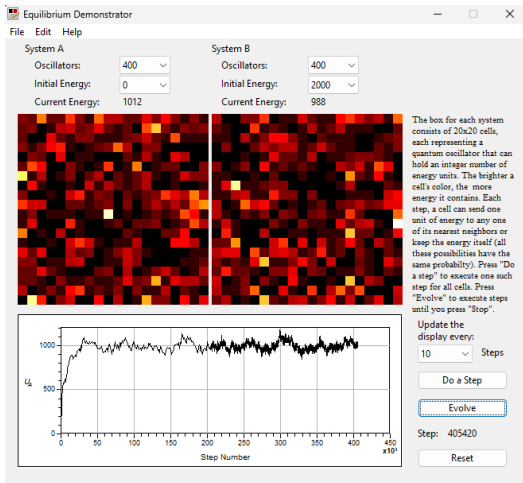


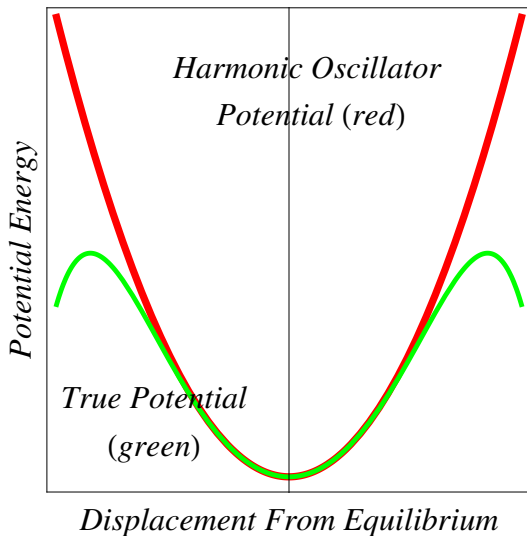


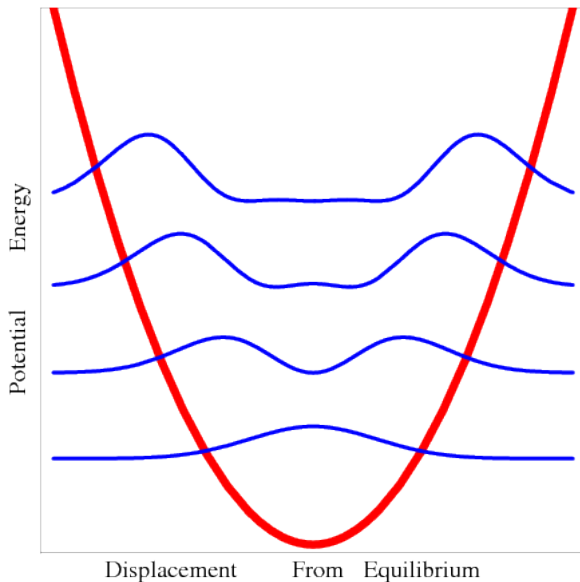
Solid	Measured Molar Specific Heat (J/K-mole)	Our Results (J/K-mole)	Our Calculation (J/K-mole)
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Tin	27.0 ± 0.6	43 ± 8	24.9
Gold	25.4 ± 0.6		24.9
Silver	25.4 ± 0.6		24.9
Iron	25.0 ± 0.6		24.9

Imagine that the entropy of a certain substance as a function of N and E is given by the formula $S = \alpha N k_b E^3$. Using the definition of temperature, find an expression for the thermal energy E of this substance in terms of its temperature T , the number of particles N , and any other necessary constants. Use this result to calculate the molar specific heat. Are these results well behaved?









Installing Capstone

- 1 Go to the website

<https://www.pasco.com/downloads/capstone>

and select the free trial for your platform (Windows or Mac). The installer will be downloaded to your machine.

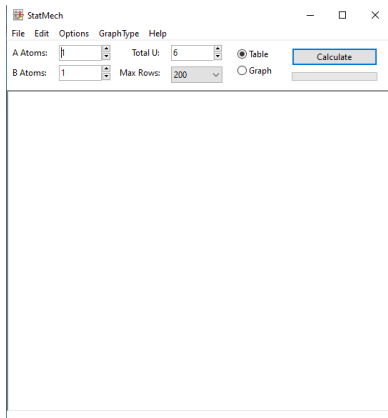
- 2 Launch the installer you just downloaded.
- 3 Accept defaults.
- 4 On first launch, enter the license key listed below.

19F5C-S10o2-4o0m0-ppip3-40qr8-ece1h

- 5 The capstone files for each lab will be linked to the lab schedule on the course website at the following location.

<https://facultystaff.richmond.edu/~ggilfoyl/genphys.html>

- 1 Go to: <http://www.physics.pomona.edu/sixideas/old/sicpr.html>
- 2 Scroll down to the section entitled “For Use With Unit T:”.
- 3 Scroll down to the paragraph that starts with “statmech 2.7”.
- 4 Scroll down to “Download for:” and right click on “Windows” or “Mac OSX” and save it to your Desktop.
- 5 On your desktop double click on the folder entitled “statmech.exe.zip”. You should see a list of the contents of the folder.
- 6 Click the “Extract All” button and then choose the Desktop (if it’s not already set) to place the files.
- 7 Double click on “*statmech.exe*” and you will now see the contents of the folder with the application.
- 8 Double click on “*statmech.exe*”. You will get a GUI like the one shown here.
- 9 You’re off.



- 1 Go to: <http://www.physics.pomona.edu/sixideas/old/sicpr.html>
- 2 Scroll down to the section entitled “For Use With Unit T:”.
- 3 Scroll down to the paragraph that starts with “Equilib 2.1”.
- 4 Scroll down to “Download for:” and right click on “Windows” and save it to your Desktop.
- 5 Double click on the folder entitled “Equil.exe.zip”. You should see a list of the contents of the folder.
- 6 Double click on “*Equilib.exe*”. You should get a GUI telling you the application may depend on other compressed files in the folder. Click the “Extract All” button and then choose the Desktop to place the files.
- 7 Double click on “*Equilib.exe*” and you will now see the contents of the folder with the application.
- 8 Double click on “*Equilib.exe*”. You will get a GUI worrying about the publisher.
- 9 Click “Run” and you’re off.