The Twins Paradox

Consider two twins. One sets out at the age of 25 on a spaceship from Earth at a speed of $0.99c$ where c is the speed of light. The Earthbound twin goes on about his business accumulating the normal acoutrements of advancing age (gray hair, drooping body parts, *etc.*). After twenty years have passed for the Earthbound twin, the spacefaring one returns. When they finally meet the voyager is NOT twenty years older! He looks only a few years older than when he left and shows few signs of age. How much has he aged during his journey?

Time Dilation

Electrons at the speed of light.

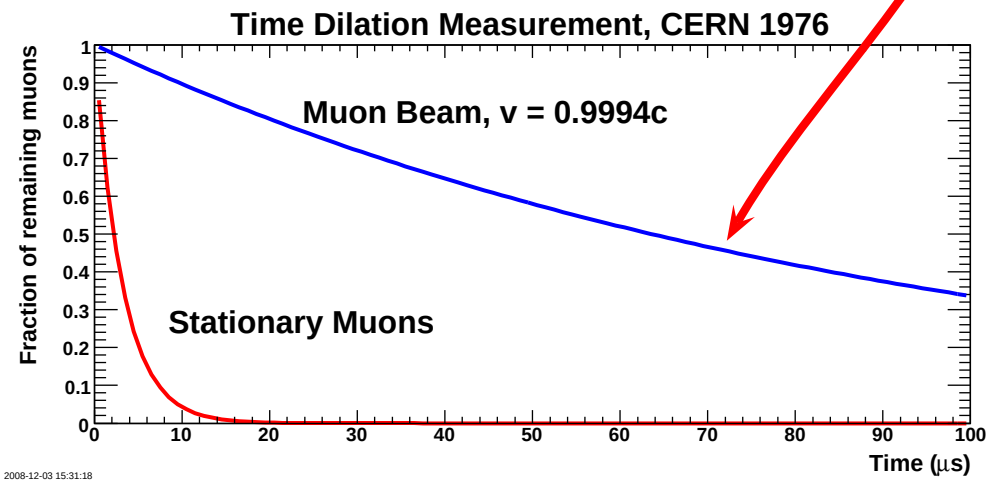
FERMIONS matter constituents spin = 1/2, 3/2, 5/2, ...

Leptons spin = 1/2

Flavor	Mass GeV/c^2	Electric charge
ν_e electron neutrino	$<1\times10^{-8}$	0
e electron	0.000511	−1
ν_μ muon neutrino	<0.0002	0
μ muon	0.106	−1
ν_τ tau neutrino	<0.02	0
τ tau	1.7771	−1

Quarks spin = 1/2

Flavor	Approx. Mass GeV/c^2	Electric charge
u up	0.003	2/3
d down	0.006	−1/3
c charm	1.3	2/3
s strange	0.1	−1/3
t top	175	2/3
b bottom	4.3	−1/3



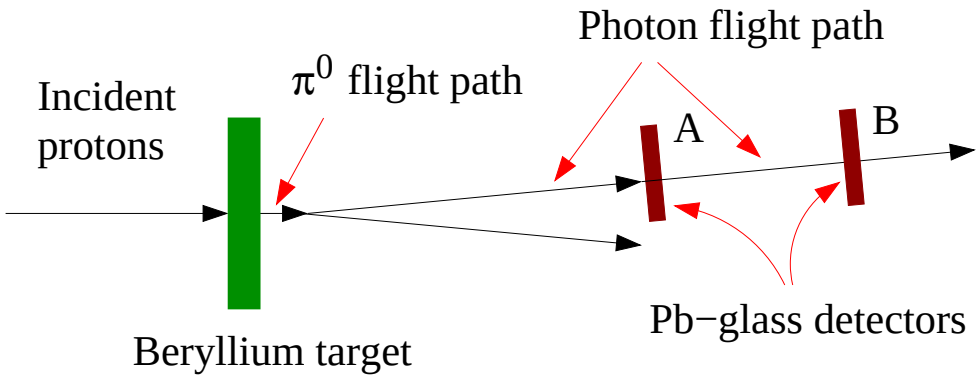
Muon half-life: $2.2 \times 10^{-6} s$

The Postulates

1. Physics is the same in all inertial reference frames (hopefully).
2. The speed of light is the same in all inertial reference frames.

Testing The Second Postulate

1. Get on a very fast train. - At CERN in 1964 T. Alvager *et al.* created a beam of π^0's moving close to the speed of light ($0.99975c$) by hitting a beryllium target with a high-energy proton beam.
2. The π^0's almost immediately decayed into particles of light called photons ($t_{1/2} = 8.64 \times 10^{-17}\ s$).
3. The photons were measured at different, known locations downstream from the target.
4. $c' = (2.9977 \pm 0.0004) \times 10^8\ m/s$ versus $2.99792458 \times 10^8\ m/s$.



Alvager et al, CERN, 1964

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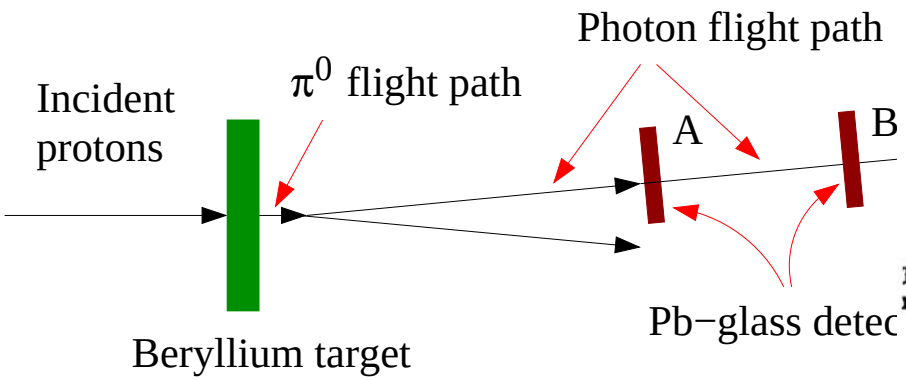

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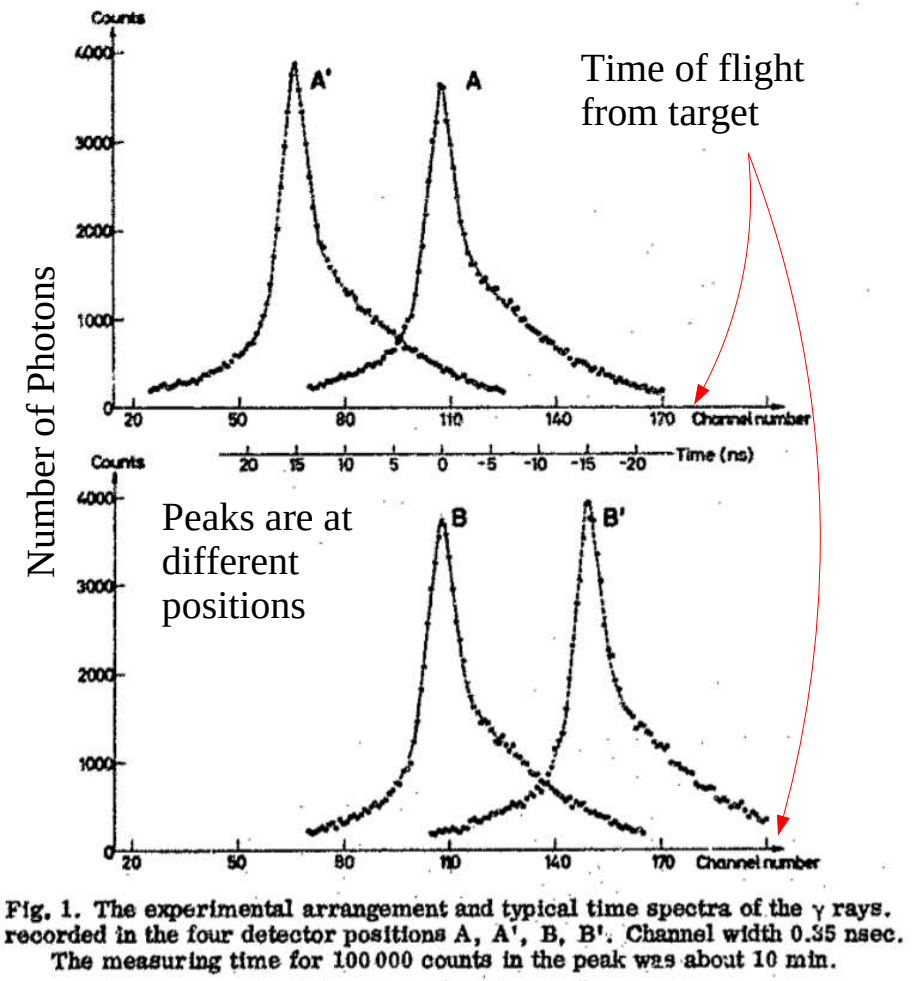
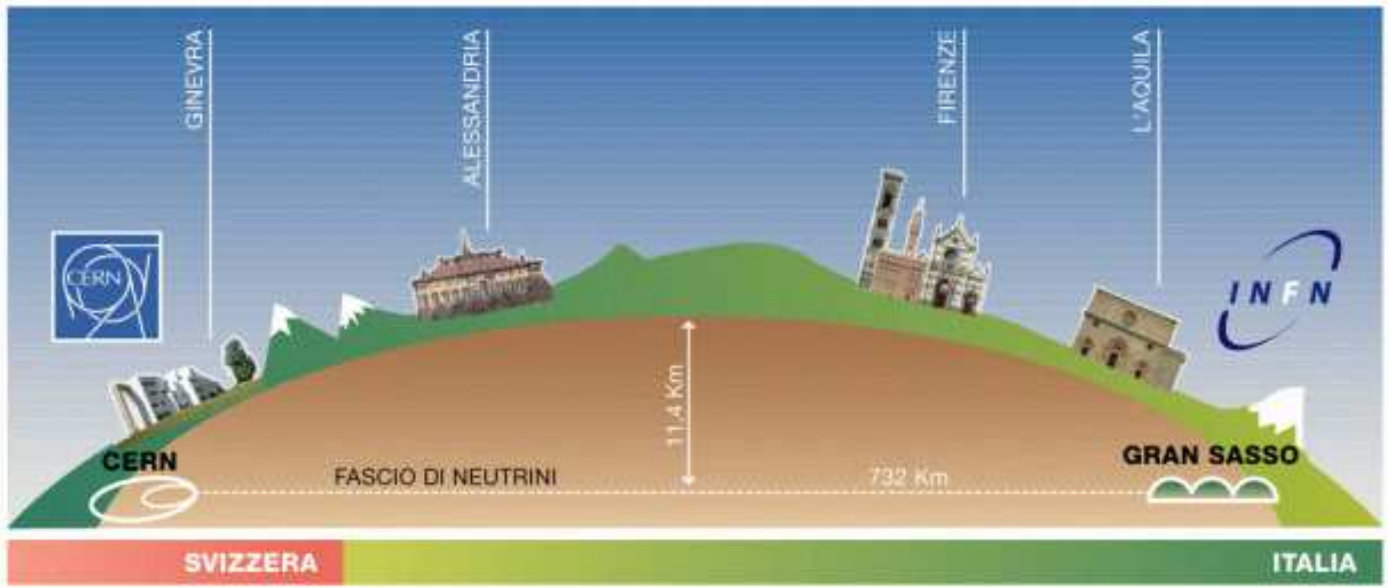


Fig. 1. The experimental arrangement and typical time spectra of the γ rays recorded in the four detector positions A, A', B, B'. Channel width 0.35 nsec. The measuring time for 100 000 counts in the peak was about 10 min.

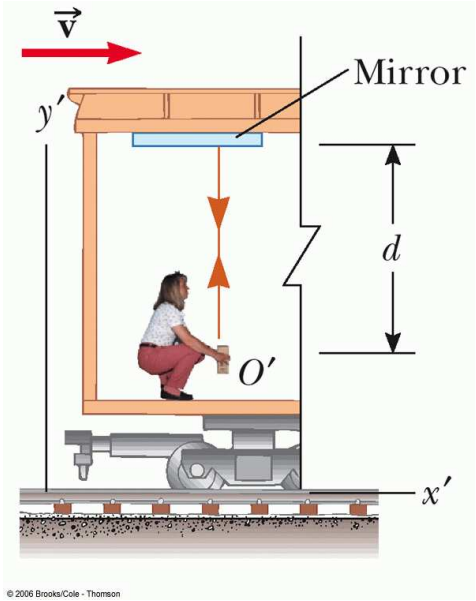
T.Alvager *et al.*, Phys. Lett. 12, 260 (1964)

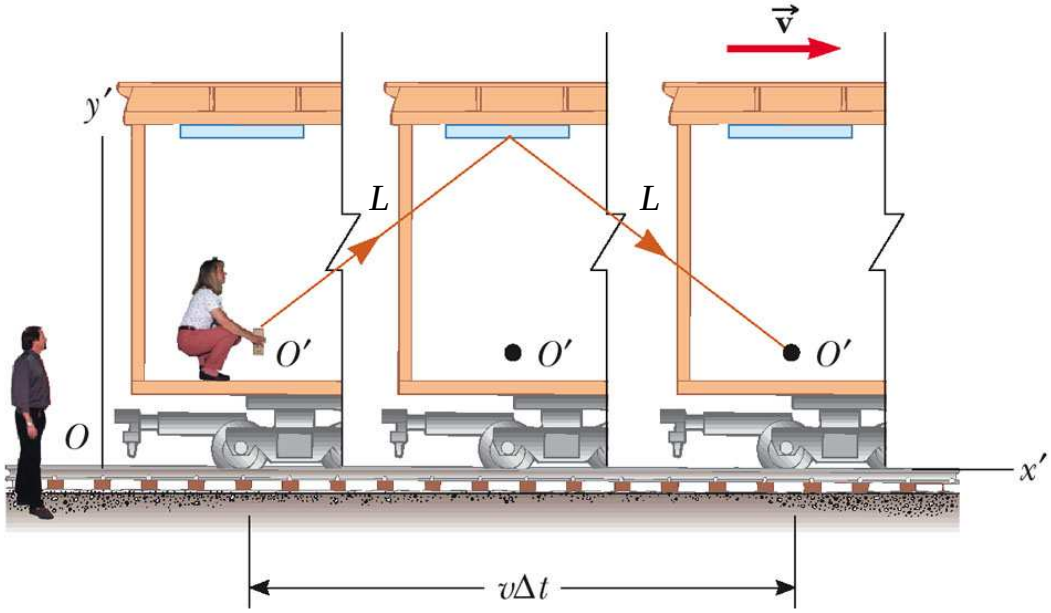
The OPERA results

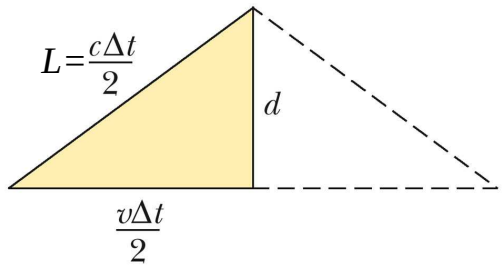
1. A recent measurement of the speed of sub-atomic particles at CERN by the OPERA Collaboration (Oscillation Project with Emulsion-tRacking Apparatus) measured neutrinos traveling slightly faster than the speed of light.
2. The Theory of Special Relativity established the speed of light in vacuum as an upper limit and has passed all previous tests made during the last 106 years.
3. High-energy protons strike a graphite target producing tau neutrinos. The protons (and many of the neutrinos) are aimed at an underground detector in San Grasso, Italy $743\ km$ away.
4. $TOF_{th} - TOF_{exp} = 57.8 \pm 7.8(\text{stat})^{+8.3}_{-5.9}(\text{syst})\ \text{ns}.$



Time Dilation







Evidence for Time Dilation

1. In 1971 Hafele and Keating at the old National Bureau of Standards (now National Institute for Standards amd Technology) took four cesium-beam atomic clocks aboard commercial airliners and flew twice around the world, first eastward, then westward, and compared the clocks against those of the United States Naval Observatory.

	nanoseconds gained			
	predicted			measured
	gravitational (general relativity)	kinematic (special relativity)	total	
eastward	144 ± 14	-184 ± 18	-40 ± 23	-59 ± 10
westward	179 ± 18	96 ± 10	275 ± 21	273 ± 7

2. Mountaintop muon decay measurements.
3. Electron beam at JLab.
4. GPS and Countless others.

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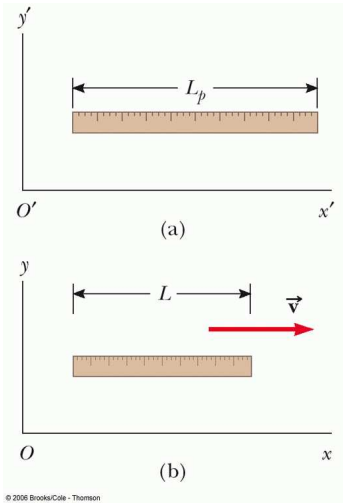
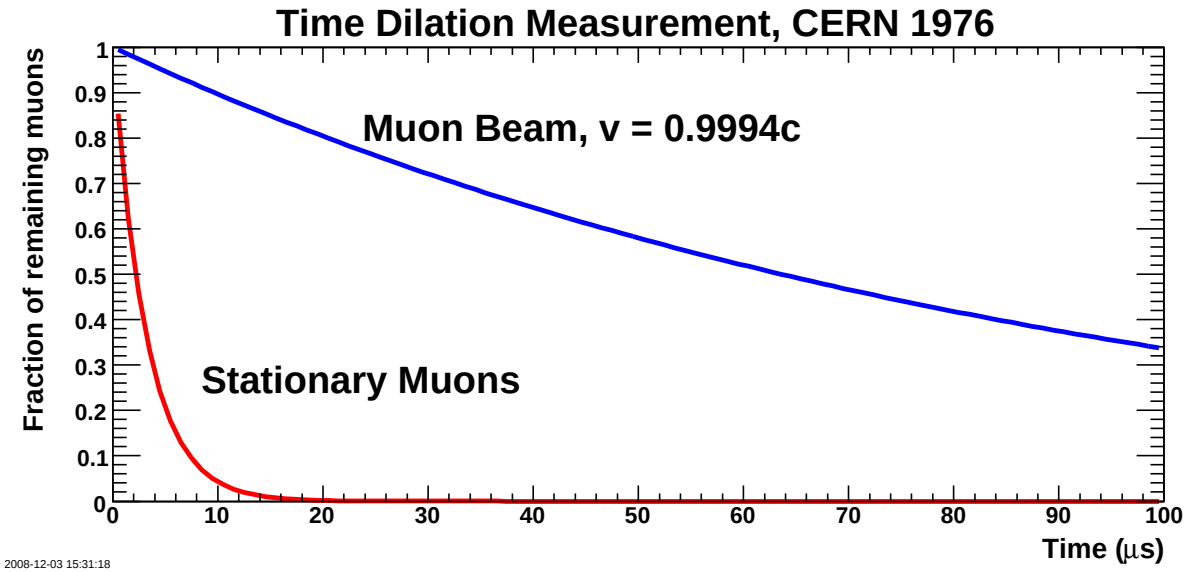
Another Twins Paradox (Length Contraction)

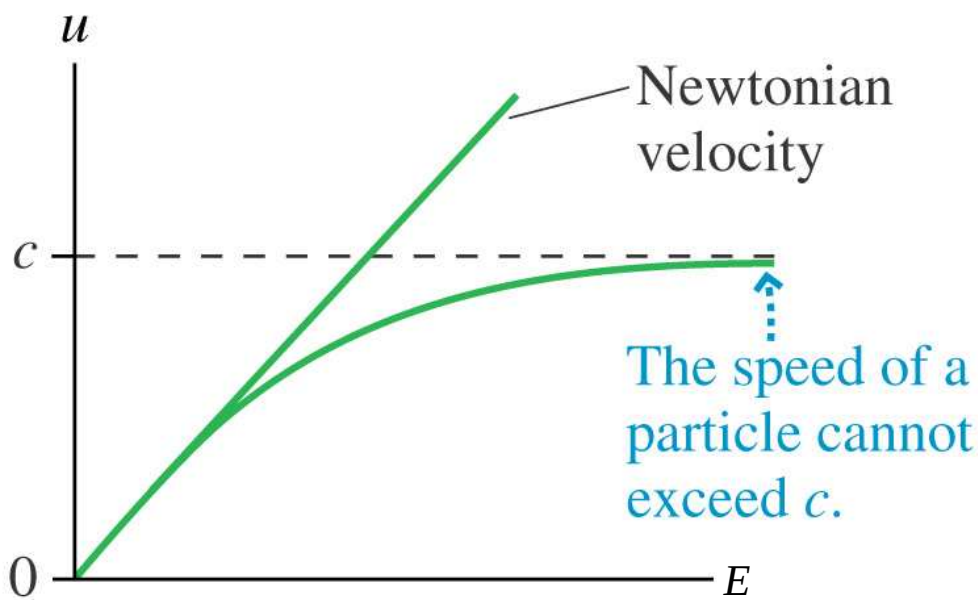
Consider the two twins again. One sets out at the age of 25 on a spaceship from Earth at a speed of $0.99c$ where c is the speed of light. After twenty years have passed for the Earthbound twin, the spacefaring one returns. What is the mileage on the spacefaring twin's spaceship? In other words, what distance did he measure in traveling outward from the Earth at 0.99c, turning around at the midpoint of his trip, and returning directly to Earth?





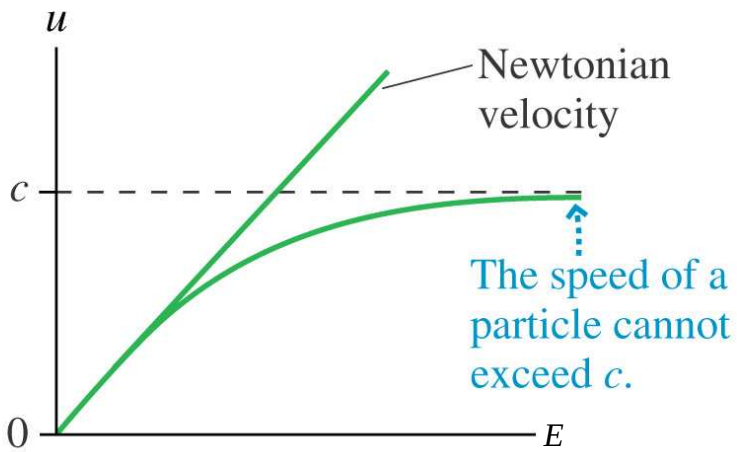
Relativistic Energy

$$E = m_R c^2 = \frac{mc^2}{\sqrt{1 - \frac{v^2}{c^2}}}$$



Relativistic Particles

An electron is accelerated to an energy $E = 6\ \mathrm{GeV}$ where $1\ \mathrm{GeV} = 10^9\ \mathrm{GeV}$ at the Thomas Jeffeson National Accelerator Facility in Newport News. What is the electron's speed, relativistic mass, and kinetic energy?



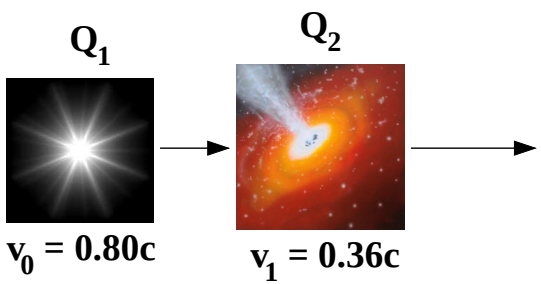
Adding Relativistic Velocities

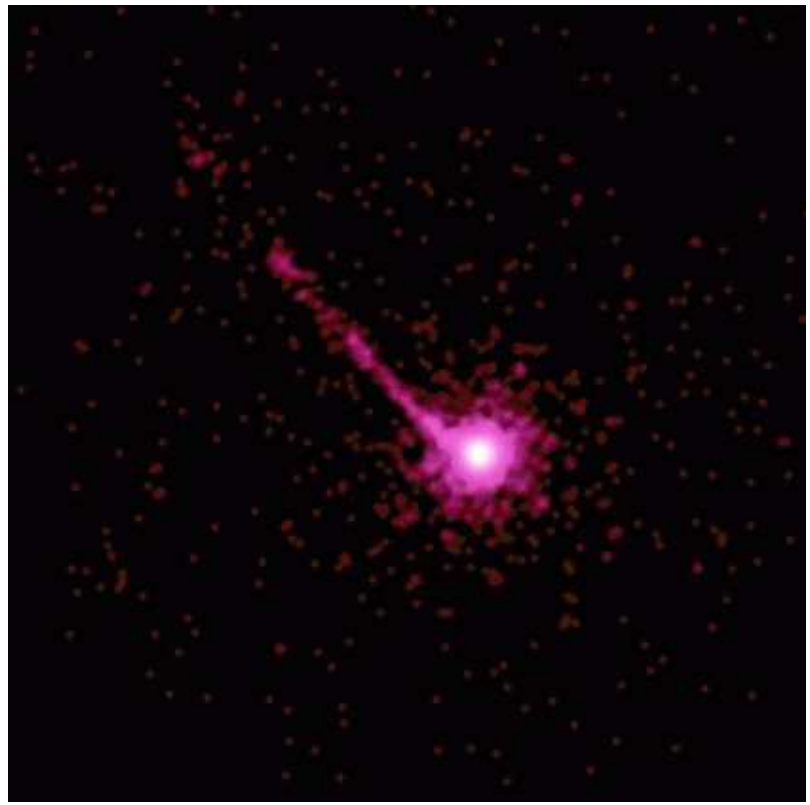
A fast-moving train with speed $v_0 = 2.5 \times 10^8\ m/s$ passes an observer standing on the ground. A girl on the train kicks a soccer ball at her big brother sitting in front of her with a speed $v_1 = 10^8\ m/s$ as measured by her father (much to his horror!). What speed does the stationary observer measure for the speed v_2 of the thrown ball?

Addition of Velocities

Quasars are galaxies in the early throes of birth (we think). They have been observed to be receding from us at high speeds and at great distances. Quasar Q_1 is found to have a recessional velocity $v_0 = 0.80c$ where c is the speed of light. An alien who lives in galaxy Q_1 measures the speed of a nearby galaxy Q_2 to be velocity $v_1 = 0.36c$ along approximately the same line of sight as measured from Earth. What is the speed v_2 of galaxy Q_2 as measured by an observer on the Earth?

Earth

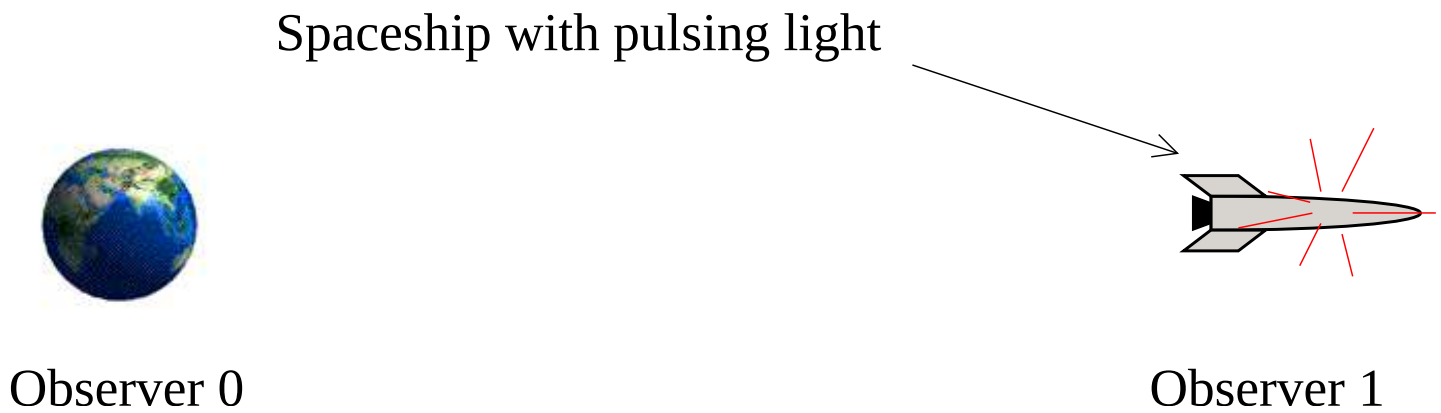




X-ray image of the quasar PKS 1127-145 10 billion light years from Earth. The jet is at least a million light years from the quasar.

The Universal Speed Limit (Part 1)

A spaceship (Observer 1 in the figure) is moving away from an Earth-bound observer (0) at a high speed v_0 as measured by Observer 0. It emits a periodic light pulse the observer on the Earth (0) detects. The time between pulses measured by Observer 1 is Δt_1. The time between pulses measured by Observer 0 is Δt_0. How is Δt_0 related to Δt_1?

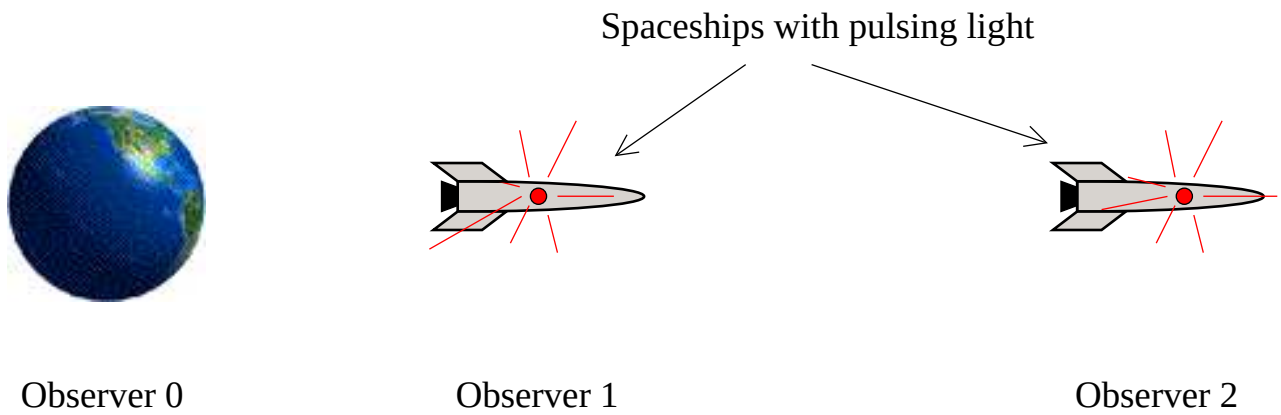


The Universal Speed Limit (Part 2)

Two spaceships (1 and 2 in the figure) are moving away from an Earth-bound observer (0) at different speeds. The fast, lead ship (2) emits a periodic light pulse the observer on the second, slow ship (1) receives and immediately relays to Earth (0). The speeds and time intervals are defined below.

v_0: speed of 1 from 0	Δt_0: time interval on 0
v_1: speed of 2 from 1	Δt_1: time interval on 1
	Δt_2: time interval on 2
v_2: speed of 2 from 0	

1. How is Δt_0 related to Δt_1?
2. How is Δt_1 related to Δt_2?
3. How is Δt_0 related to Δt_2?
4. What is v_2 in terms of v_0 and v_1?



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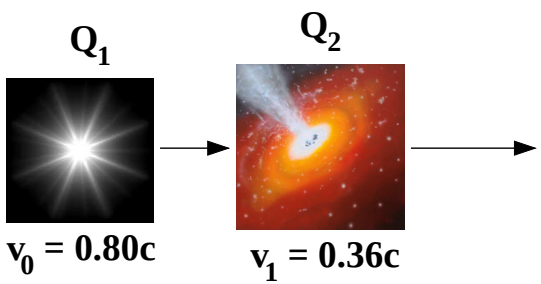

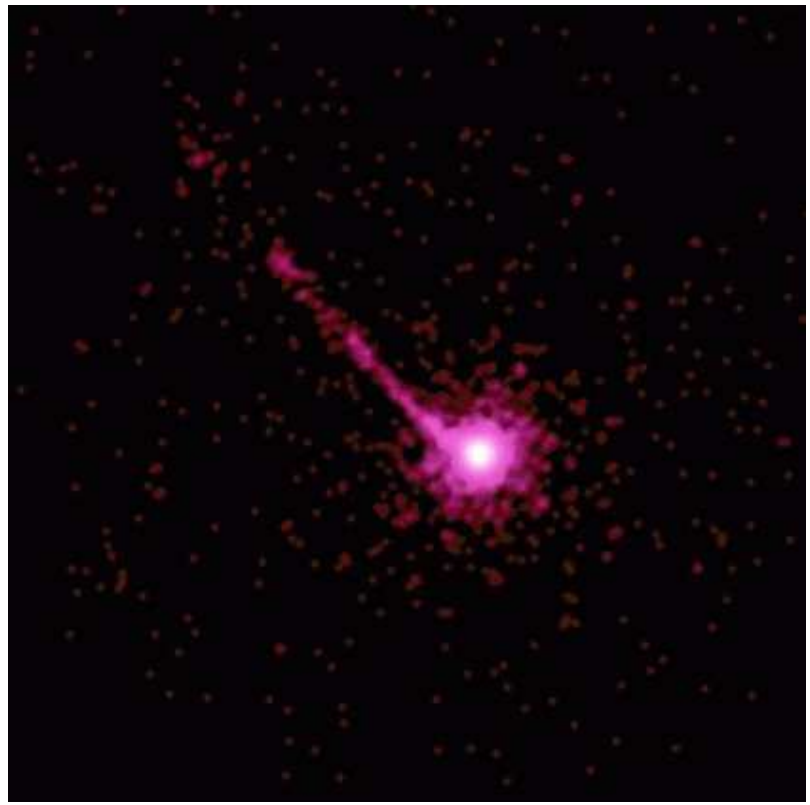

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