The Arctic is Melting

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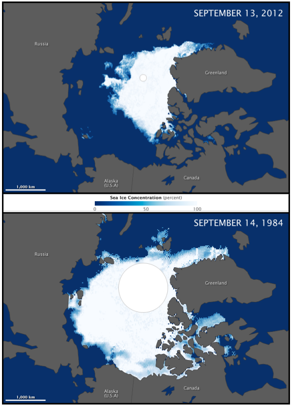

Japanese and US satellite data

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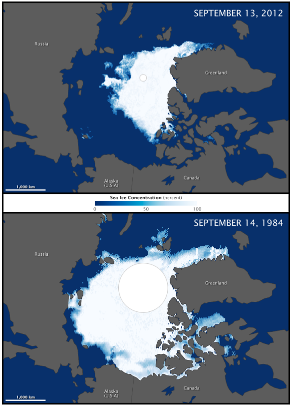

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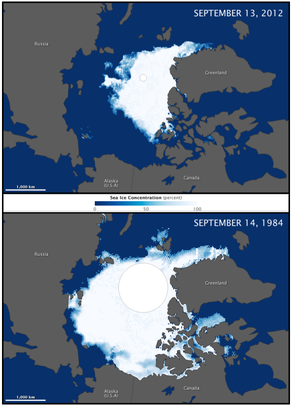

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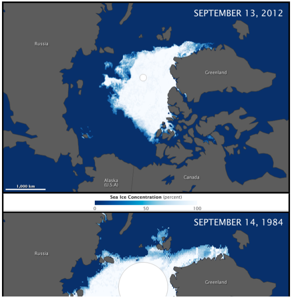

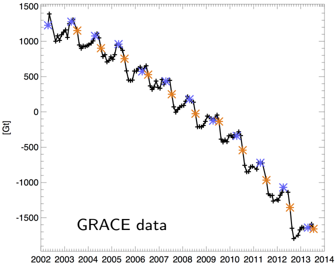

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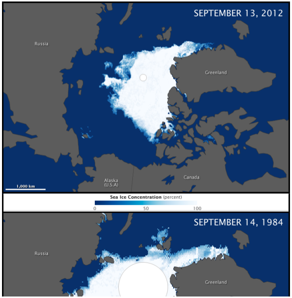
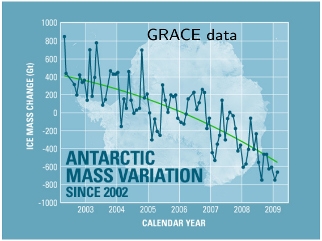
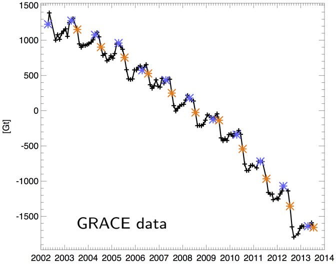
5. The ice in and around Antarctica is shrinking too.



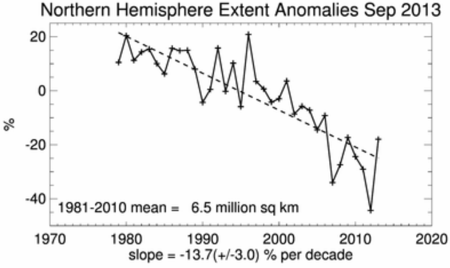




How Do You Measure Climate Change?

Much of the Earth's weather is driven by the Earth's rotation and reflects the interplay of air, water, and ice. This interplay might be used as a tool to measure the impact of climate change. Suppose you are developing a proposal for the National Science Foundation to study the impact of climate change by measuring the change in the Earth's rotation, *i.e.* the length of day (LOD). Your high-precision clock can detect changes in the LOD as small as 0.005 ms ($1\ ms = 10^{-3}\ s$).

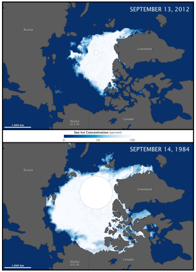
The measured rate of loss of Arctic sea ice is shown in the plot. Will your instrument be able to measure any change in the average size of the Arctic sea ice in one year of operation?

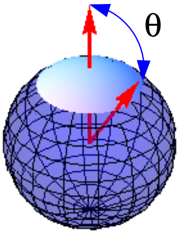


National Snow and Ice Data Center

Melting Ice - Rotating Earth

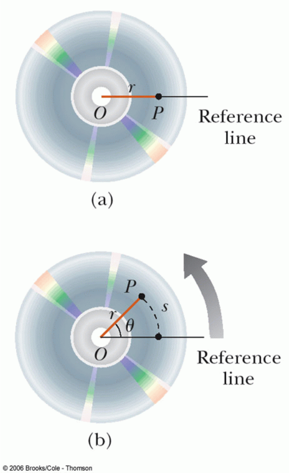
Consider a model of the Earth consisting of a uniform sphere covered by water with a thin, circular ice cap at the North Pole floating on the water and symmetric about the rotation axis. If the ice melts, the water is evenly distributed over the all-ocean Earth. The initial ice cap mass is $m_1 = 2.1 \times 10^{16}\ kg$, its surface mass density is $\sigma = 3.2 \times 10^3\ kg/m^2$, and its angular size is $\theta_1 = 13.0^\circ$ (see figure). The density of water is $\rho_{water} = 10^3\ kg/m^3$. The moment of inertia of the ice cap is $I_c = (8/3)\pi\sigma R_E^4(2+\cos\theta_1)\sin^4(\theta_1/2)$.

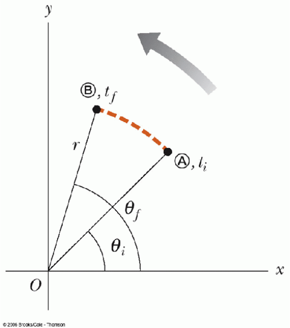




1. The ice cap shrinks by 0.1° and loses $\Delta m = 3 \times 10^{14}\ kg$. How much does the radius change?
2. How much does I_E of the Earth change?
3. How much does I_c of the ice cap change?
4. What happens to the LOD after the melting?
5. Should NSF fund you?

Rotational Quantities



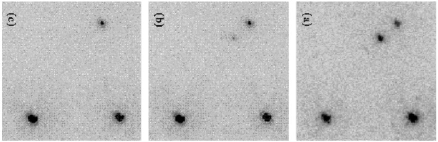


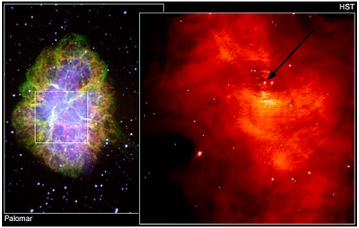
Linear $\rightarrow$ Rotational Quantities

Linear Quantity	Connection	Rotational Quantity
s	$s = r\theta$	$\theta = \frac{s}{r}$
v_T	$v_T = r\omega$	$\omega = \frac{v_T}{r} = \frac{d\theta}{dt}$
a	$a_T = r\alpha$	$\alpha = \frac{a_T}{r} = \frac{d\omega}{dt}$
$\vec{F} = m\vec{a}$	$\tau = rF_\perp$	$\vec{\tau} = I\vec{\alpha}$
$KE = \frac{1}{2}mv^2$		$KE_R = \frac{1}{2}I\omega^2$
$\vec{p} = m\vec{v}$	$\vec{L} = \vec{r} \times \vec{p}$	$\vec{L} = I\vec{\omega}$

How Fast Will the Star Spin?

The pulsar in the Crab nebula has a period $T_0 = 0.033\ s$ and this period has been observed to be increasing by $\Delta T = 1.26 \times 10^{-5}\ s$ each year. Assuming constant angular acceleration what is the expression for the angular displacement of the pulsar? What are the values of the parameters in that expression? What is the torque exerted on the pulsar? The moment of inertia of a uniform sphere is $I_s = 2mr^2/5$.

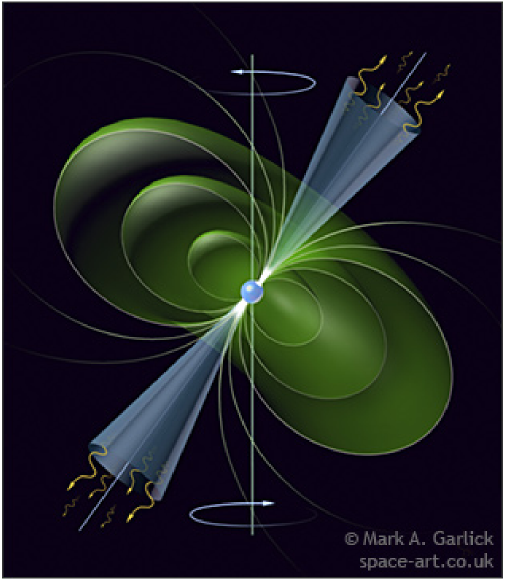




$$m_C = 3.4 \times 10^{30}\ kg$$

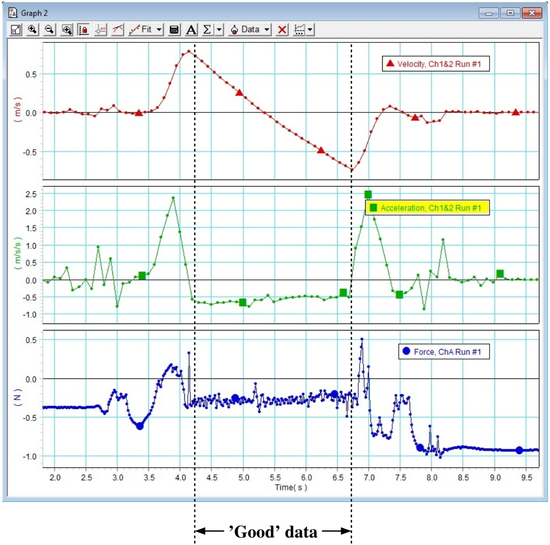
$$r_C = 25 \times 10^{3}\ m$$

A Pulsar



$\vec{F} \propto \vec{a} \rightarrow \vec{F} = m\vec{a}$

Force and Motion 2



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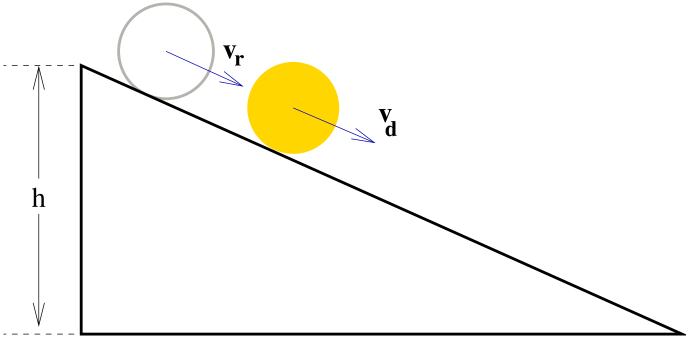
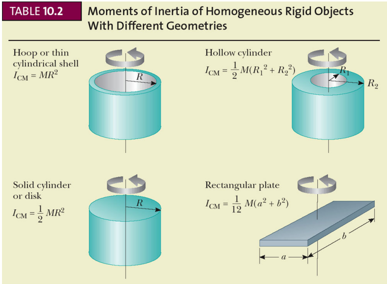
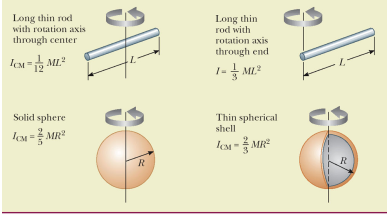
Which One Wins?

A wooden disk and a metal ring have the same mass m and radius r and start from rest and roll down an inclined plane (see figure). What are the kinetic energies at the bottom in terms of the height of the incline h, m, r, and any other constants? Which one is going faster at the bottom of the incline and gets to the bottom in the shortest time?



Moments of Inertia





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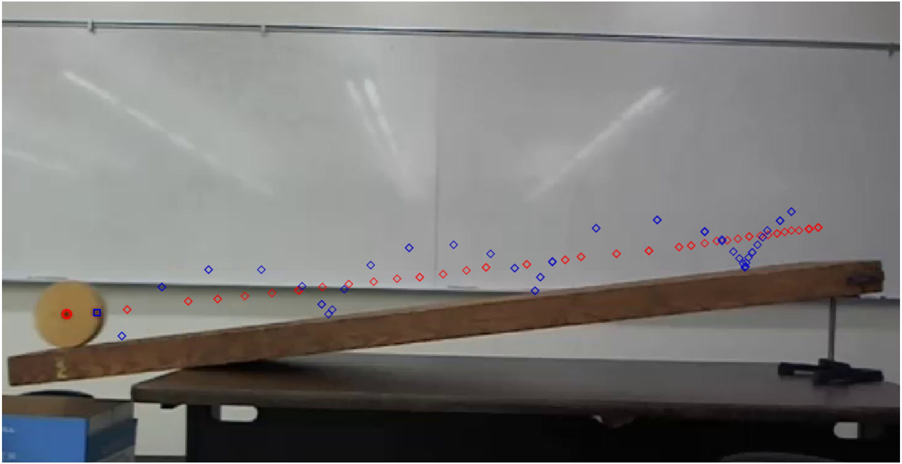
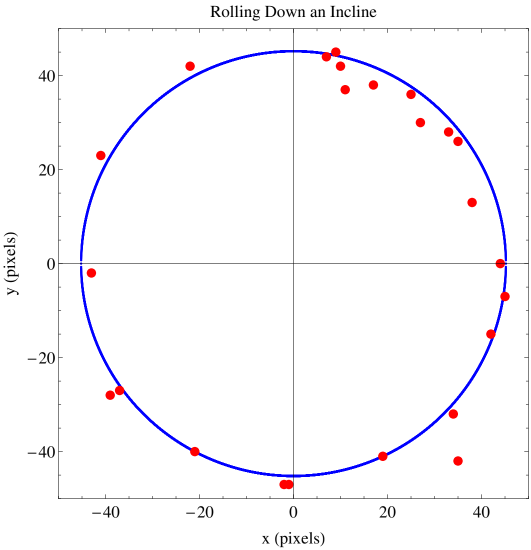
Rolling Down an Incline - 1



Rolling Down an Incline - 2



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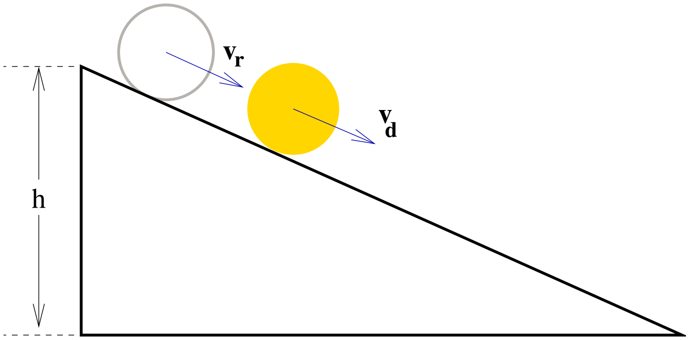
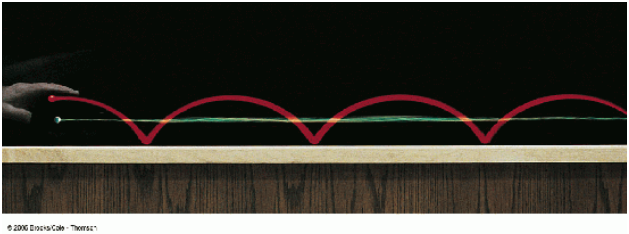
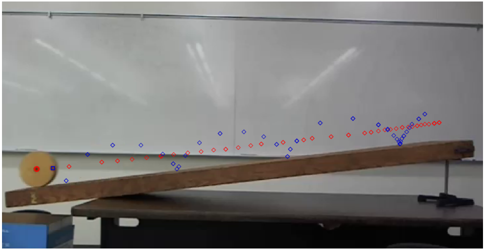


Rolling





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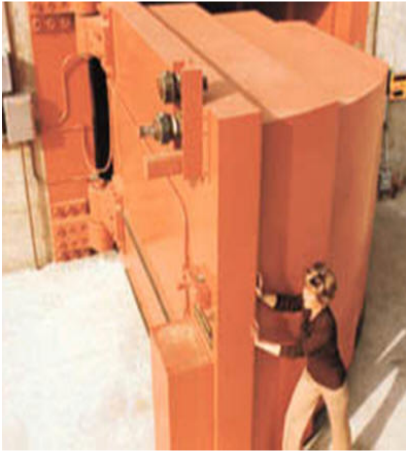
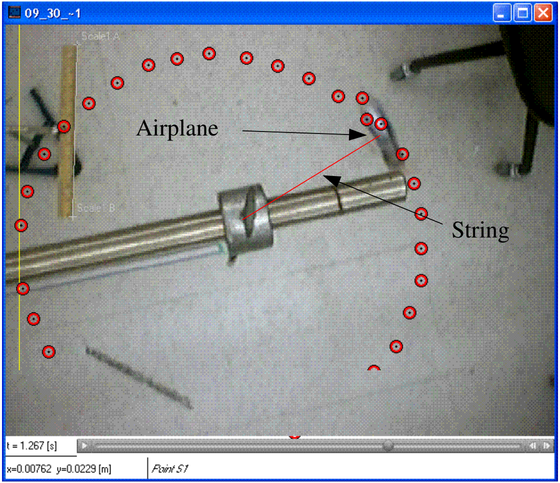
Torque - Rotational Force

The shield door at a neutron test facility at Lawrence Livermore Laboratory is possibly the world's heaviest hinged door. It has a mass $m = 44,000\ kg$, a rotational inertia about a vertical axis through its hinges of $I = 8.7 \times 10^4\ kg - m^2$, and a (front) face width of $w = 2.4\ m$. A steady force $\vec{F}_a = 72\ N$, applied at its outer edge and perpendicular to the plane of the door, can move it from rest through an angle $\theta = 90^\circ$ in $\Delta t = 30\ s$. What is the torque exerted by the friction in the hinges? If the hinges have a radius $r_h = 0.1\ m$ what is the friction force?



Torque - Rotational Equivalent of Force



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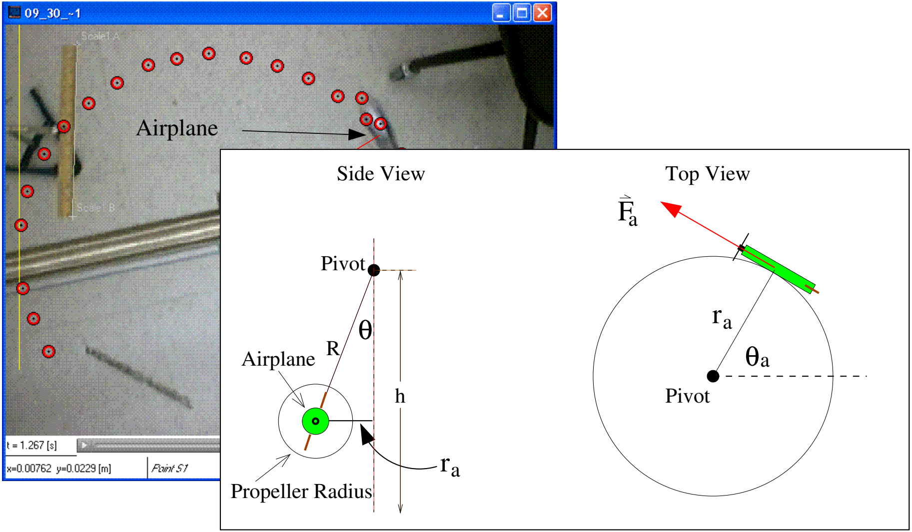

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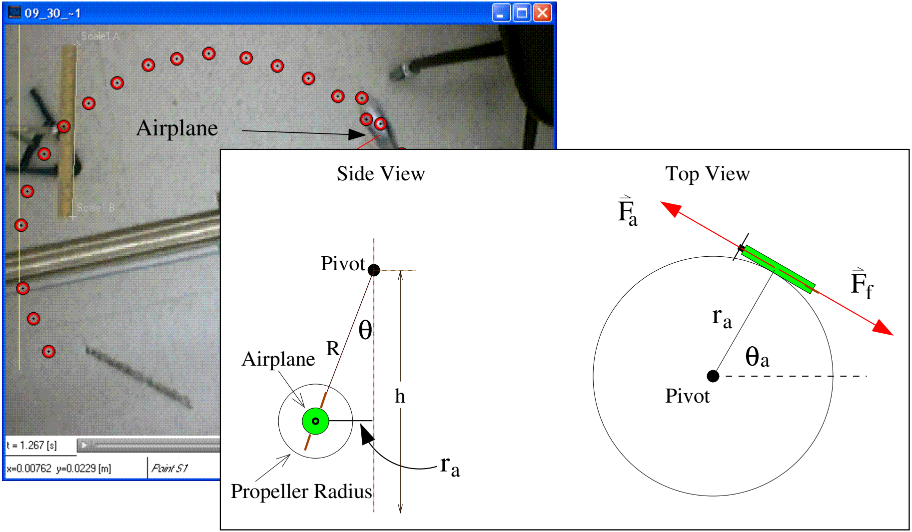
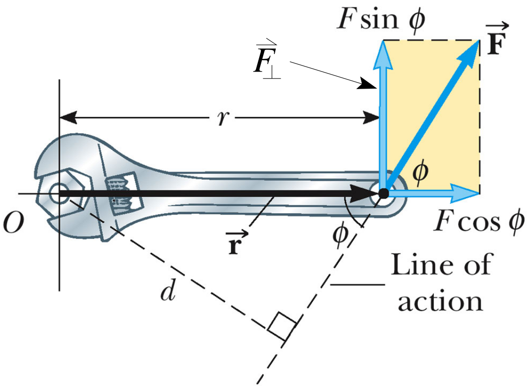


Torque - Rotational Equivalent of Force

$$\vec{F} = m\vec{a} \rightarrow \vec{\tau} = r\vec{F}_{\perp}$$



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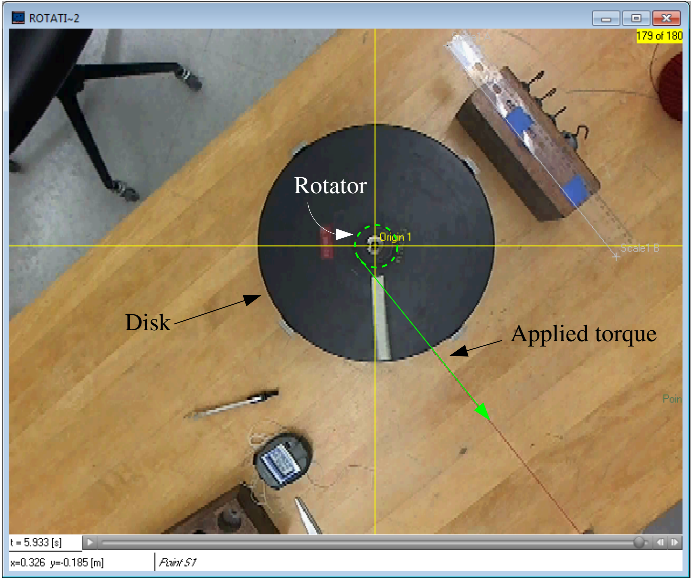
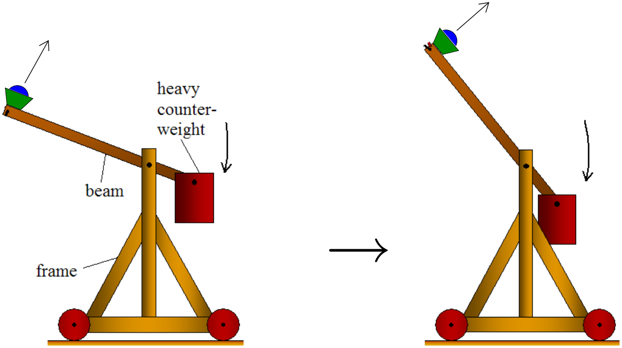
$\vec{p} = m\vec{v}$	$L = rp_\perp$	$\vec{L} = I\vec{\omega}$

Rotational Form of $\vec{F} = m\vec{a}$

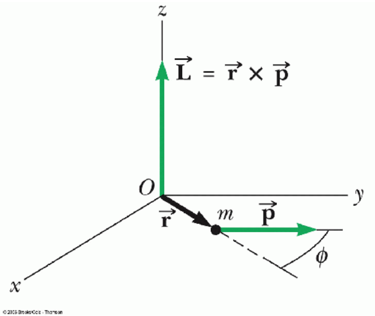


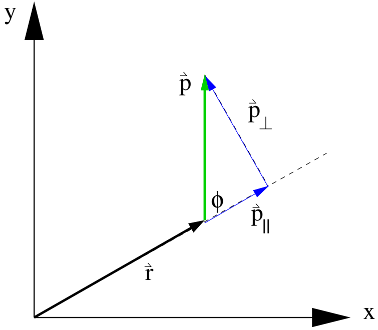
Torque and Rotational Energy - An Application

A trebuchet is a device used in the Middle Ages to throw big rocks at castles and is now used to throw other things like pumpkins, pianos, ... Consider the figures below. The trebuchet has a stiff wooden beam of mass $m_b = 15\ kg$ and length $l_b = 5\ m$ with masses $m_c = 700\ kg$ (the counterweight) and $m_p = 0.1\ kg$ (the payload) on it's ends. Treat these two masses as point particles. A frictionless axle is located a distance $d = 0.15\ m$ from the counterweight. The beam is released from rest in a horizontal position. We will launch the payload from a bucket at the end of the beam . What is the maximum speed the payload can reach before it leaves the bucket?



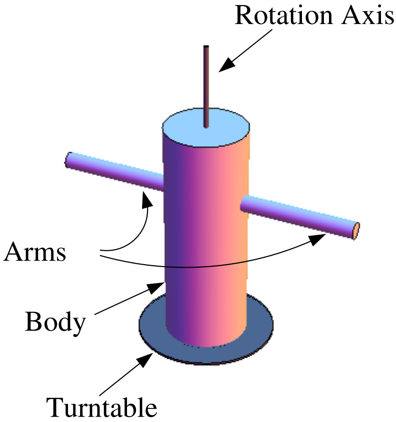
Angular Momentum



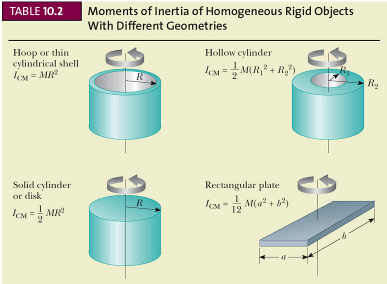


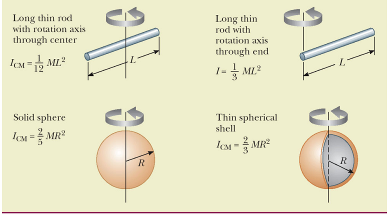
Twirling Student

A student volunteer/victim is spinning around on a turntable with her arms outstretched. She is spinning initially at a rate of ω_0 and then drops her arms flat to her side at a distance r_b from the axis. Treat the student's body as a cylinder of mass m_b and radius r_b with thin rods for arms of length l_a and mass m_a. The turntable has a moment of inertia of I_t. What is her final rotation rate ω_1 in terms of the quantities listed above?



Moments of Inertia





Linear $\rightarrow$ Rotational Quantities

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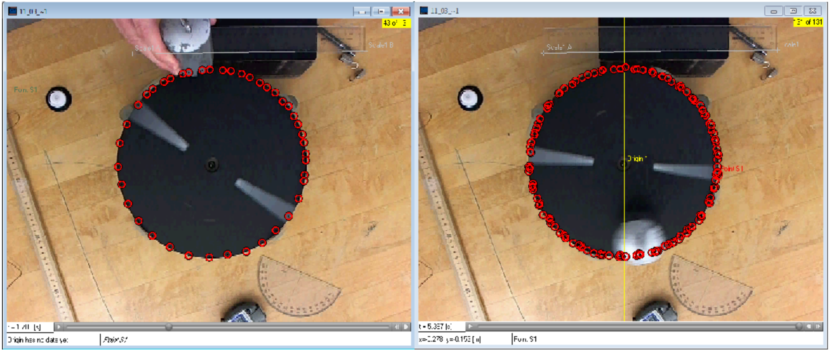
a	$a = r\alpha$	$\alpha = \frac{a}{r} = \frac{d\omega}{dt}$
$\vec{F} = m\vec{a} = \frac{d\vec{p}}{dt}$	$\tau = rF_\perp$	$\vec{\tau} = I\vec{\alpha} = \frac{d\vec{L}}{dt}$
$KE = \frac{1}{2}mv^2$		$KE_R = \frac{1}{2}I\omega^2$
$\vec{p} = m\vec{v}$	$L = rp_\perp$	$\vec{L} = I\vec{\omega}$

Angular Momentum Conservation



Angular Momentum Conservation Lab - Inputs

To get the movie or data file go to the following website

`https://facultystaff.richmond.edu/~ggilfoyl/genphys/131/links2.html`

right-click on the appropriate link and save it to the Desktop.

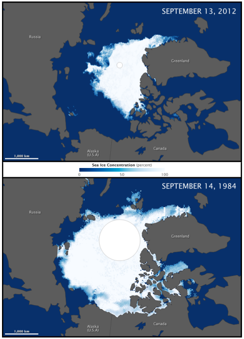
disk diameter	0.253 m	weight diameter	0.0525 m
mass of disk	5.10 kg	mass of weight	1.0 kg
I of rotator	0.0020 $kg-m^2$	center-to-edge-of-weight	0.062 m

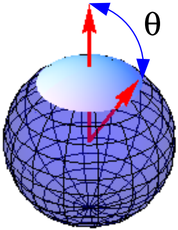
Melting Ice - Rotating Earth - 1

Consider a model of the Earth consisting of a uniform sphere of water with a thin, circular ice cap at the North Pole symmetric about the rotation axis. Treat the ice cap as infinitely thin and sitting on top of the water. If the ice melts, the water is evenly distributed over the all-ocean Earth. The initial ice cap mass is $m_1 = 2.07 \times 10^{16}\ kg$, its surface mass density is $\sigma = 3.17 \times 10^3\ kg/m^2$, and its angular size is $\theta_1 = 13.0^\circ$ (see figure). The density of water is $\rho_{water} = 10^3\ kg/m^3$. The moment of inertia of the ice cap is

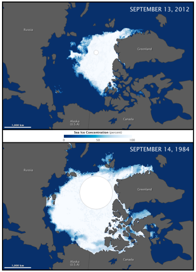
$$I_c = (8/3)\pi\sigma R_E^4(2 + \cos\theta_1)\sin^4(\theta_1/2)$$

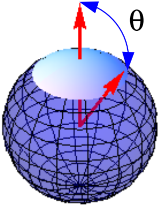
where R_E is the Earth's radius. Let the initial mass of the Earth m_E be just the water part and not include the Arctic ice cap. Your high-precision clock can detect changes in the LOD as small as 0.005 ms ($1\ ms = 10^{-3}\ s$).





1. What is the initial moment of inertia I_{E1} of the water-only part of the Earth?
2. What is the initial moment of inertia I_{C1} of the ice cap?
3. The ice cap melts, θ shrinks by 0.1° and its mass drops by $\Delta m = -3 \times 10^{14}\ kg$. How much does the added water change the radius of the Earth?
4. How much does I_E of the Earth change?
5. How much does I_c of the ice cap change?
6. What happens to the LOD after the melting?
7. Should NSF fund you?





Initial ice cap mass	$m_1 = 2.1 \times 10^{16}\ kg$	Ice cap angle 1	$\theta_1 = 13.0^\circ$
Surface mass density	$\sigma = 3.2 \times 10^3\ kg/m^2$	Ice cap angle 2	$\theta_1 = 12.9^\circ$
Mass loss	$\Delta m = -3 \times 10^{14}\ kg$	Water density	$\rho_w = 10^3 kg/m^3$

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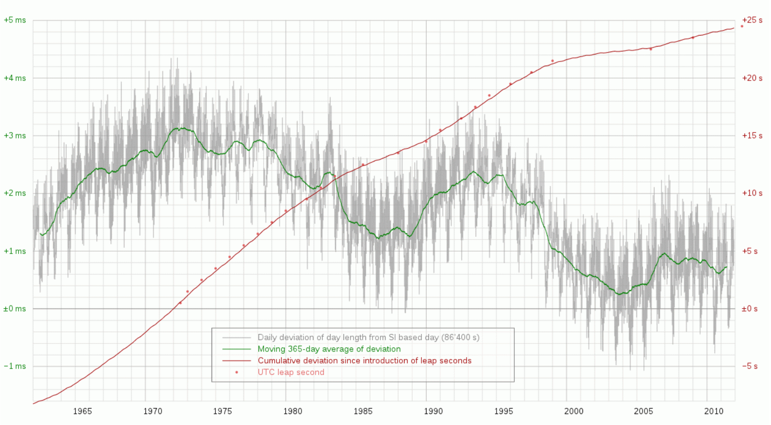
2. The resolution of the proposed instrument is $5\ \mu s$.
3. Is the signal big enough to detect?
4. What is the background?



International Earth Rotation and Reference Systems Service data.