A Star Is Born!

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The photograph below shows a cloud of molecules called Bernard 68 (B68). It is located about 300 light-years ($2.8 \times 10^{15}\ km$) away from us in the constellation Ophiuchus and is about 1.6 trillion kilometers across. It is made of molecules like CS, $\mathrm{N_2H}$, $\mathrm{H_2}$, and CO and is slowly rotating ($\omega = 9.4 \times 10^{-14}\ rad/s$). The internal gravitational attraction of B68 may make the molecular cloud collapse far enough so it will ignite the nuclear fires and B68 will begin to shine.

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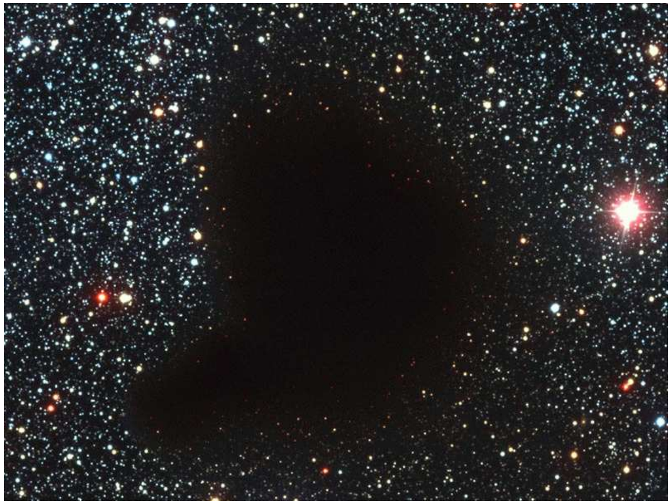
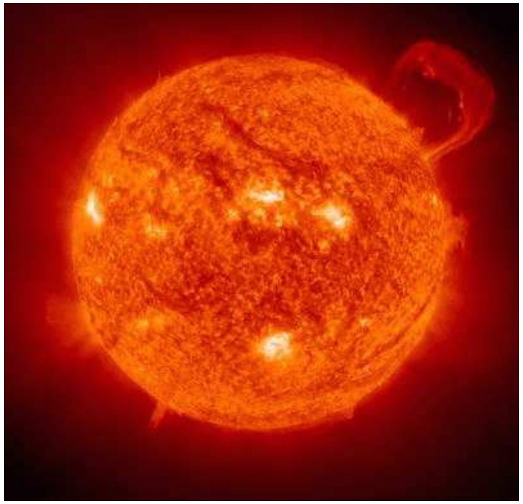
The molecular cloud B68 in the constellation Ophiuchus is rotating with an angular speed $\omega = 9.4 \times 10^{-14}\ rad/s$. The gravitational attraction among the atoms in the cloud may make it collapse until the core is hot enough to ignite nuclear reactions and B68 will begin to shine. If the final properties of B68 are the same as our Sun, *i.e.*, the same mass and size, then what will be its final angular velocity and period? Assume the lost mass carries away very little angular momentum. Compare this with the angular velocity of the Sun. Is your result reasonable? Why or why not?

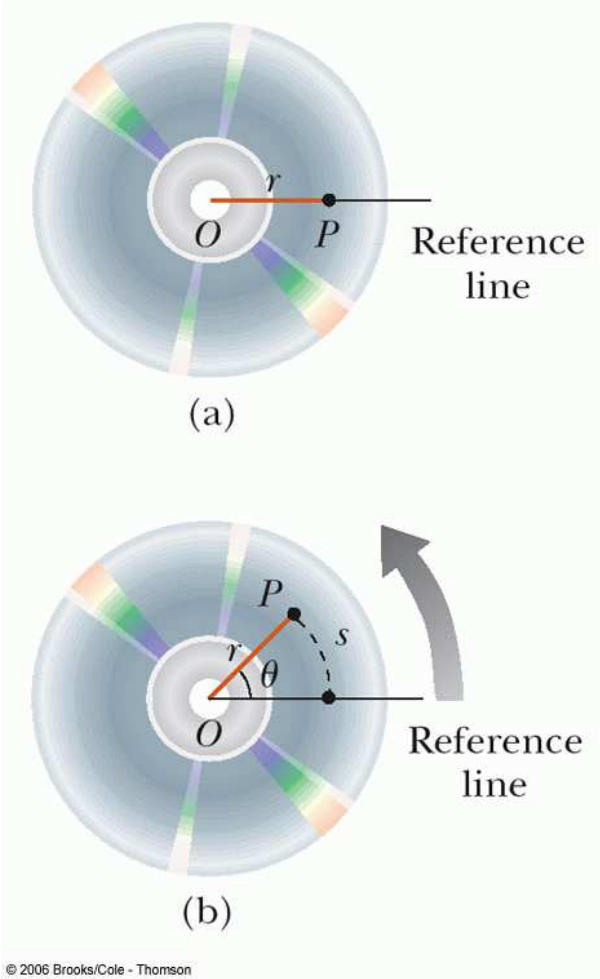
$$M_{B68} = 6.04 \times 10^{30}\ kg \qquad I_{B68} = 2.7 \times 10^{54}\ kg - km^2 \qquad M_{Sun} = 1.989 \times 10^{30}\ kg$$

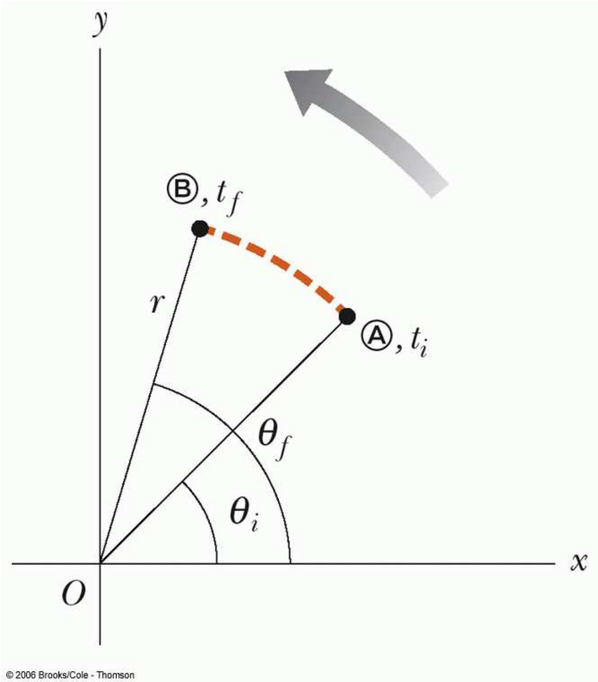
$$R_{Sun} = 6.96 \times 10^{5}\ km \qquad T_{Sun} = 25.4\ d$$





Rotational Quantities



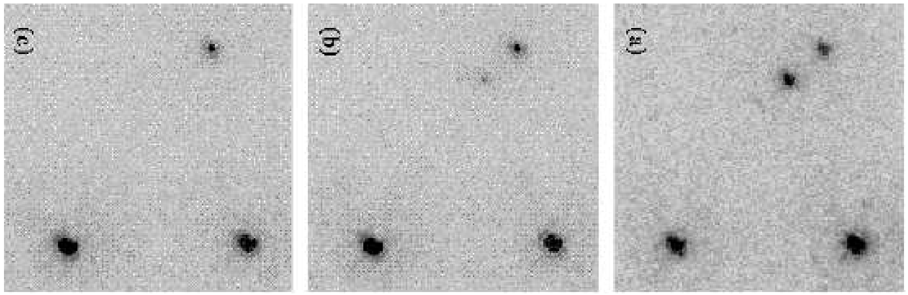


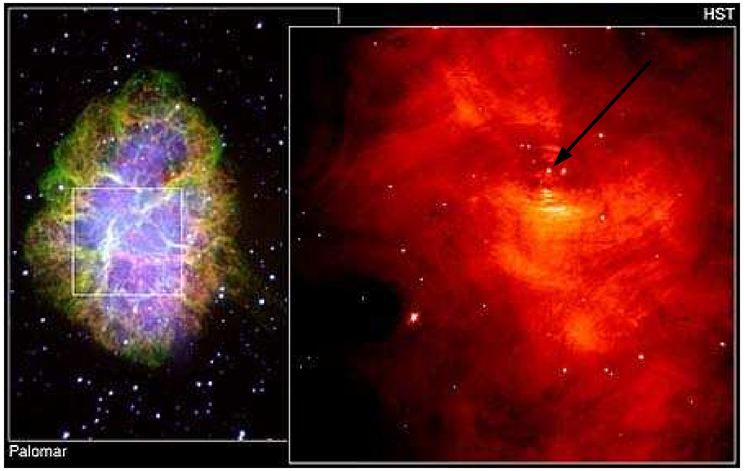
Linear $\rightarrow$ Rotational Quantities

Linear Quantity	Connection	Rotational Quantity
s	$s = r\theta$	$\theta = \frac{s}{r}$
v_T	$v_T = r\omega$	$\omega = \frac{v_T}{r} = \frac{d\theta}{dt}$
a	$a_T = r\alpha$	$\alpha = \frac{a_T}{r} = \frac{d\omega}{dt}$
$KE = \frac{1}{2}mv^2$		$KE_R = \frac{1}{2}I\omega^2$
$\vec{F} = m\vec{a}$	$\tau = rF_\perp$	$\vec{\tau} = I\vec{\alpha}$
$\vec{p} = m\vec{v}$	$\vec{L} = \vec{r} \times \vec{p}$	$\vec{L} = I\vec{\omega}$

How Fast Will the Star Spin?

The pulsar in the Crab nebula has a period $T_0 = 0.033\ s$ and this period has been observed to be increasing by $\Delta T = 1.26 \times 10^{-5}\ s$ each year. Assuming constant angular acceleration what is the expression for the angular displacement of the pulsar? What are the values of the parameters in that expression? What is the torque exerted on the pulsar?



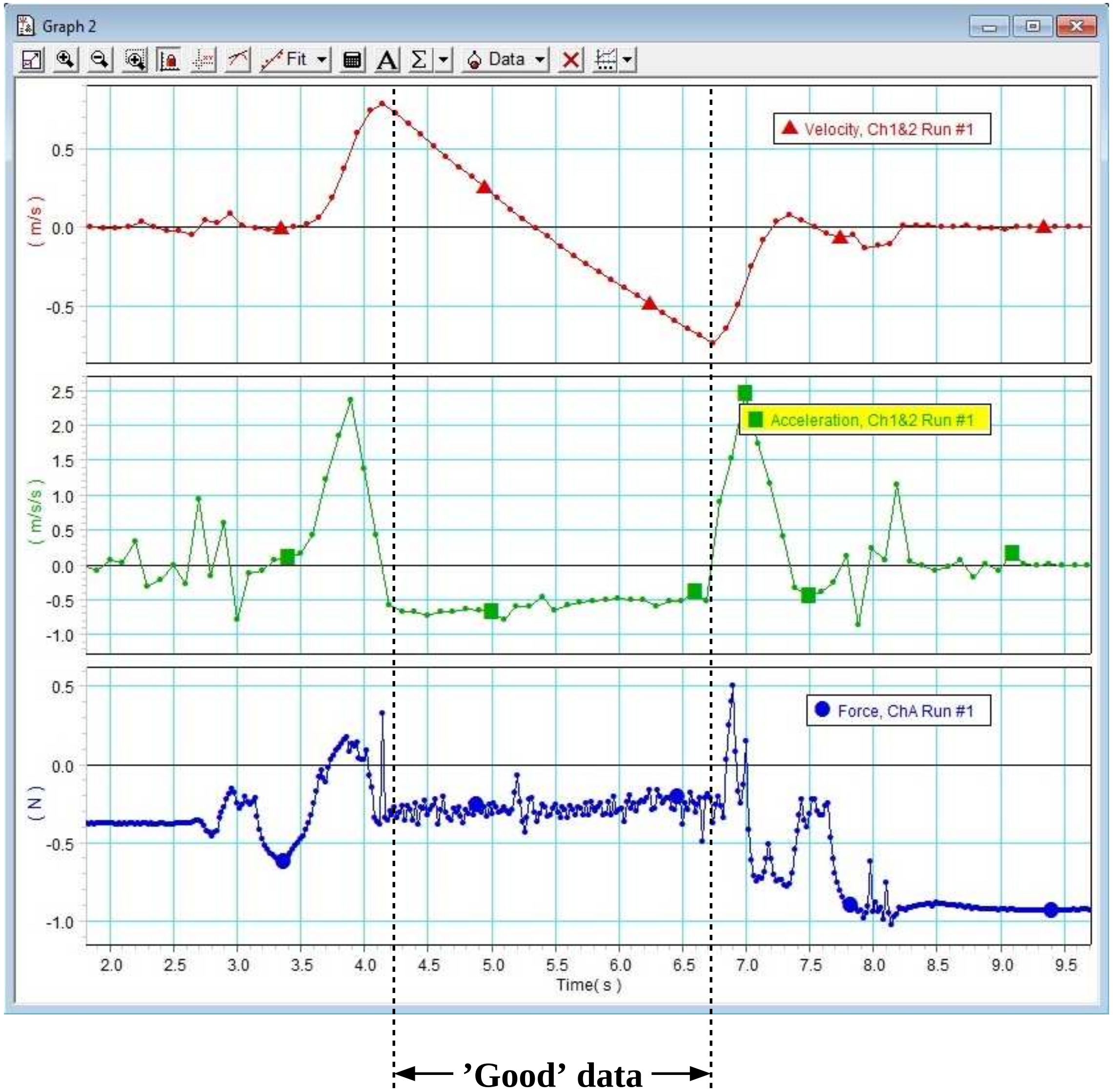


$$m_C = 3.4 \times 10^{30}\ kg$$

$$r_C = 25 \times 10^{3}\ m$$

$$\vec{F} \propto \vec{a} \rightarrow \vec{F} = m\vec{a}$$

Force and Motion 2



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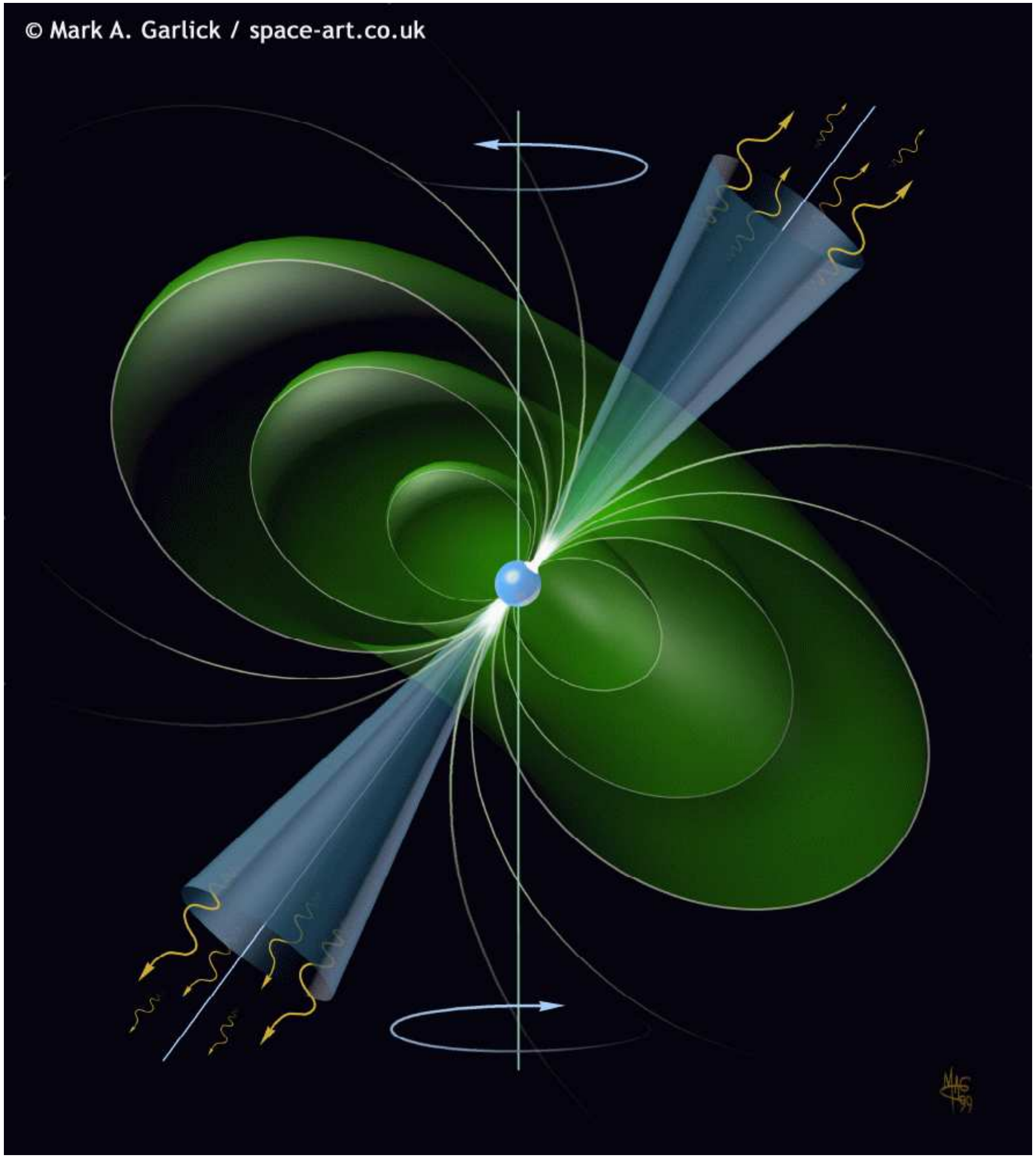
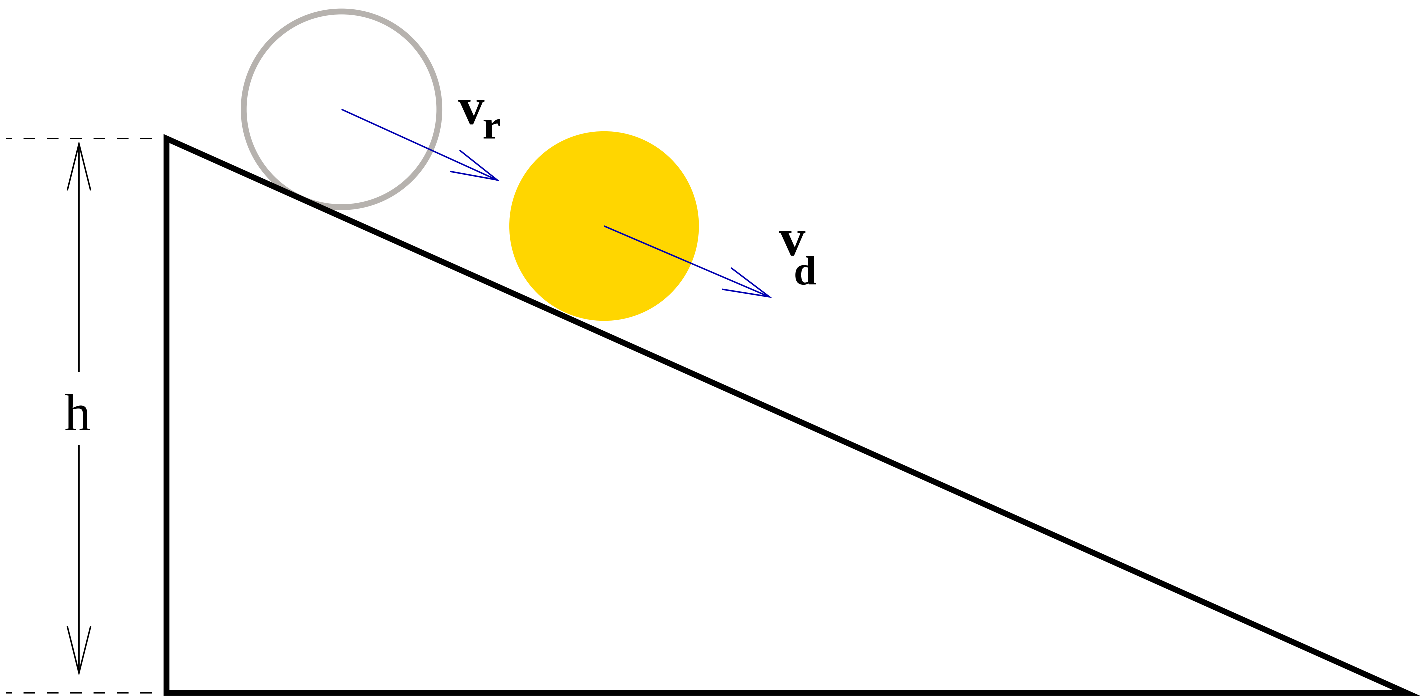
$\vec{p} = m\vec{v}$	$\vec{L} = \vec{r} \times \vec{p}$	$\vec{L} = I\vec{\omega}$

A Pulsar

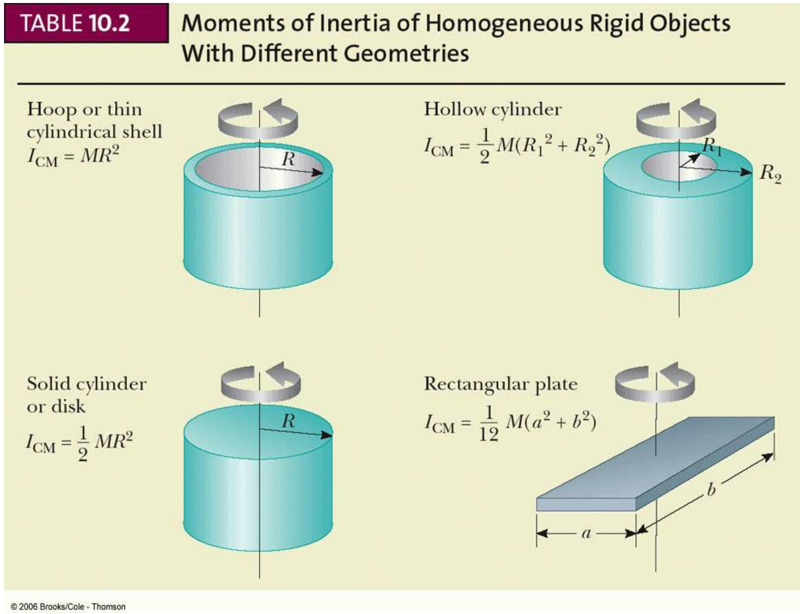


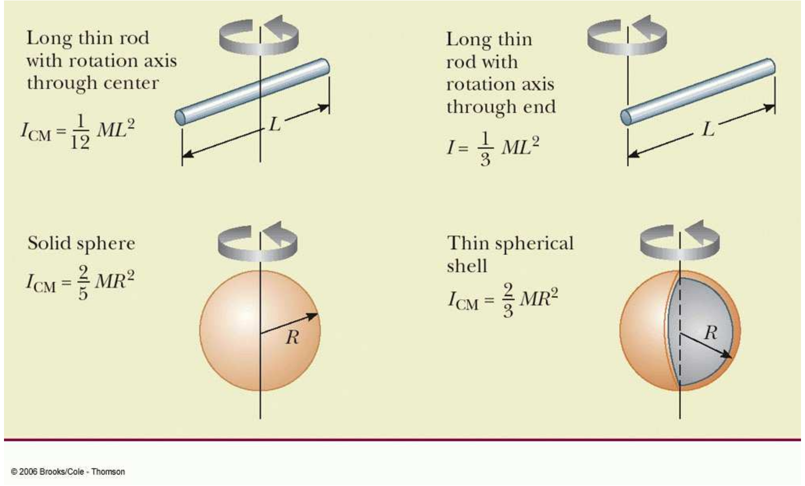
Which One Wins?

A wooden disk and a metal ring have the same mass m and radius r and start from rest and roll down an inclined plane (see figure). What are the kinetic energies at the bottom in terms of the height of the incline h, m, r, and any other constants? Which one is going faster at the bottom of the incline and gets to the bottom in the shortest time?

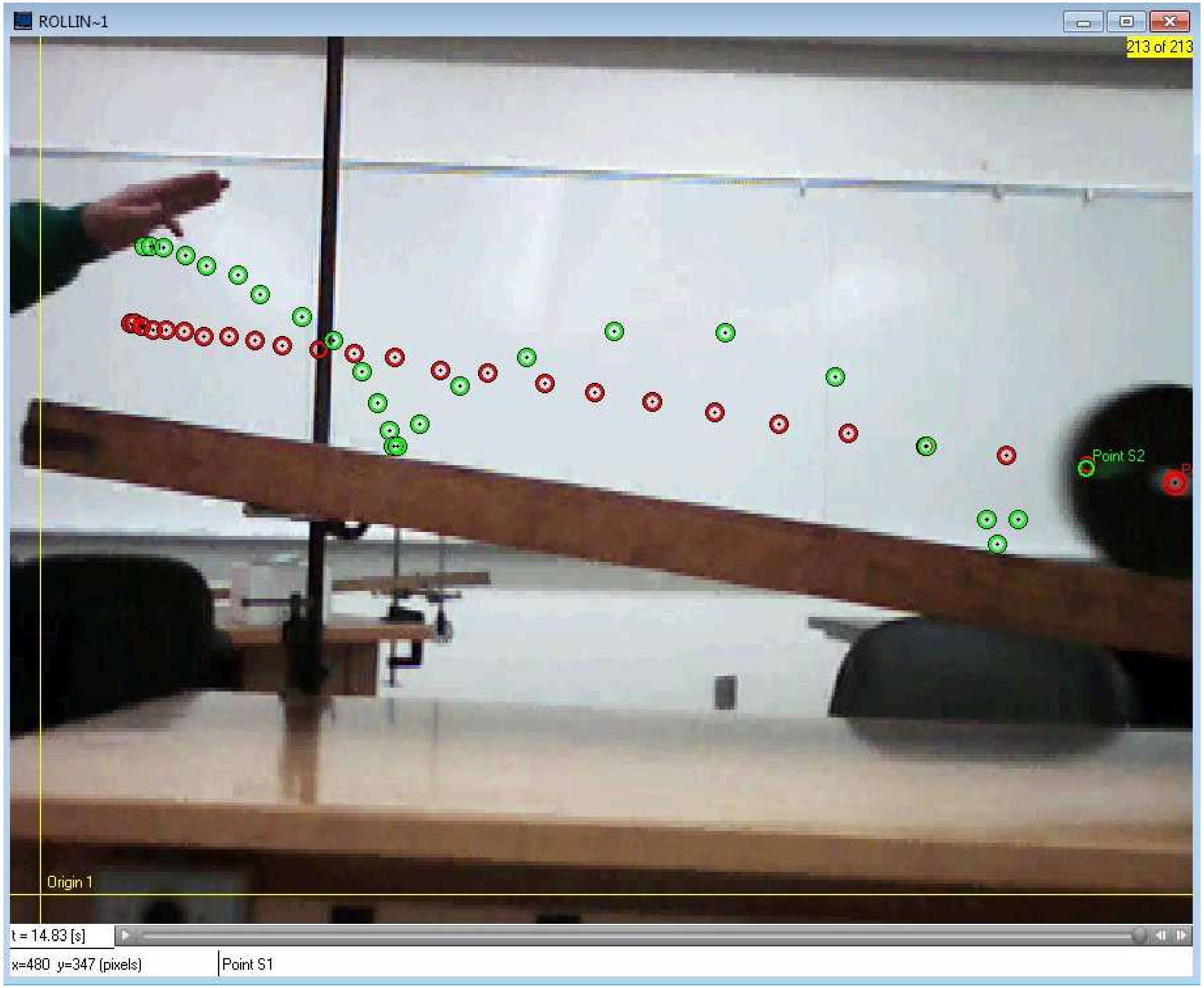


Moments of Inertia

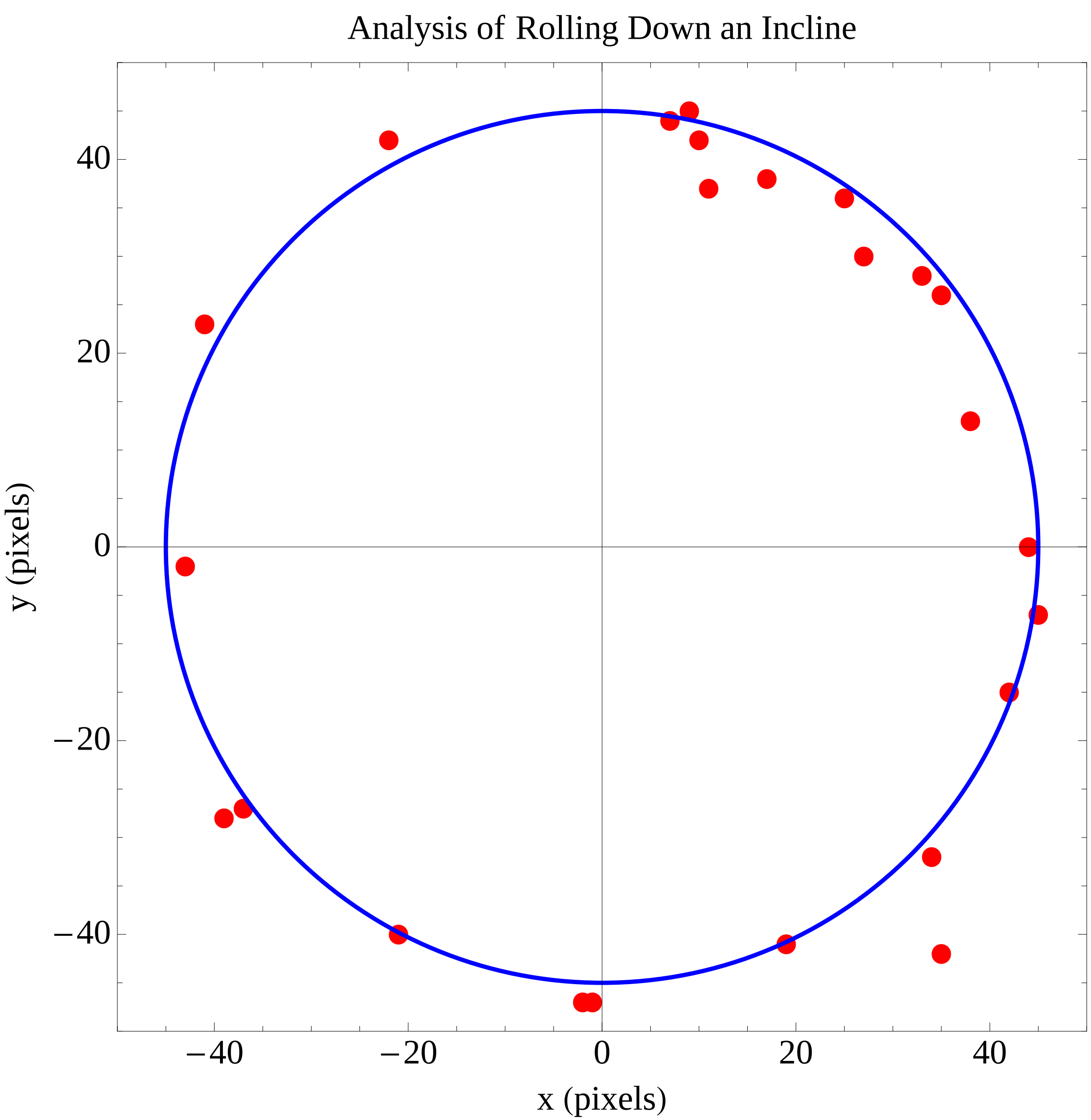




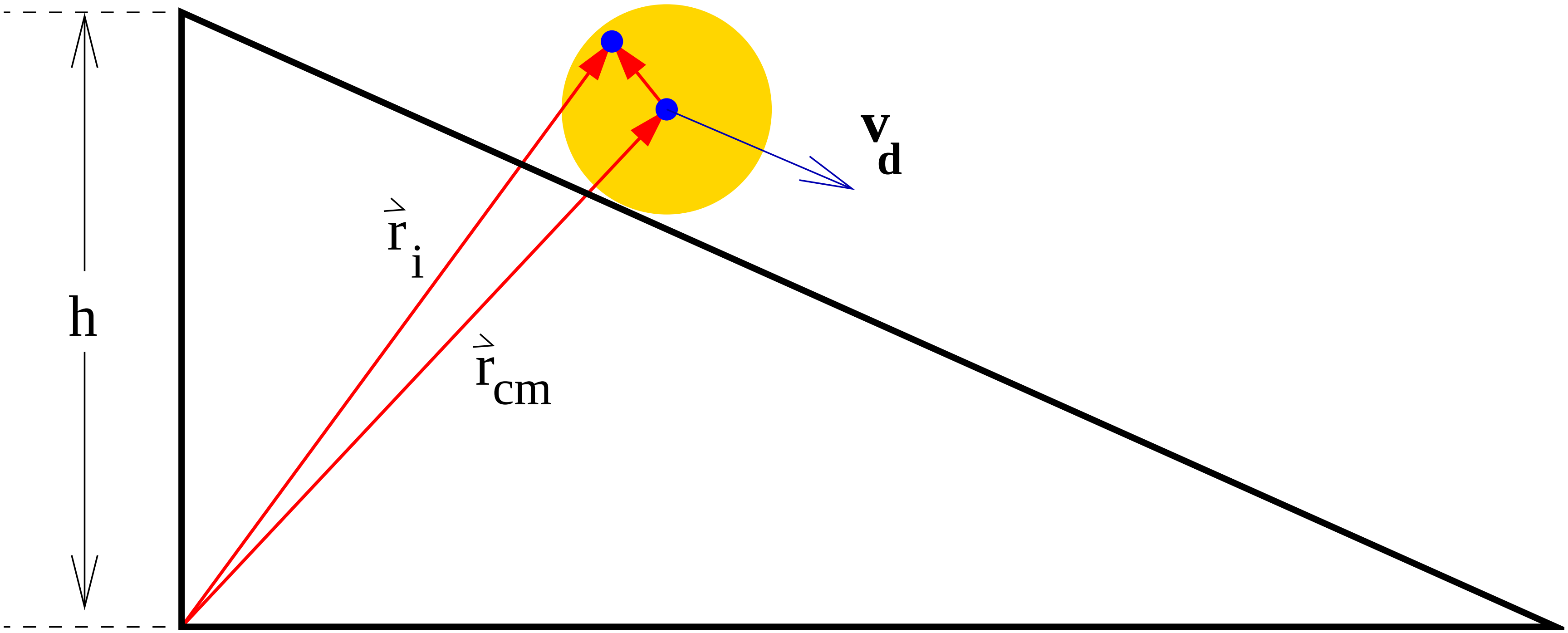
Rolling Down an Incline - 1



Rolling Down an Incline - 2



Rolling Down an Incline - 3



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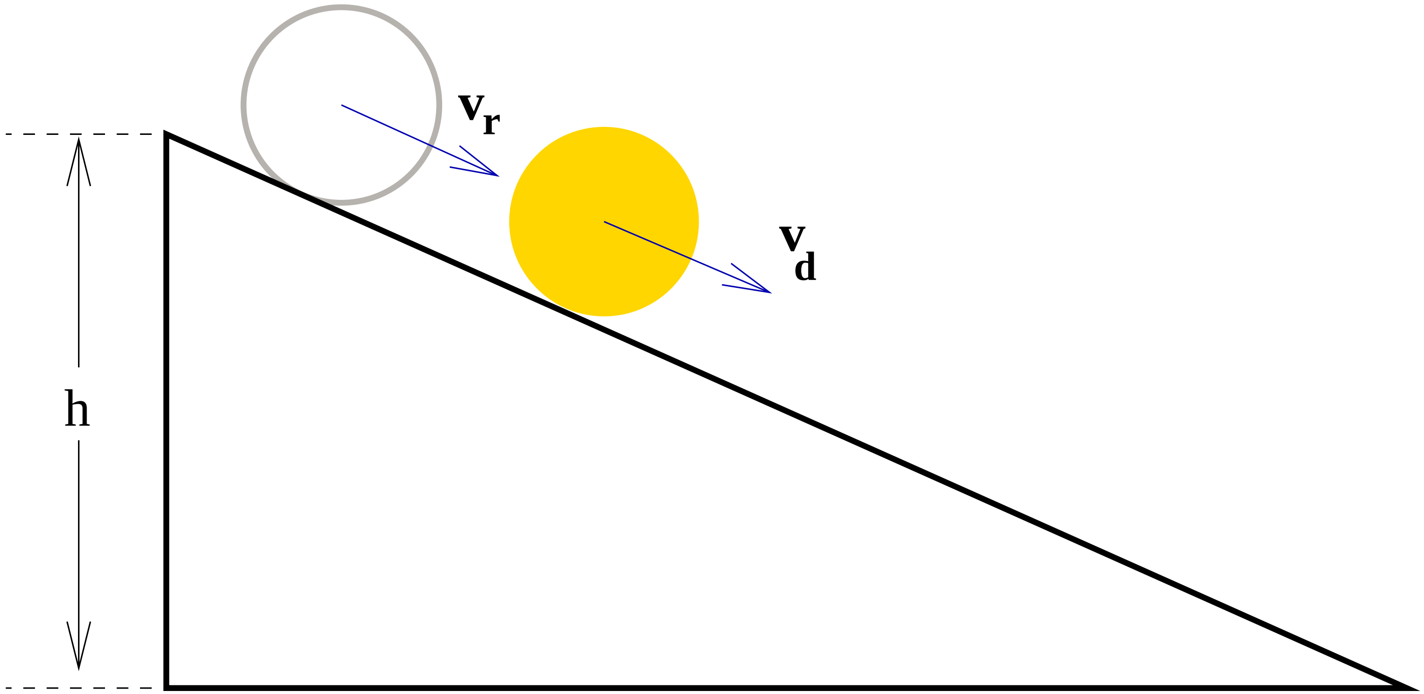
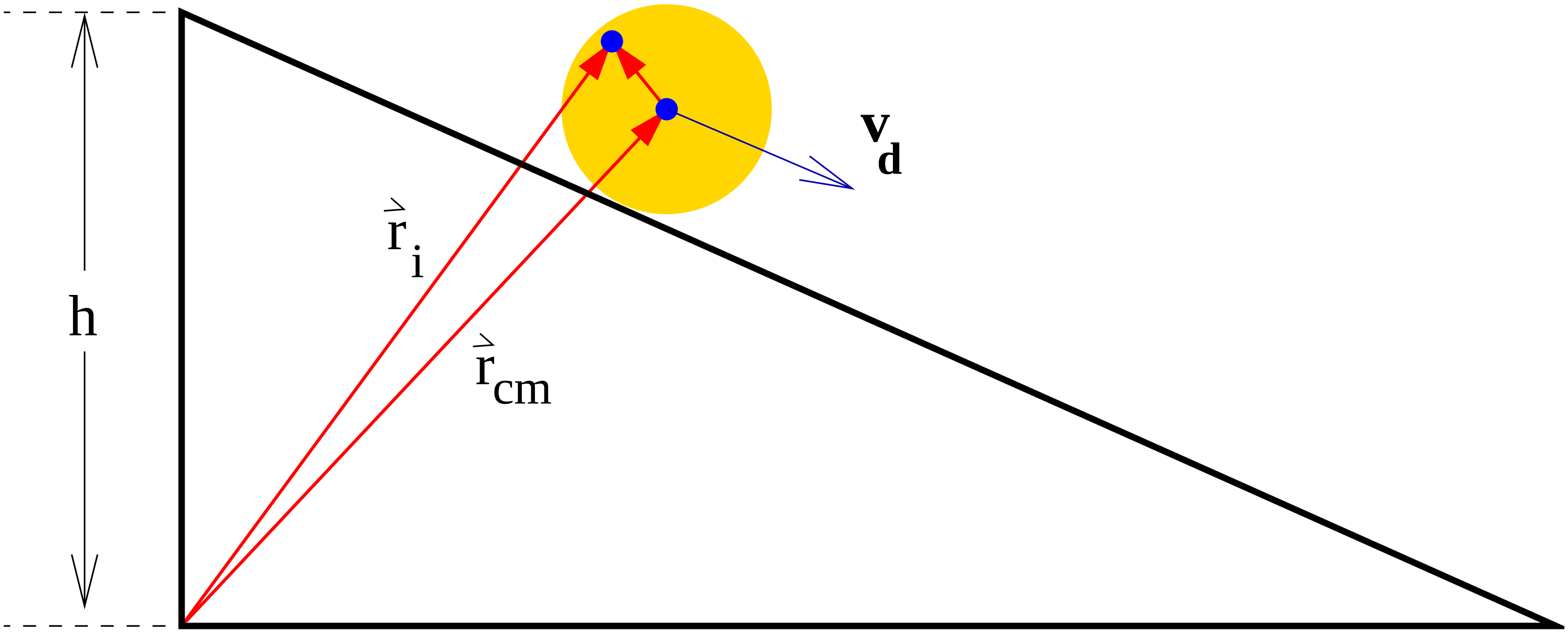


Rolling Down an Incline



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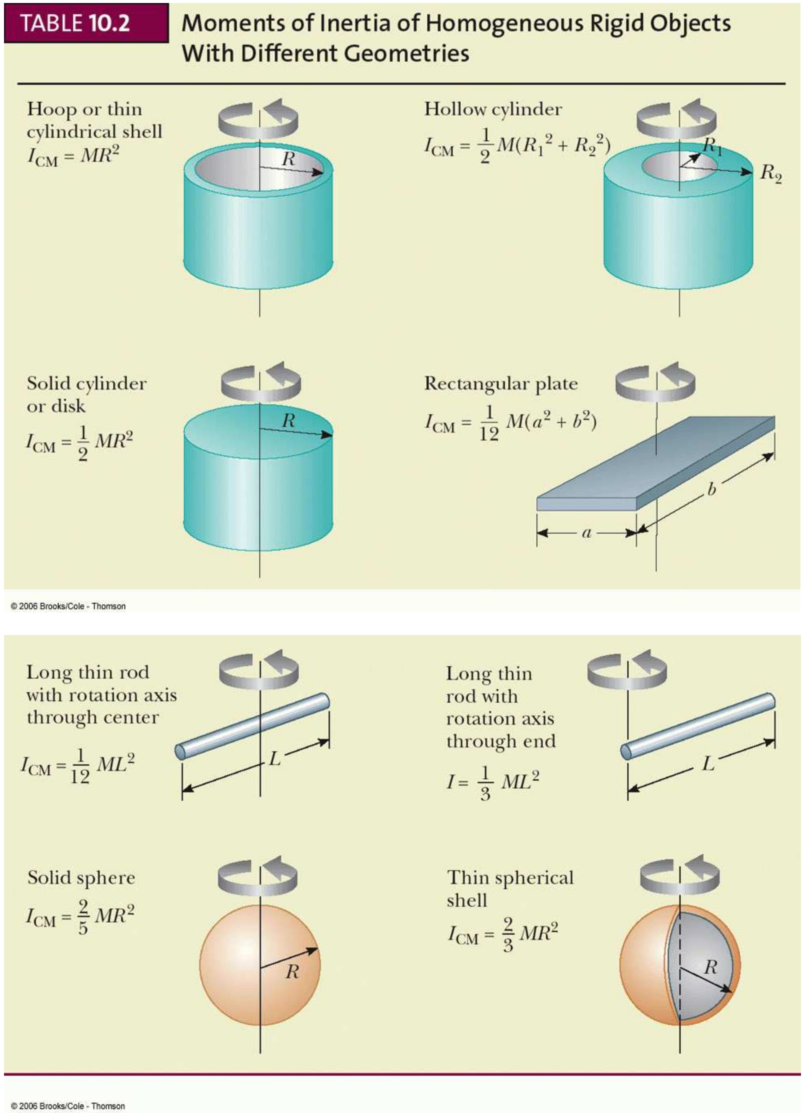
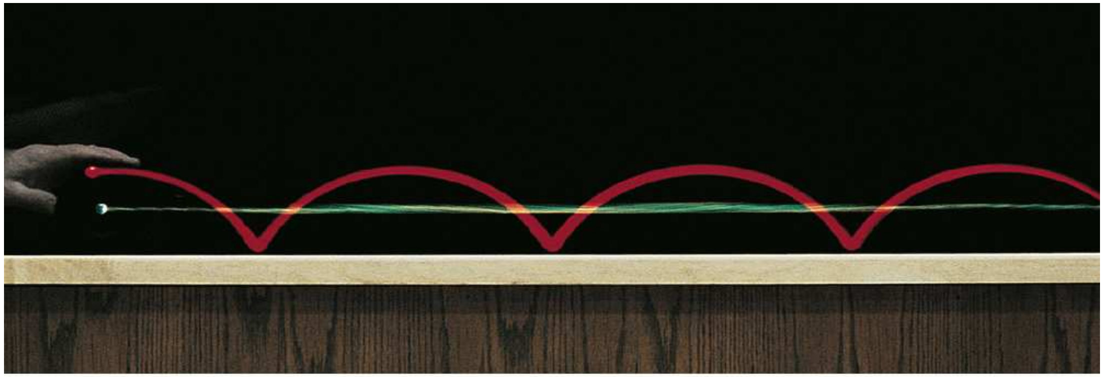
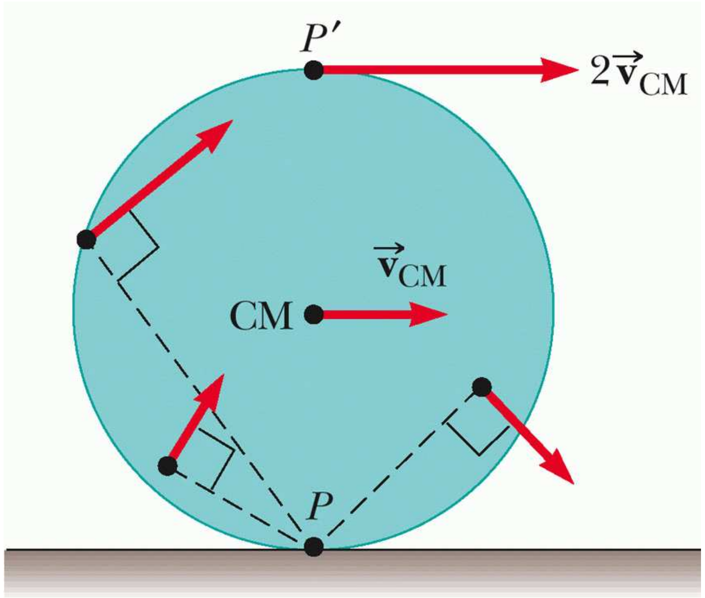


Rolling





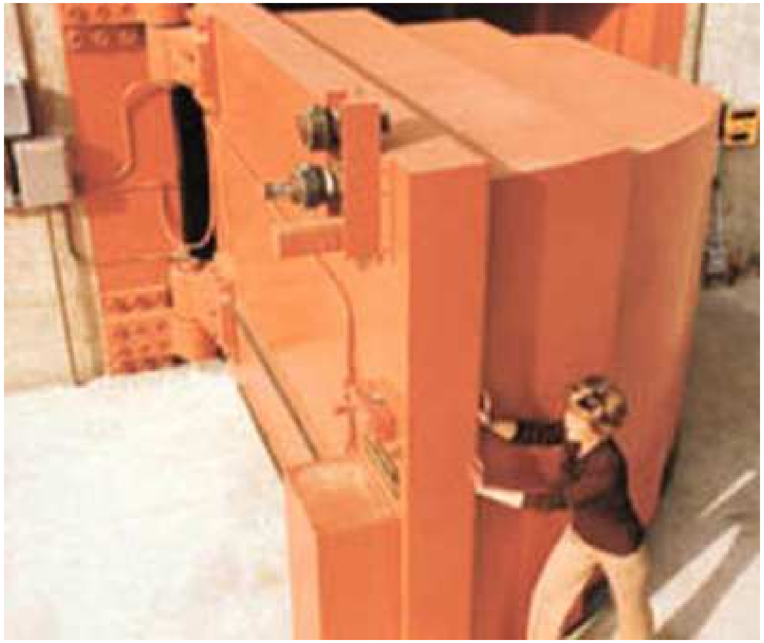
Rotational Energy

Where is most of the Earth's kinetic energy? Is it in the orbital motion around the Sun or in the spin about the Earth's axis?

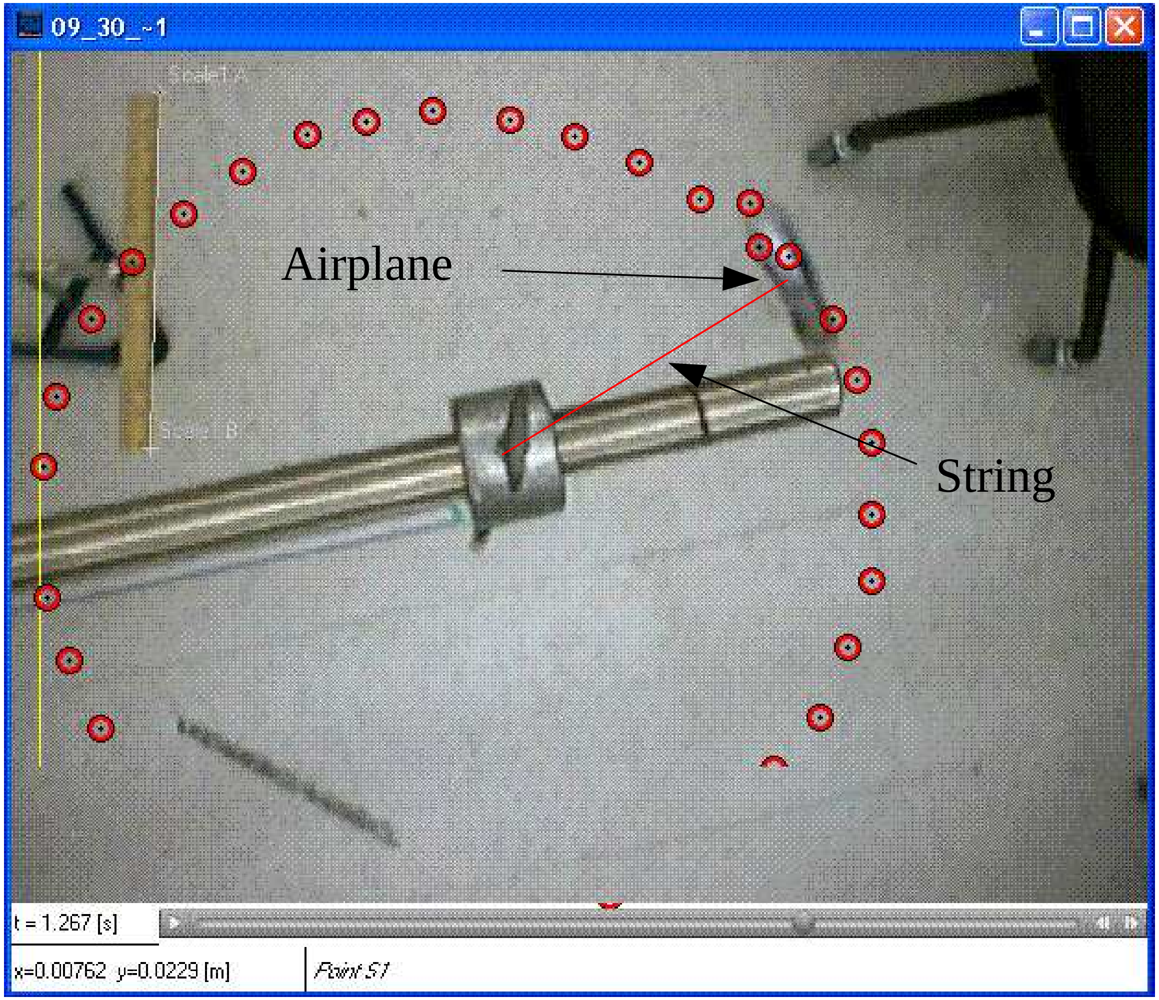
Earth's radius	$6.37 \times 10^{6}\ m$
Earth-Sun distance	$1.5 \times 10^{11}\ m$

Torque - Rotational Force

The shield door at a neutron test facility at Lawrence Livermore Laboratory is possibly the world's heaviest hinged door. It has a mass $m = 44,000\ kg$, a rotational inertia about a vertical axis through its hinges of $I = 8.7 \times 10^4\ kg - m^2$, and a (front) face width of $w = 2.4\ m$. A steady force $\vec{F}_a = 73\ N$, applied at its outer edge and perpendicular to the plane of the door, can move it from rest through an angle $\theta = 90^\circ$ in $\Delta t = 30\ s$. What is the torque exerted by the friction in the hinges? If the hinges have a radius $r_h = 0.1\ m$ what is the friction force?



Torque - Rotational Equivalent of Force



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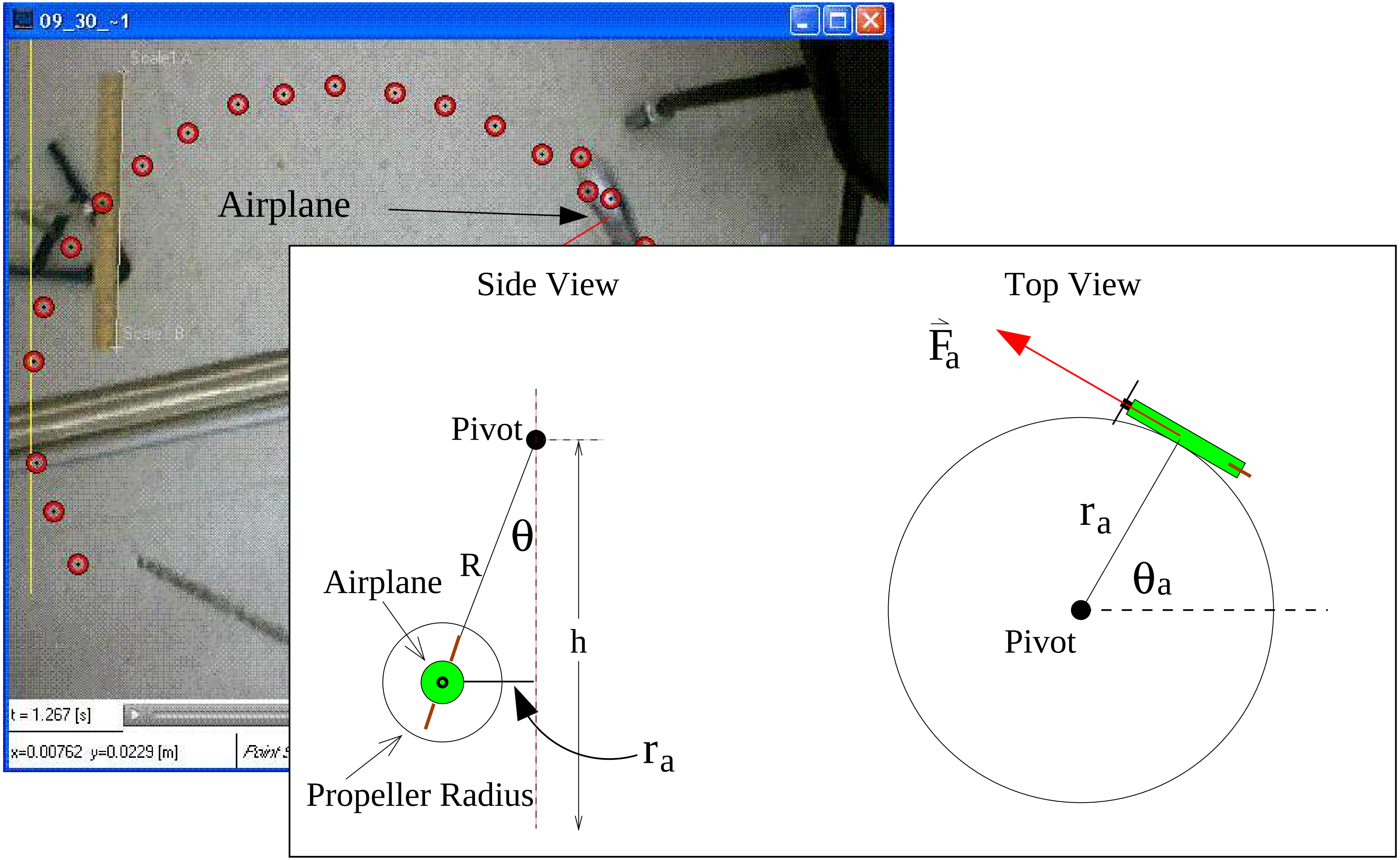

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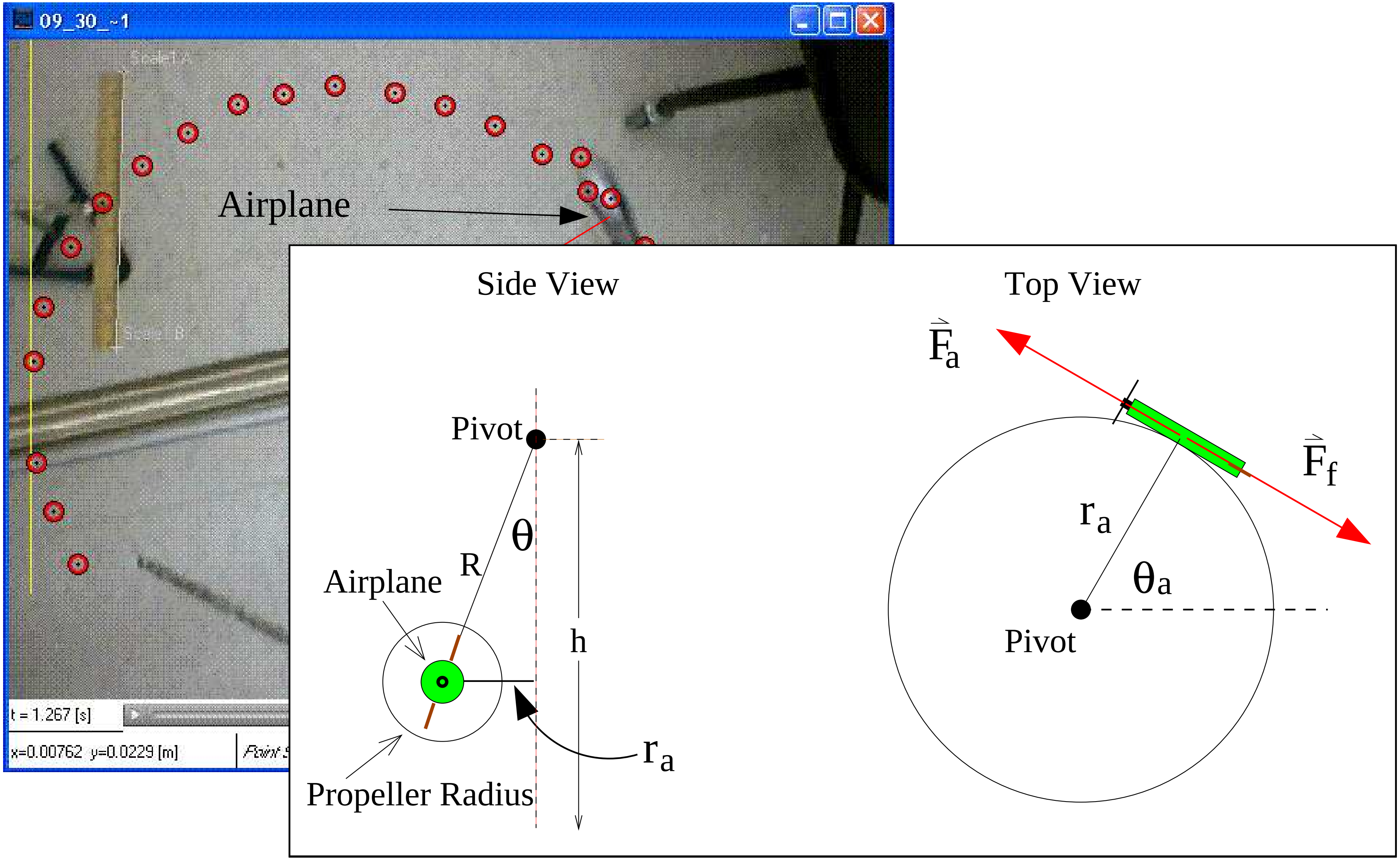
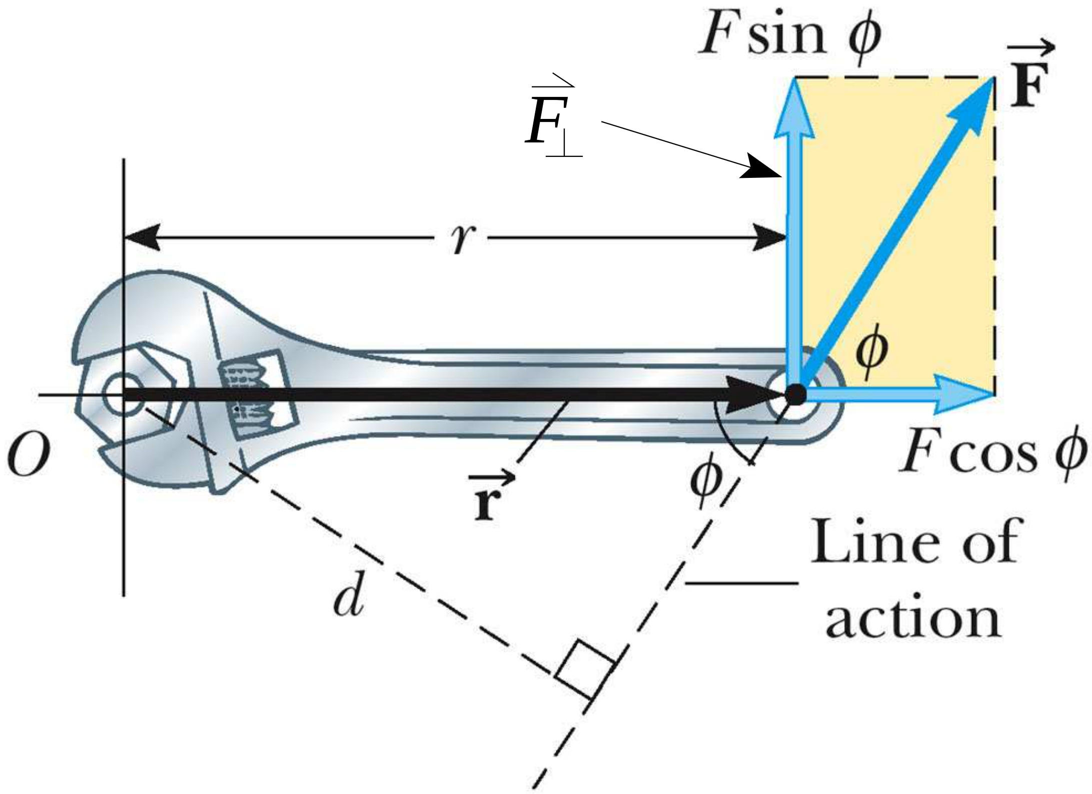


Torque - Rotational Equivalent of Force

$$\vec{F} = m\vec{a} \rightarrow \vec{\tau} = r\vec{F}_{\perp}$$



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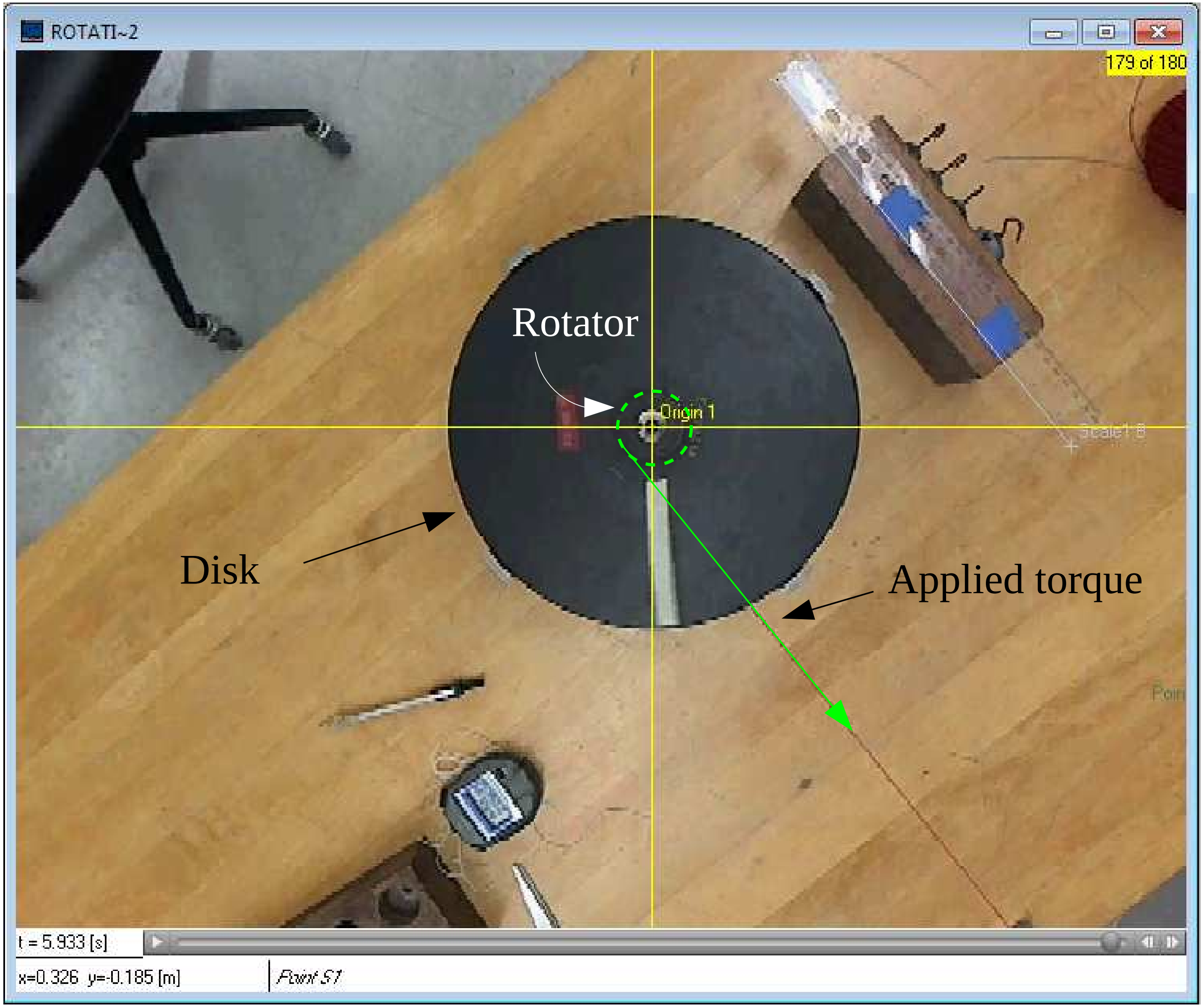
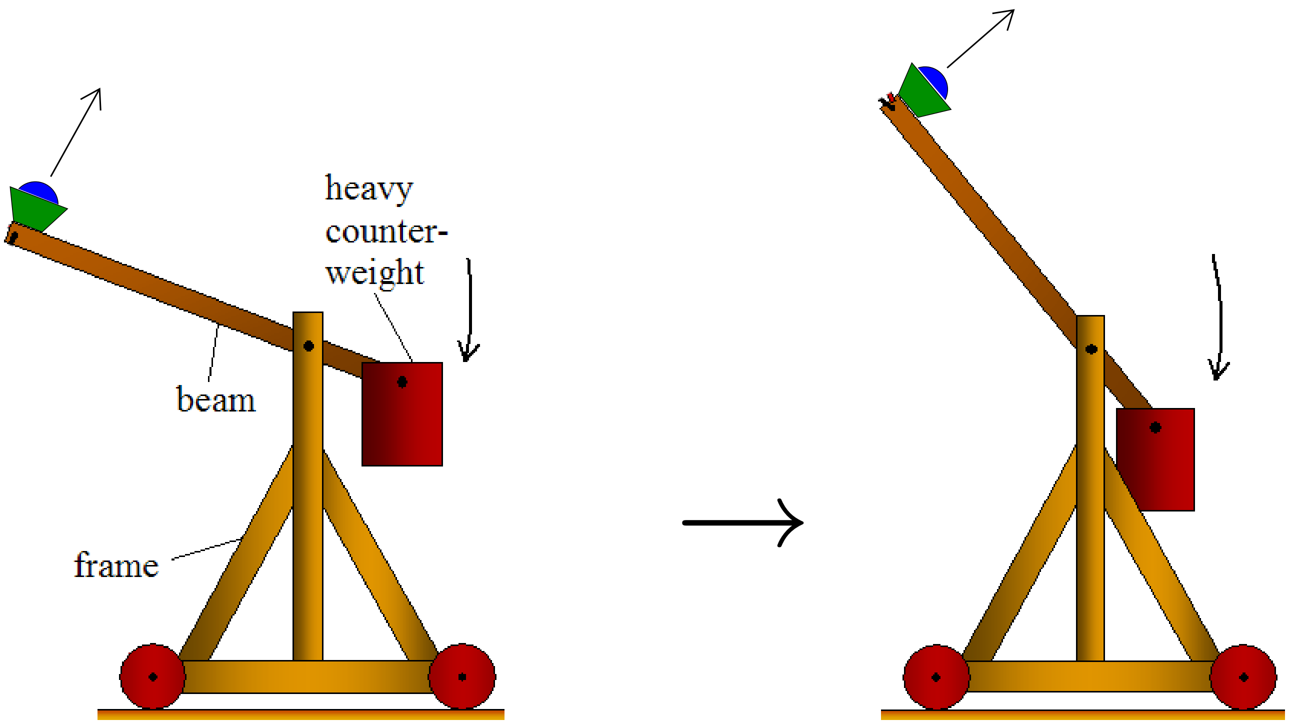
Rotational Form of $\vec{F} = m\vec{a}$



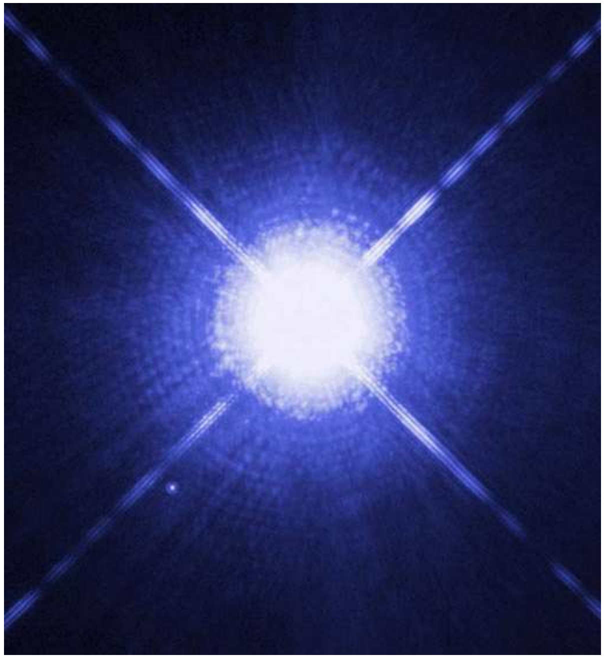
Torque and Rotational Energy - An Application

A trebuchet is a device used in the Middle Ages to throw big rocks at castles and is now used to throw other things like pumpkins, pianos, ... Consider the figures below. The trebuchet has a stiff wooden beam of mass $m_b = 15\ kg$ and length $l_b = 5\ m$ with masses $m_c = 700\ kg$ (the counterweight) and $m_p = 0.1\ kg$ (the payload) on it's ends. Treat these two masses as point particles. A frictionless axle is located a distance $d = 0.15\ m$ from the counterweight. The beam is released from rest in a horizontal position. We will launch the payload from a bucket at the end of the beam . What is the maximum speed the payload can reach before it leaves the bucket?

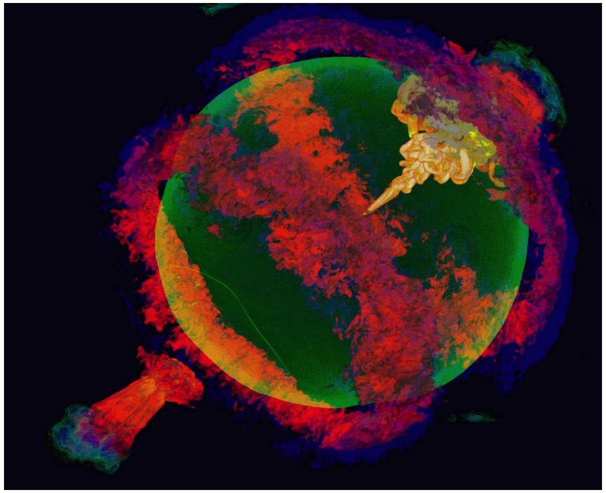


Collapsing Stars

Most stars in our galaxy will eventually run out of nuclear fuel and collapse to form a white dwarf star. The upper figure shows a white dwarf (the swall white dot at lower left) orbiting the star Sirius. The lower one shows a simulation of the explosion a white dwarf performed at the the University of Chicago. The yellow and orange represent the flame that pops out of the star, while the blue marks the surface of the star. The star is approximately the size of the Earth, but contains a mass greater than the Sun's.

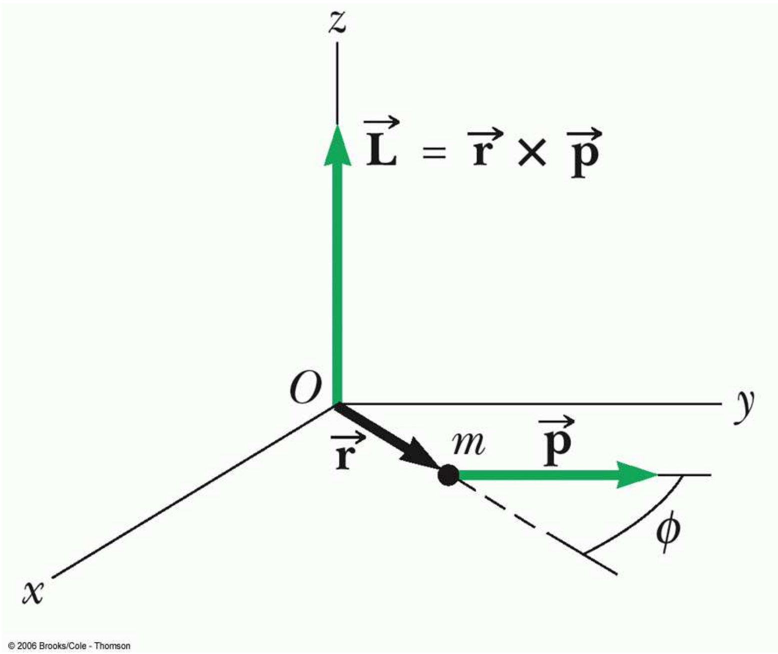


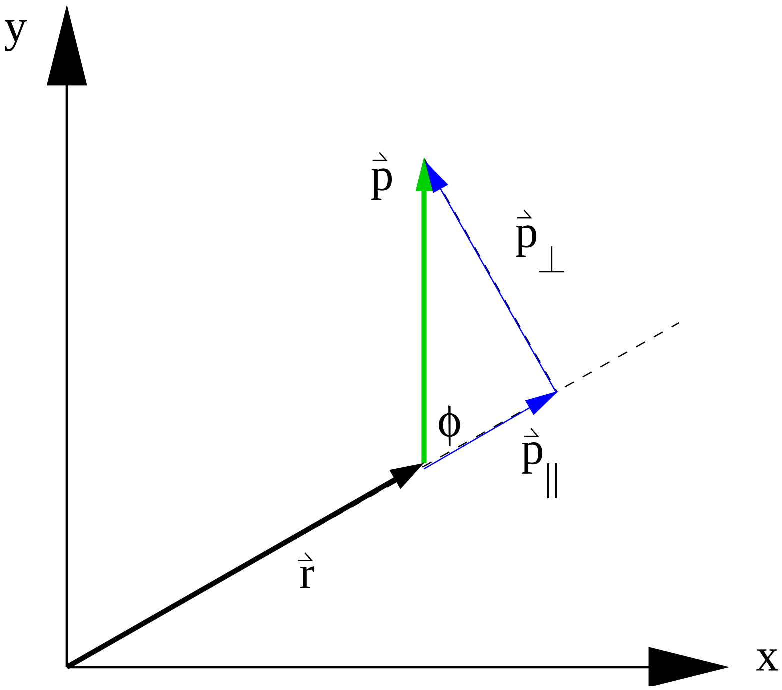
Suppose the Sun runs out of nuclear fuel and collapses into a white dwarf star with a radius equal to the radius of the Earth. What would be the rotation period T_f after the collapse? Treat the stars as uniform spheres.



Sun radius:	$6.96 \times 10^8\ m$
Earth radius:	$6.37 \times 10^6\ m$
$\mathrm{T}_{\mathrm{Sun}}$:	$24.5\ d$

Angular Momentum

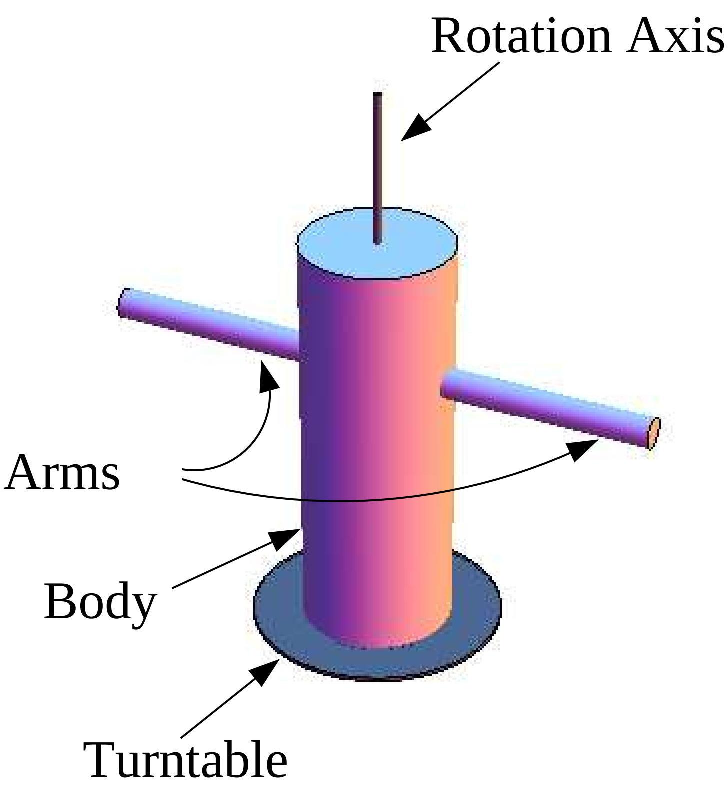




Twirling Student

A student volunteer/victim is spinning around on a turntable with her arms outstretched. She is spinning initially at a rate of $0.6\ revolutions/s$ and then drops her arms flat to her side at a distance $r_b = 0.20\ m$ from the axis. What is her final rotation rate? Treat the student's body as a cylinder with thin rods for arms. The turntable has a moment of inertia of $I_t = 1\ kg - m^2$.

Arm length:	$0.45\ m$
Arm mass:	$8\ kg$
Cylinder mass:	$55\ kg$



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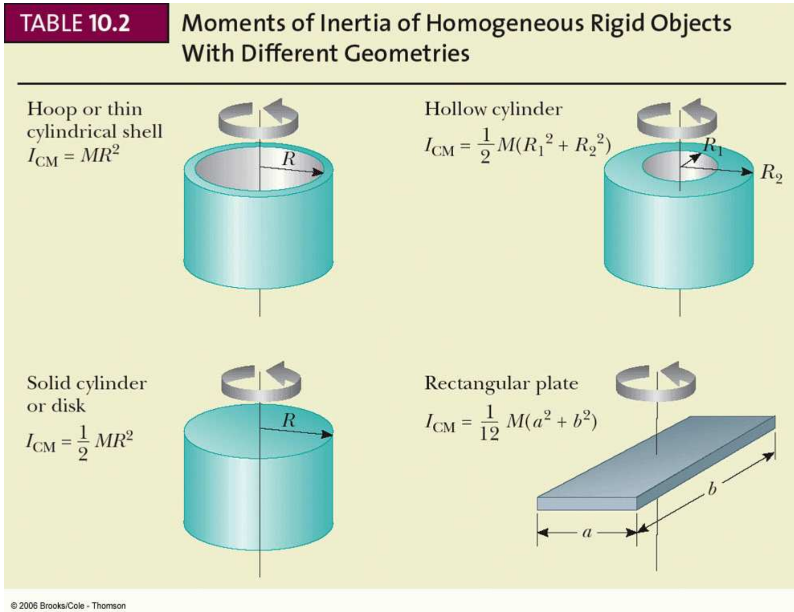

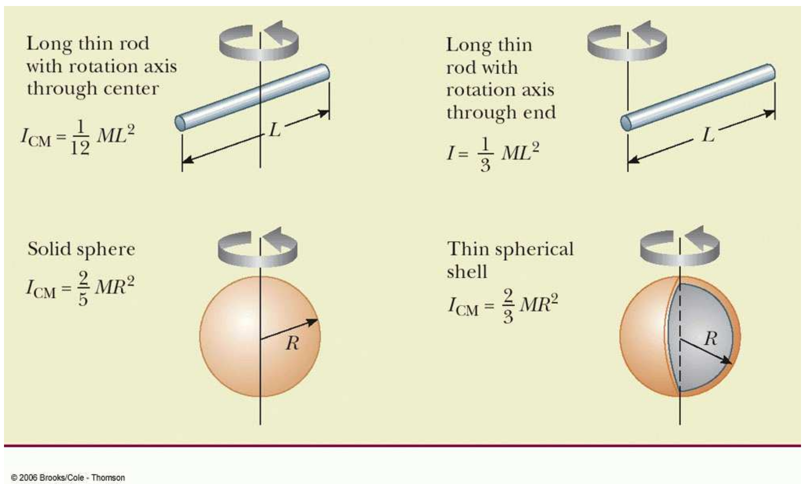

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a	$a = r\alpha$	$\alpha = \frac{a}{r} = \frac{d\omega}{dt}$
$\vec{F} = m\vec{a} = \frac{d\vec{p}}{dt}$	$\vec{\tau} = \vec{r} \times \vec{F}$	$\vec{\tau} = I\vec{\alpha} = \frac{d\vec{L}}{dt}$
$KE = \frac{1}{2}mv^2$		$KE_R = \frac{1}{2}I\omega^2$
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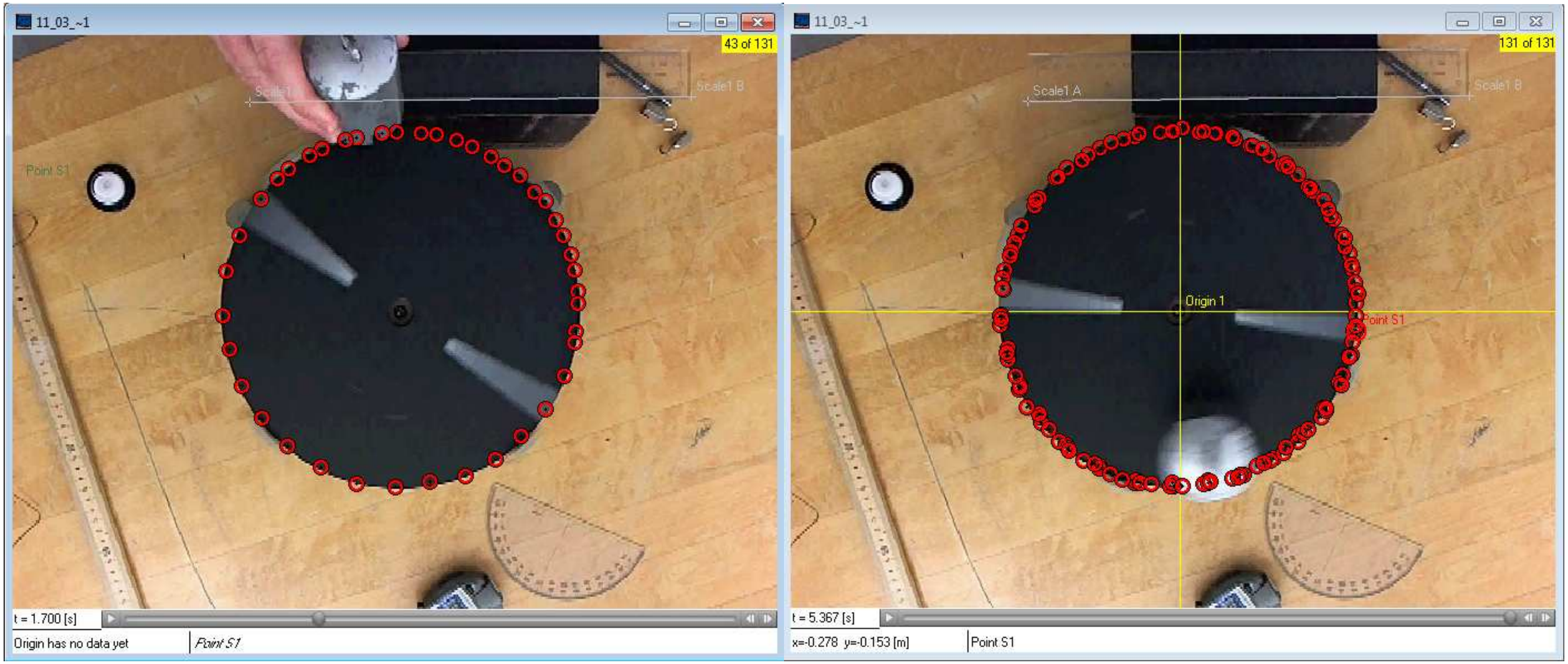
Angular Momentum Conservation

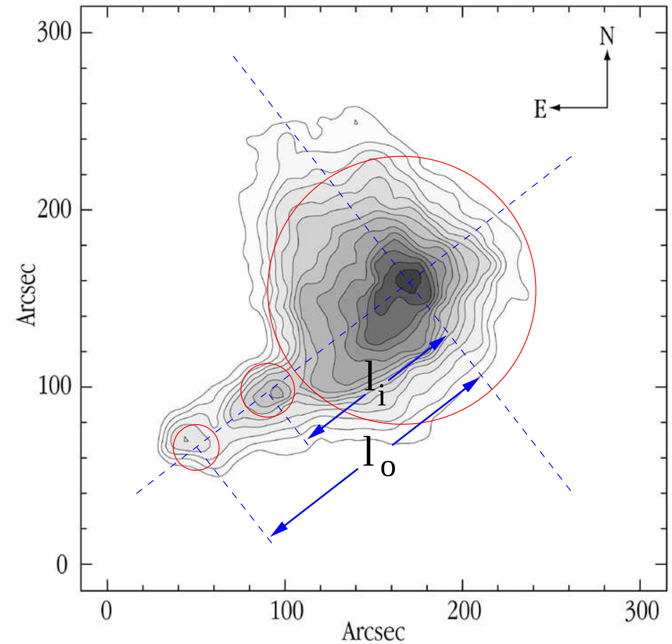


The Shape It's In

The plot below shows the 'obscuration' in the angular area around B68 based on measurements of background stars. The light in the center is 10^{14} dimmer than outside the edge of the cloud. To make life simple we will treat the mass distribution of B68 as three, rigid, uniform spheres that lie along the axis shown in the figure and rotate with $\omega = 9.4 \times 10^{-14}\ rad/s$. The spheres do NOT rotate independently of the rest of the cloud. The origin is at the center of the central lobe. What is the moment of inertia of the cloud?

Lobe	Radius (km)	Mass (kg)
central	$R_c = 1.0 \times 10^{12}$	$m_c = 6.0 \times 10^{30}$
inner	$R_i = 2.0 \times 10^{11}$	$m_i = 4.6 \times 10^{28}$
outer	$R_o = 1.7 \times 10^{11}$	$m_o = 2.9 \times 10^{28}$

origin - inner cloud center	$l_i = 1.4 \times 10^{12}\ km$
origin - outer cloud center	$l_o = 2.0 \times 10^{12}\ km$



Map of the Obscuration in the Dark Cloud B68

ESO PR Photo 29c/99 (2 July 1999)

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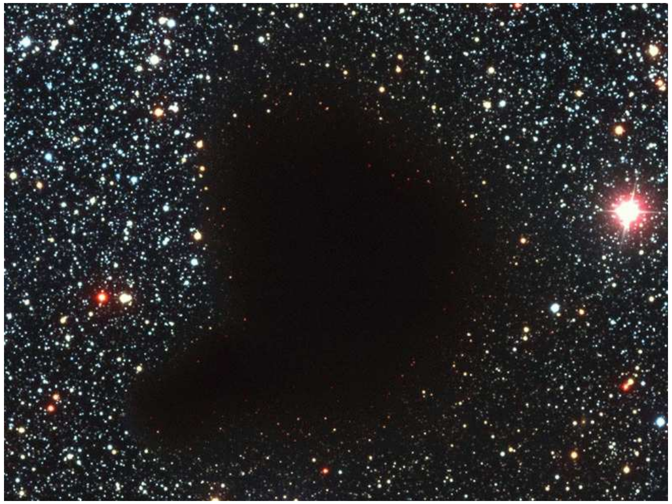

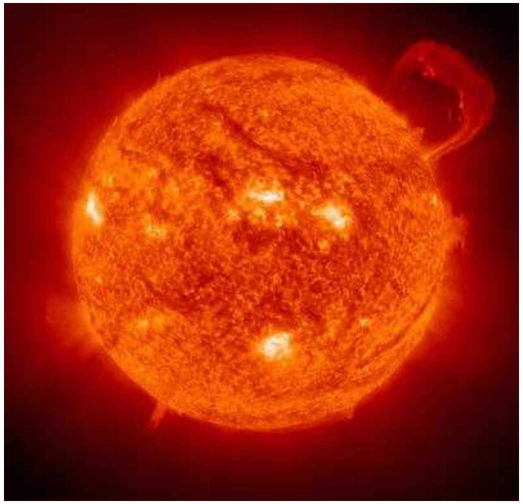

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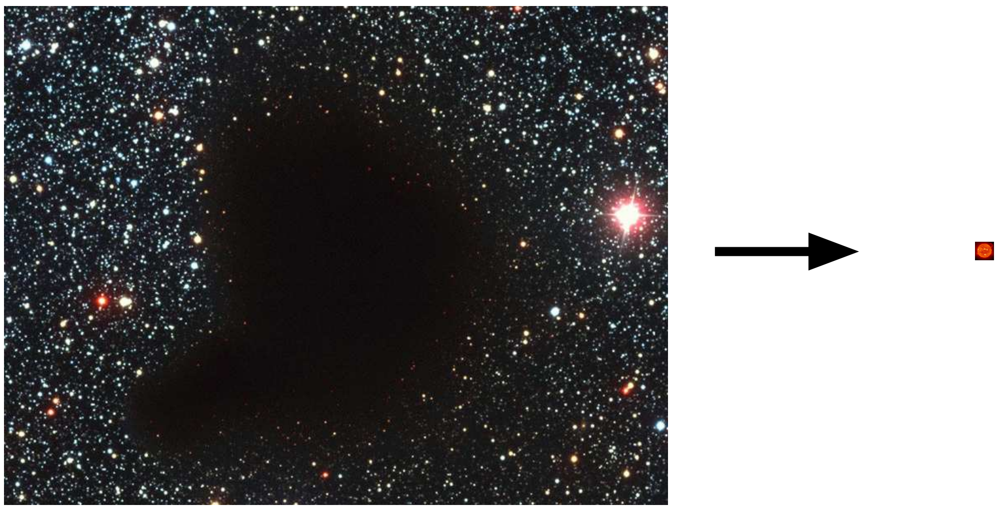
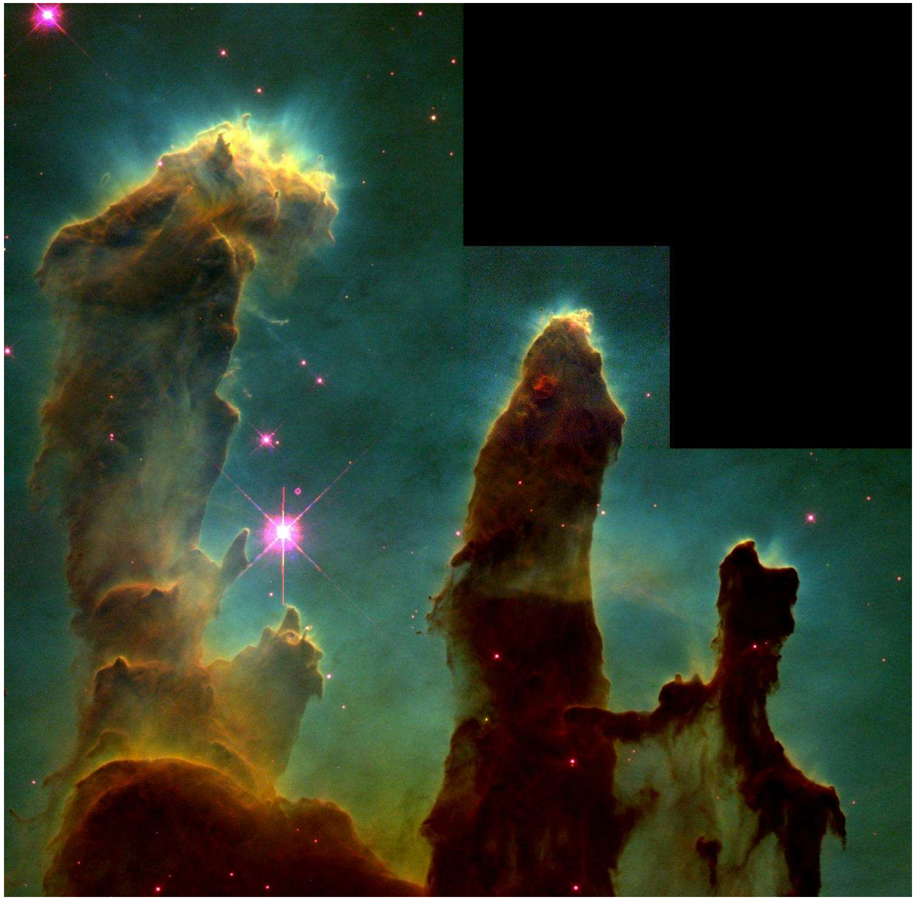


More Stars Are Born - The Eagle Nebula

The dark pillar-like structures are columns of cool interstellar hydrogen gas and dust that protrude from the interior wall of a dark molecular cloud. They have survived longer than their surroundings in the face of a flood of ultraviolet light from hot, massive newborn stars (off the top edge of the picture). The tallest pillar (left) is about about 4 light-years long from base to tip. As the pillars are eroded away by the ultraviolet light, small globules of even denser gas buried within the pillars are uncovered. Forming inside at least some of the globules are embryonic stars. The picture was taken on April 1, 1995 with the Hubble Space Telescope Wide Field and Planetary Camera 2.



Credit: Jeff Hester and Paul Scowen (Arizona State University), and NASA/ESA.