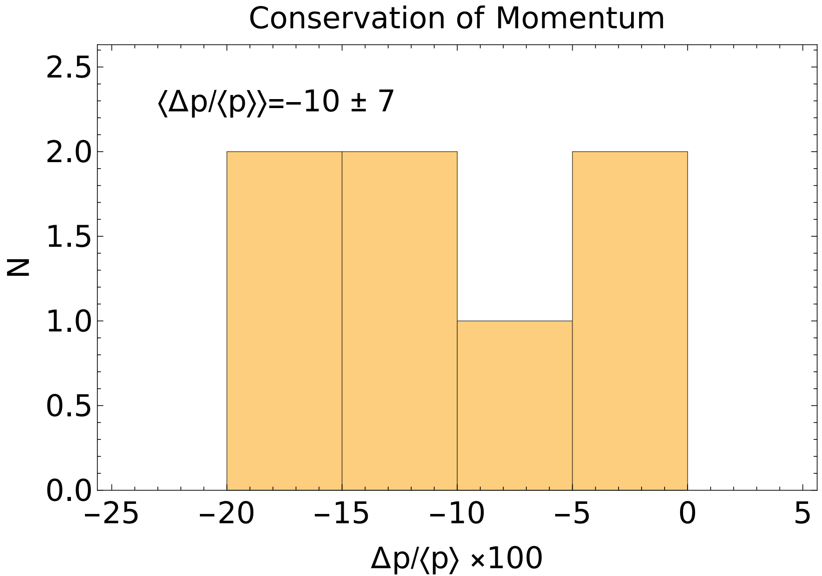

Conservation of Momentum
⟨Δp/⟨p⟩⟩=−10 ± 7
2.5
2.0
1.5
1.0
0.5
0.0
N
−25
−20
−15
−10
−5
0
5
Δp/⟨p⟩ ×100

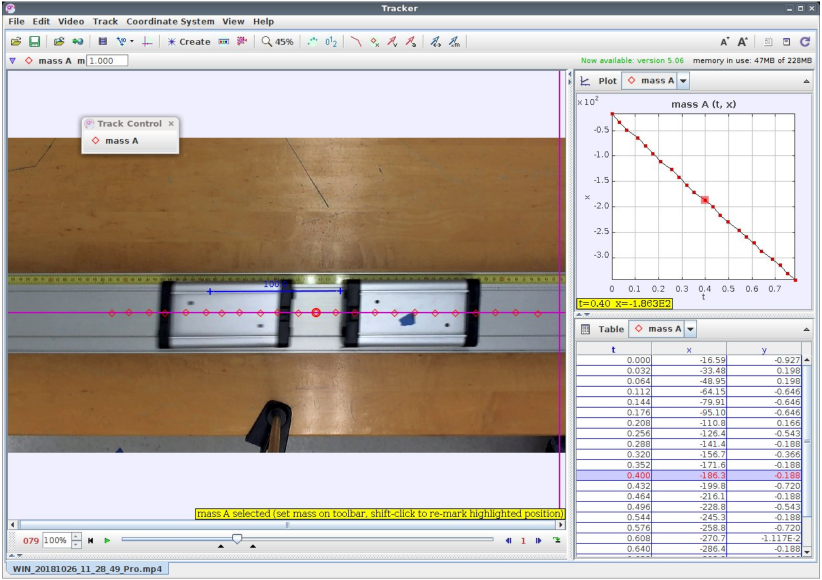
1. For Activities 3b and 3d do NOT make a video of your own. Go to the course webpage here or enter the address below.

   `https://facultystaff.richmond.edu/~ggilfoyl/genphys.html`

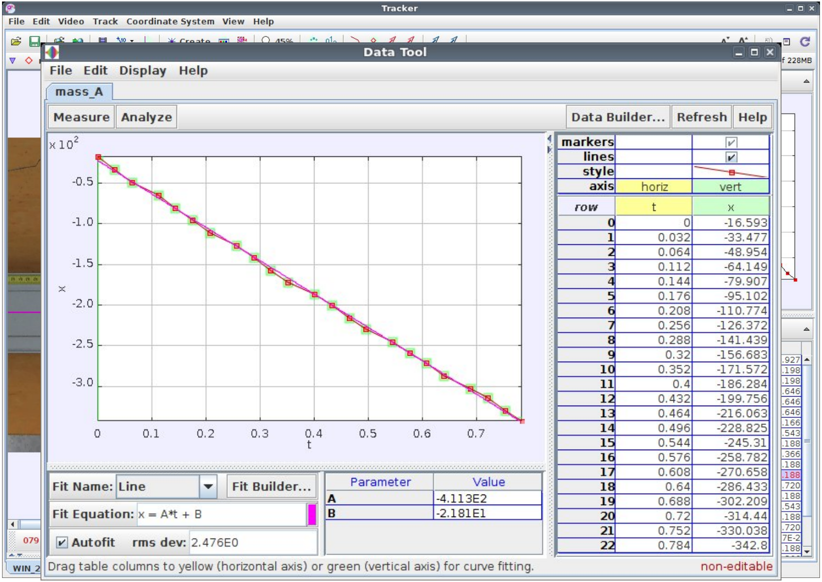
2. Then under *General Physics I (Physics 131)* navigate to **General Physics I links** and then to the item **Videos for Activities 3.b and 3.d in Momentum Conservation and Center-of-Mass**.

3. There will be links for the videos for Activities 3b and 3d. Analyze these.

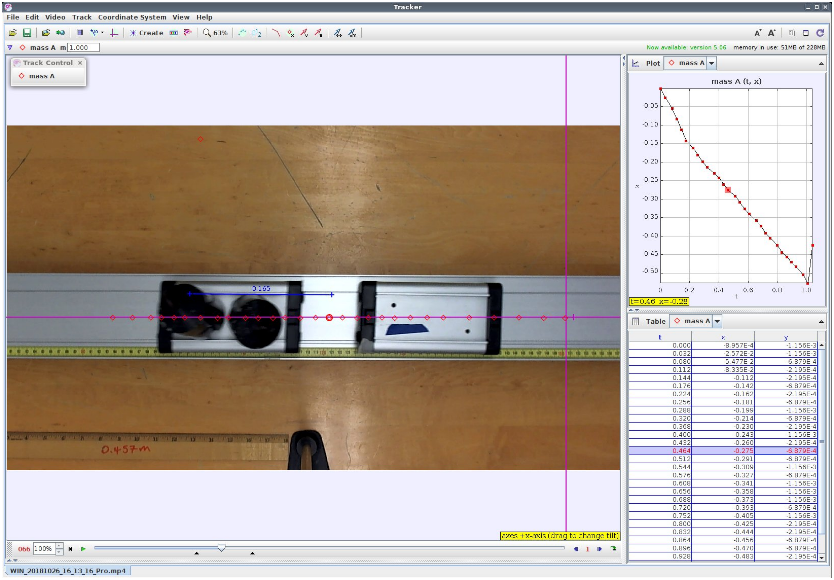
Center-of-Mass Motion - Equal Massses

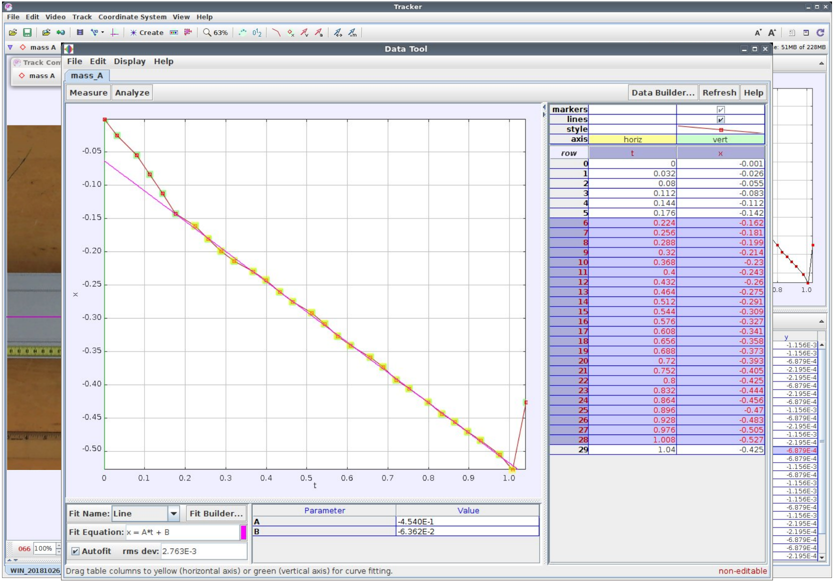


Center-of-Mass Motion - Equal Massses

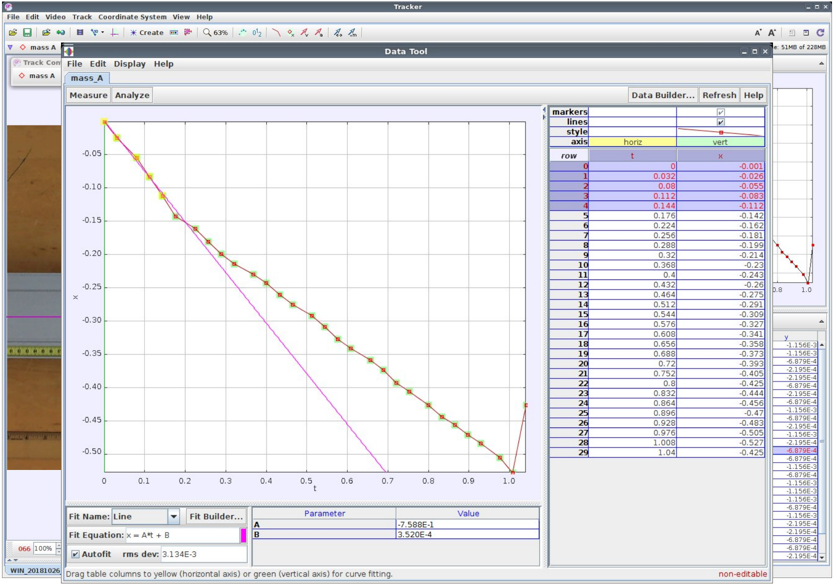


Center-of-Mass Motion - Different Massses

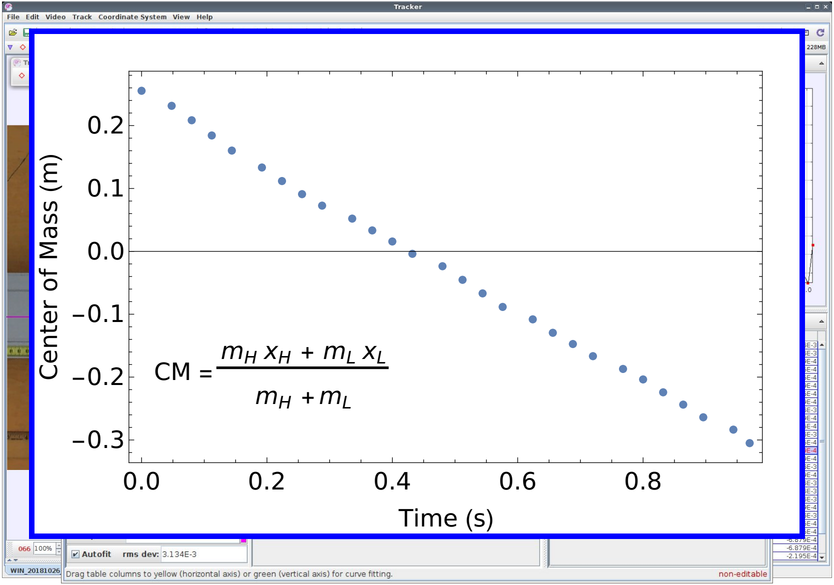


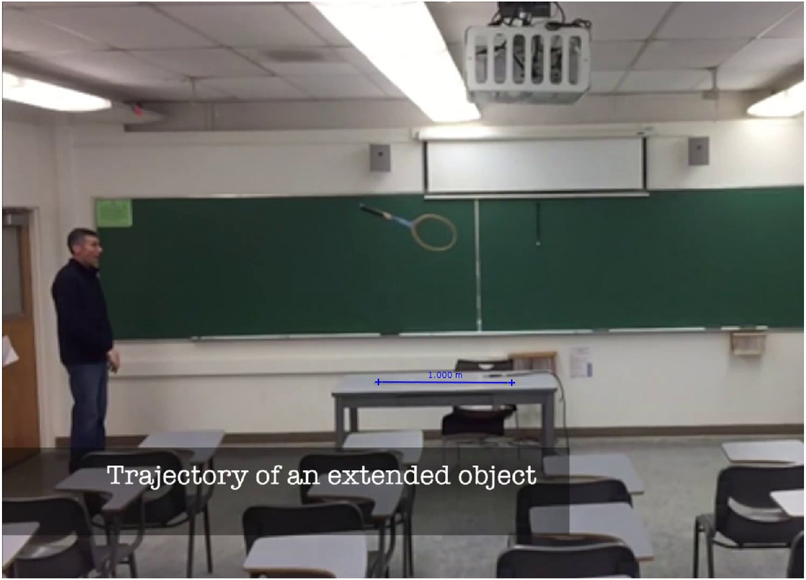


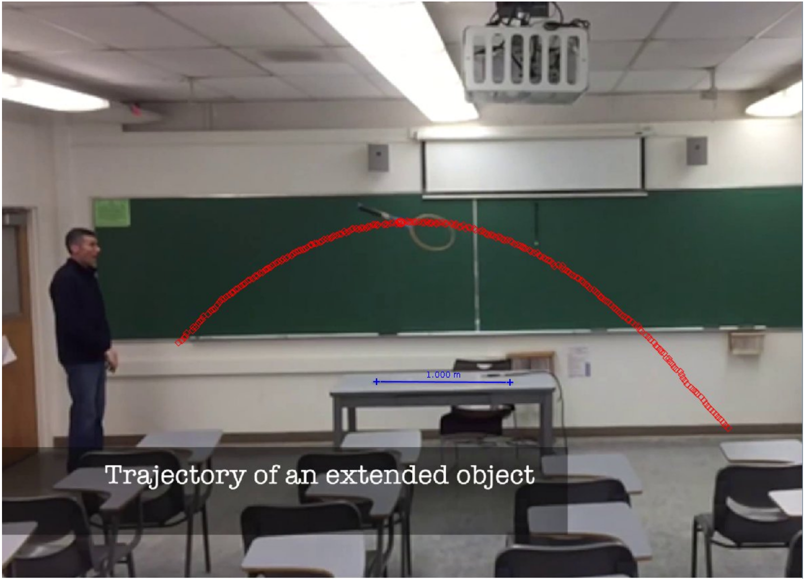
Tracker
File Edit Video Track Coordinate System View Help
Data Tool
File Edit Display Help
mass_A
Measure Analyze
Data Builder... Refresh Help
markers
lines
style
axis horiz vert
row t x
0 0 -0.001
1 0.032 -0.026
2 0.08 -0.055
3 0.112 -0.083
4 0.144 -0.112
5 0.176 -0.142
6 0.224 -0.162
7 0.256 -0.181
8 0.288 -0.199
9 0.32 -0.214
10 0.368 -0.23
11 0.4 -0.243
12 0.432 -0.26
13 0.464 -0.275
14 0.512 -0.291
15 0.544 -0.309
16 0.576 -0.327
17 0.608 -0.341
18 0.656 -0.358
19 0.688 -0.373
20 0.72 -0.393
21 0.752 -0.405
22 0.8 -0.425
23 0.832 -0.444
24 0.864 -0.456
25 0.896 -0.47
26 0.928 -0.483
27 0.976 -0.505
28 1.008 -0.527
29 1.04 -0.425
Fit Name: Line
Fit Builder...
Fit Equation: x = A*t + B
Autofit rms dev: 2.763E-3
Parameter Value
A -4.540E-1
B -6.362E-2
Drag table columns to yellow (horizontal axis) or green (vertical axis) for curve fitting.
non-editable

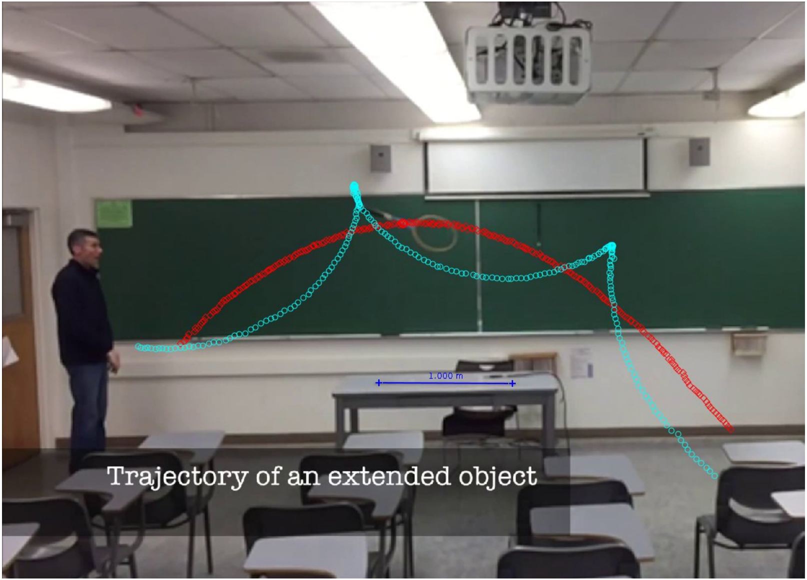


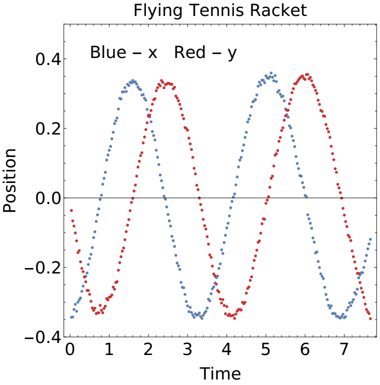
Tracker
File Edit Video Track Coordinate System View Help
Data Tool
File Edit Display Help
mass_A
Measure Analyze
Data Builder... Refresh Help
markers
lines
style
axis horiz vert
row t x
0 0 -0.001
1 0.032 -0.026
2 0.08 -0.055
3 0.112 -0.083
4 0.144 -0.112
5 0.176 -0.142
6 0.224 -0.162
7 0.256 -0.181
8 0.288 -0.199
9 0.32 -0.214
10 0.368 -0.23
11 0.4 -0.243
12 0.432 -0.26
13 0.464 -0.275
14 0.512 -0.291
15 0.544 -0.309
16 0.576 -0.327
17 0.608 -0.341
18 0.656 -0.358
19 0.688 -0.373
20 0.72 -0.393
21 0.752 -0.405
22 0.8 -0.425
23 0.832 -0.444
24 0.864 -0.456
25 0.896 -0.47
26 0.928 -0.483
27 0.976 -0.505
28 1.008 -0.527
29 1.04 -0.425
Fit Name: Line
Fit Builder...
Fit Equation: x = A*t + B
Autofit rms dev: 3.134E-3
Parameter Value
A -7.588E-1
B 3.520E-4
Drag table columns to yellow (horizontal axis) or green (vertical axis) for curve fitting.
non-editable

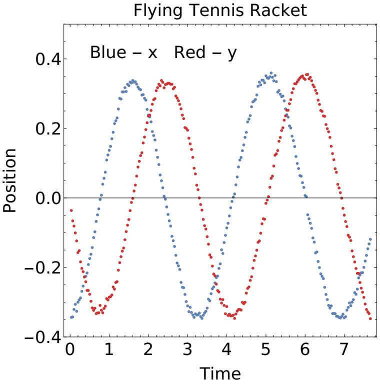

Tracker
File Edit Video Track Coordinate System View Help
Center of Mass (m)
0.2
0.1
0.0
−0.1
−0.2
−0.3
0.0
0.2
0.4
0.6
0.8
Time (s)
CM = $\frac{m_H x_H + m_L x_L}{m_H + m_L}$
Autofit rms dev: 3.134E-3
Drag table columns to yellow (horizontal axis) or green (vertical axis) for curve fitting.
non-editable

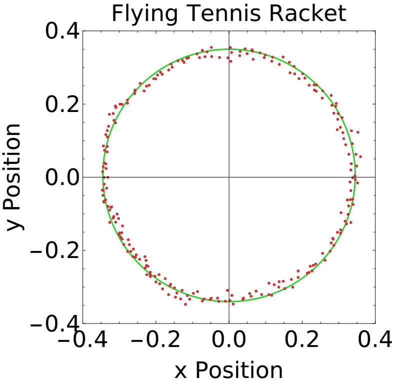

1.000 m
Trajectory of an extended object


1.000 m
Trajectory of an extended object


1.000 m
Trajectory of an extended object


Flying Tennis Racket
Blue – x Red – y
Position
Time
0.4
0.2
0.0
−0.2
−0.4
0 1 2 3 4 5 6 7


Flying Tennis Racket
Blue – x Red – y
Position
Time
0.4
0.2
0.0
−0.2
−0.4
0 1 2 3 4 5 6 7

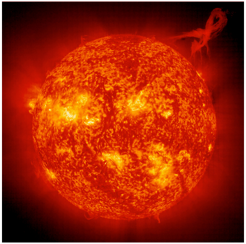

Flying Tennis Racket
y Position
x Position
0.4
0.2
0.0
−0.2
−0.4
−0.4 −0.2 0.0 0.2 0.4

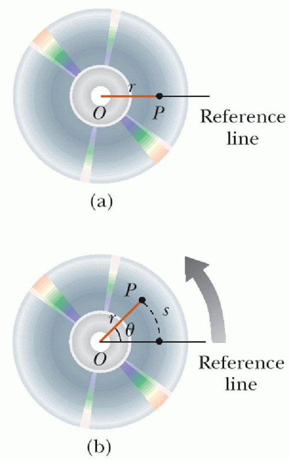
The photograph below shows a cloud of molecules called Bernard 68 (B68). It is located about 300 light-years (2.8×10^{15} km) away from us in the constellation Ophiuchus and is about 1.6 trillion kilometers across. It is made of molecules like CS, $\mathrm{N_2H}$, $\mathrm{H_2}$, and CO and is slowly rotating ($\omega = 9.4 \times 10^{-14}$ rad/s). The internal gravitational attraction of B68 may make the molecular cloud collapse far enough so it will ignite the nuclear fires and B68 will begin to shine.

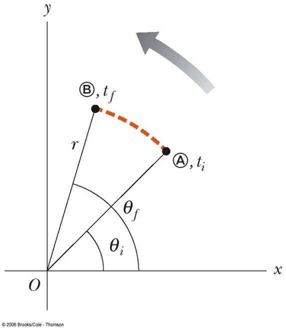
The molecular cloud B68 in the constellation Ophiuchus is rotating with an angular speed $\omega = 9.4 \times 10^{-14}$ rad/s. The gravitational attraction among the atoms in the cloud may make it collapse until the core is hot enough to ignite nuclear reactions and B68 will begin to shine. If the final properties of B68 are the same as our Sun, *i.e.*, the same mass and size, then what will be its final angular velocity and period? Assume the lost mass carries away very little angular momentum. Compare this with the angular velocity of the Sun. Is your result reasonable? Why or why not?

$M_{B68} = 6.04 \times 10^{30}$ kg $\quad I_{B68} = 2.7 \times 10^{54}$ $kg - km^2$ $\quad M_{Sun} = 1.989 \times 10^{30}$ kg

$R_{Sun} = 6.96 \times 10^{5}$ km $\quad T_{Sun} = 25.4$ d




r
O
P
Reference line
(a)
P
s
r
θ
O
Reference line
(b)

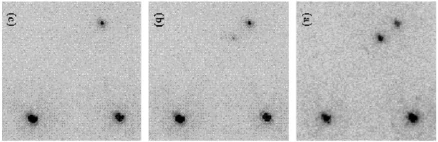

y
Ⓑ, t_f
r
Ⓐ, t_i
θ_f
θ_i
x
O
© 2006 Brooks/Cole - Thomson

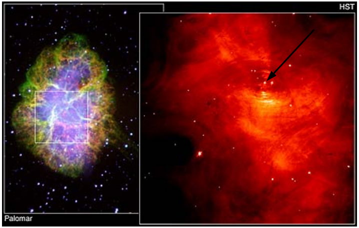
Linear Quantity	Connection	Rotational Quantity
s	$s = r\theta$	$\theta = \frac{s}{r}$
v_T	$v_T = r\omega$	$\omega = \frac{v_T}{r} = \frac{d\theta}{dt}$
a_T	$a_T = r\alpha$	$\alpha = \frac{a_T}{r} = \frac{d\omega}{dt}$
$KE = \frac{1}{2}mv^2$		$KE_R = \frac{1}{2}I\omega^2$
$\vec{F} = m\vec{a}$	$\tau = rF_\perp$	$\vec{\tau} = I\vec{\alpha}$
$\vec{p} = m\vec{v}$	$\vec{L} = \vec{r} \times \vec{p}$	$\vec{L} = I\vec{\omega}$

$$\vec{r}_{cm} = \frac{\sum m_i \vec{r}_i}{\sum m_i}$$

How Fast Will the Star Spin?

The pulsar in the Crab nebula has a period $T_0 = 0.033\ s$ and this period has been observed to be increasing by $\Delta T = 1.26 \times 10^{-5}\ s$ each year. Assuming constant angular acceleration what is the expression for the angular displacement of the pulsar? What are the values of the parameters in that expression? What is the torque exerted on the pulsar?

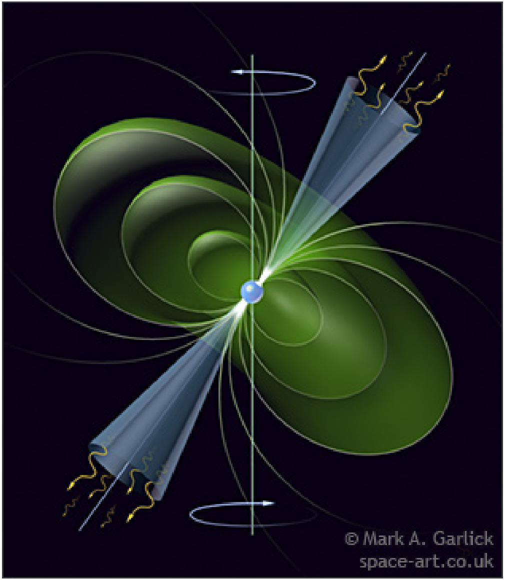




$$m_C = 3.4 \times 10^{30}\ kg$$

$$r_C = 25 \times 10^{3}\ m$$

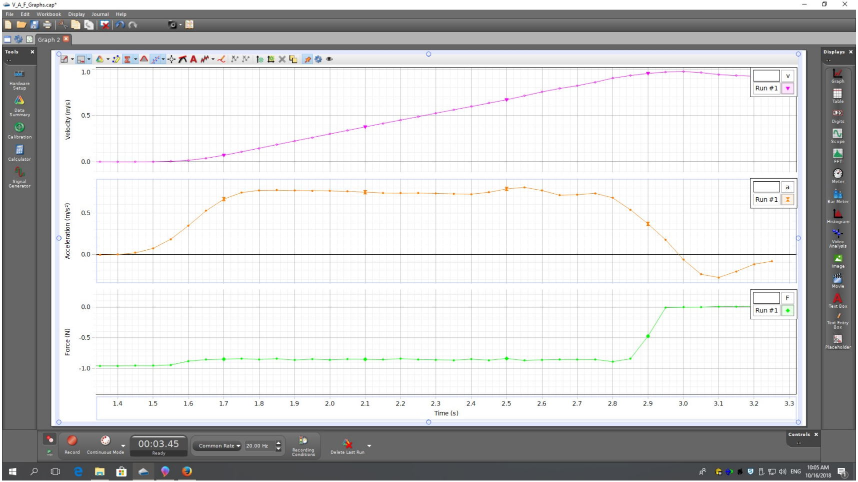
A Pulsar

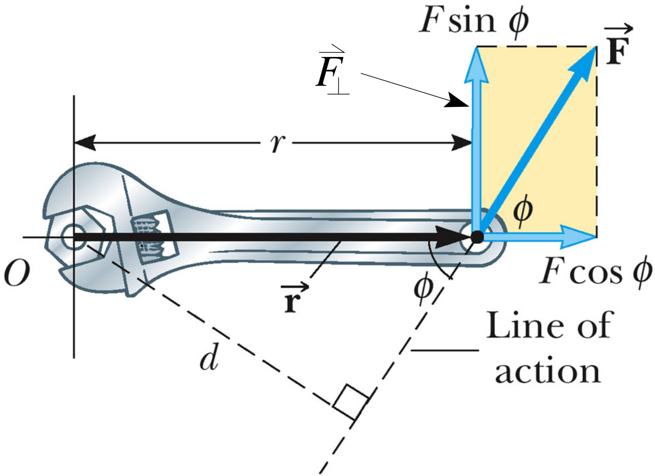


Linear Quantity	Connection	Rotational Quantity
s	$s = r\theta$	$\theta = \frac{s}{r}$
v_T	$v_T = r\omega$	$\omega = \frac{v_T}{r} = \frac{d\theta}{dt}$
a	$a_T = r\alpha$	$\alpha = \frac{a_T}{r} = \frac{d\omega}{dt}$
$KE = \frac{1}{2}mv^2$		$KE_R = \frac{1}{2}I\omega^2$
$\vec{F} = m\vec{a}$	$\tau = rF_\perp$	$\vec{\tau} = I\vec{\alpha}$
$\vec{p} = m\vec{v}$	$\vec{L} = \vec{r} \times \vec{p}$	$\vec{L} = I\vec{\omega}$

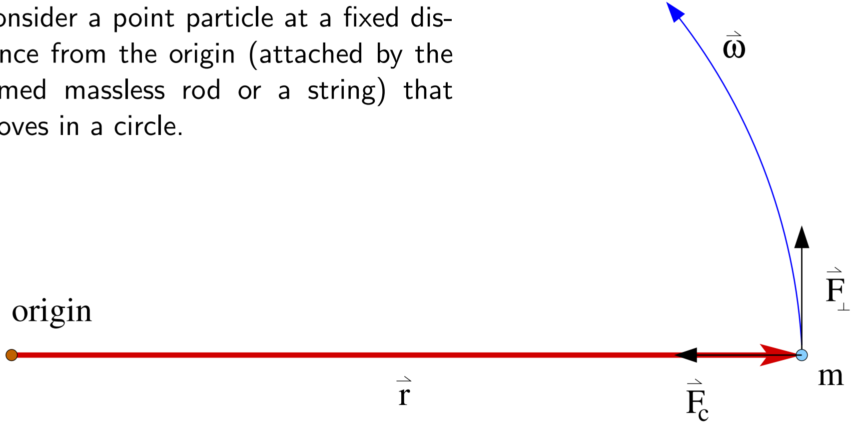
$$\vec{r}_{cm} = \frac{\sum m_i \vec{r}_i}{\sum m_i}$$

Force and Motion 1

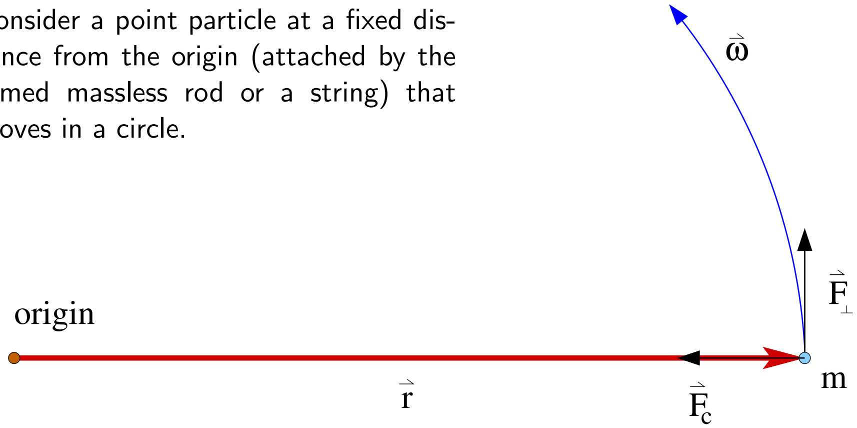



$F \sin \phi$
$\vec{\mathbf{F}}$
$\vec{F}_{\perp}$
r
ϕ
O
$\vec{\mathbf{r}}$
ϕ
$F \cos \phi$
Line of action
d

Consider a point particle at a fixed distance from the origin (attached by the famed massless rod or a string) that moves in a circle.

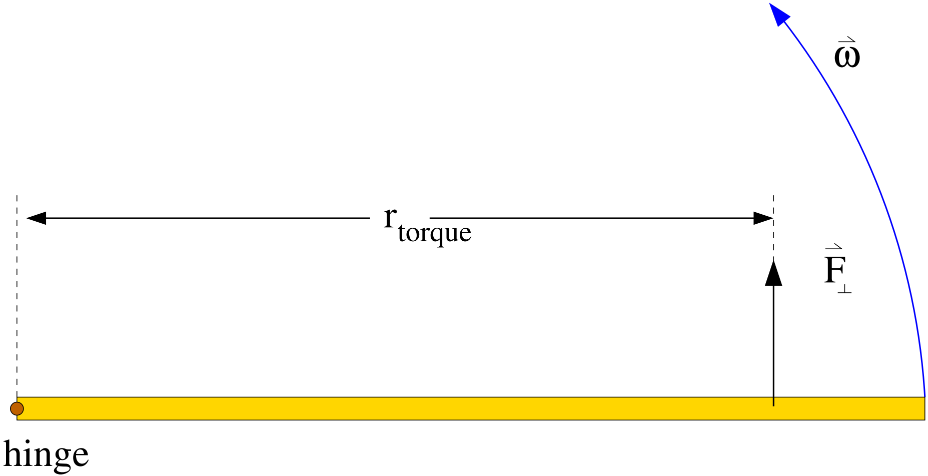


Consider a point particle at a fixed distance from the origin (attached by the famed massless rod or a string) that moves in a circle.

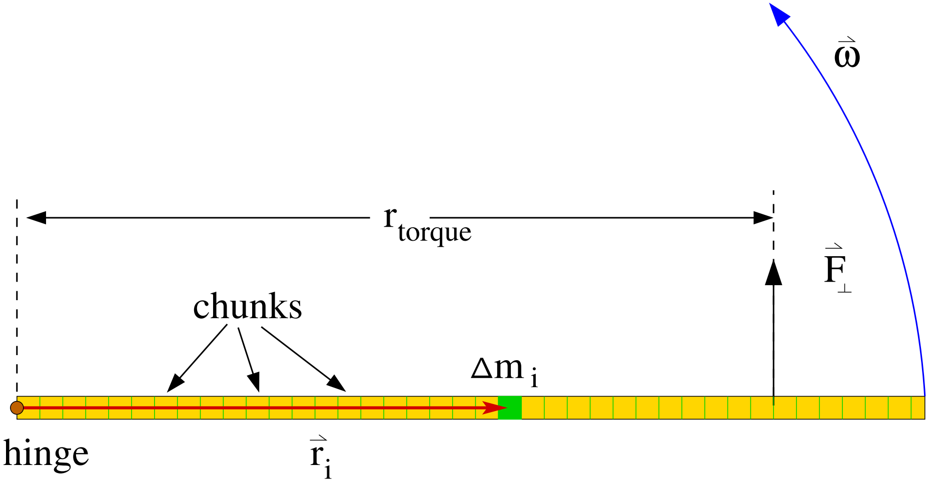


$$\vec{F} = m\vec{a} \rightarrow \vec{\tau} = r\vec{F}_{\perp}$$

Torque - Rotating a Rigid Body (a door)



Torque - Rotating a Rigid Body (a door)



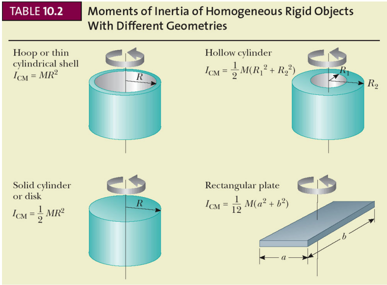
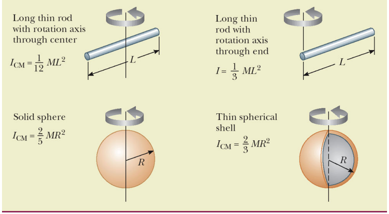


TABLE 10.2
Moments of Inertia of Homogeneous Rigid Objects With Different Geometries
Hoop or thin cylindrical shell
$I_{CM} = MR^2$
R
Hollow cylinder
$I_{CM} = \frac{1}{2}M(R_1^2 + R_2^2)$
R_1
R_2
Solid cylinder or disk
$I_{CM} = \frac{1}{2}MR^2$
R
Rectangular plate
$I_{CM} = \frac{1}{12}M(a^2 + b^2)$
a
b

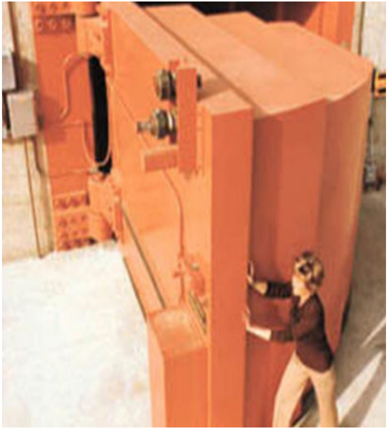


Long thin rod with rotation axis through center
$I_{CM} = \frac{1}{12}ML^2$
L
Long thin rod with rotation axis through end
$I = \frac{1}{3}ML^2$
L
Solid sphere
$I_{CM} = \frac{2}{5}MR^2$
R
Thin spherical shell
$I_{CM} = \frac{2}{3}MR^2$
R

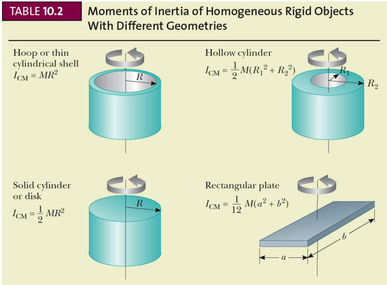
Linear Quantity	Connection	Rotational Quantity
s	$s = r\theta$	$\theta = \frac{s}{r}$
v_T	$v_T = r\omega$	$\omega = \frac{v_T}{r} = \frac{d\theta}{dt}$
a	$a_T = r\alpha$	$\alpha = \frac{a_T}{r} = \frac{d\omega}{dt}$
$KE = \frac{1}{2}mv^2$		$KE_R = \frac{1}{2}I\omega^2$
$\vec{F} = m\vec{a}$	$\tau = rF_\perp$	$\vec{\tau} = I\vec{\alpha}$
$\vec{p} = m\vec{v}$	$L = rp_\perp$	$\vec{L} = I\vec{\omega}$

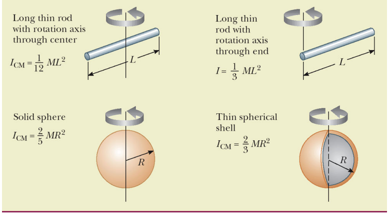
$$\vec{r}_{cm} = \frac{\sum m_i \vec{r}_i}{\sum m_i}$$

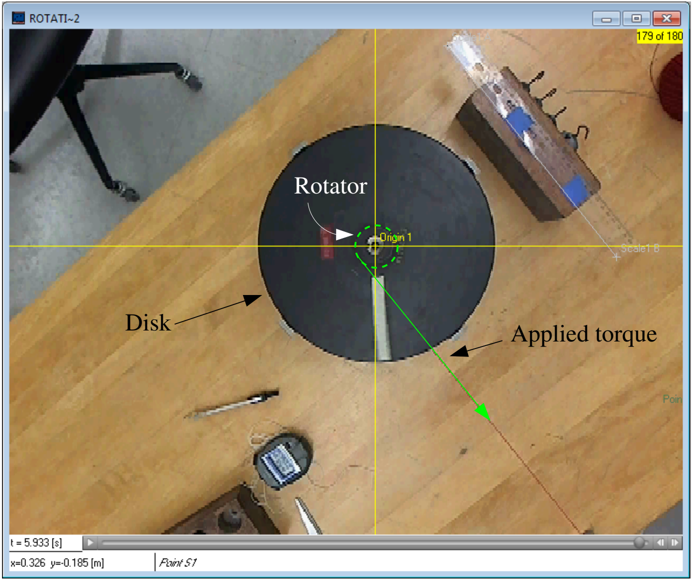
The shield door at a neutron test facility at Lawrence Livermore Laboratory is possibly the world's heaviest hinged door. It has a mass $m = 44,000\ kg$, a rotational inertia about a vertical axis through its hinges of $I = 8.7 \times 10^4\ kg - m^2$, and a (front) face width of $w = 2.4\ m$. A steady force $\vec{F}_a = 73\ N$, applied at its outer edge and perpendicular to the plane of the door, can move it from rest through an angle $\theta = 90^\circ$ in $\Delta t = 75\ s$. What is the torque exerted by the friction in the hinges?



Moments of Inertia

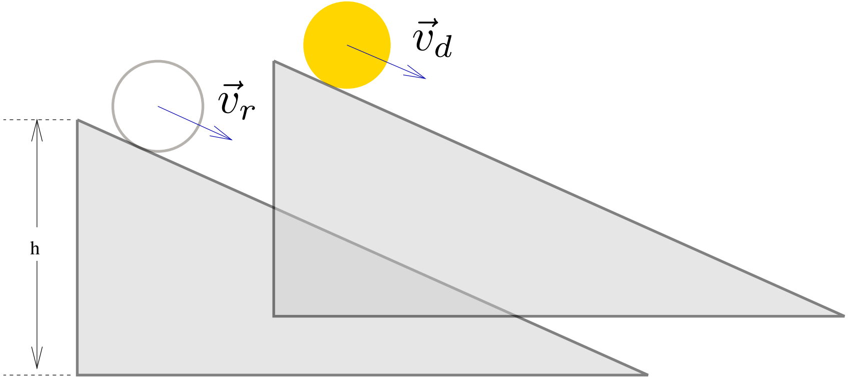




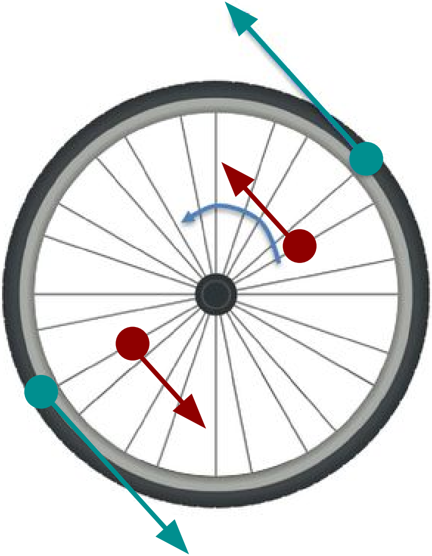


ROTATI~2
179 of 180
Rotator
Origin 1
Scale1 B
Disk
Applied torque
t = 5.933 [s]
x=0.326 y=-0.185 [m]

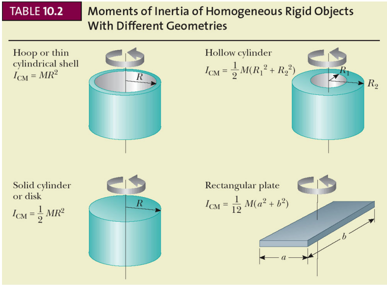
A wooden disk and a metal ring have the same mass *m* and radius *r*, start from rest, and roll down identical inclined planes (see figure). Which one wins?

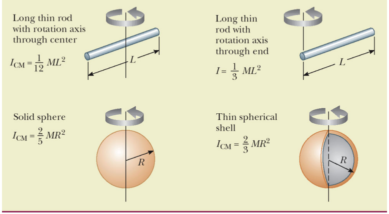


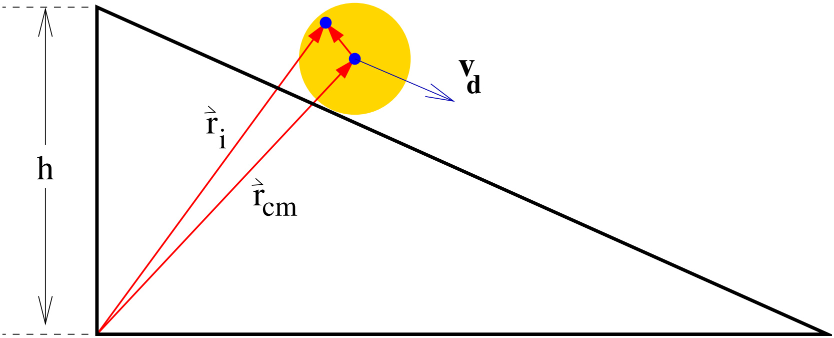
Which One Wins?



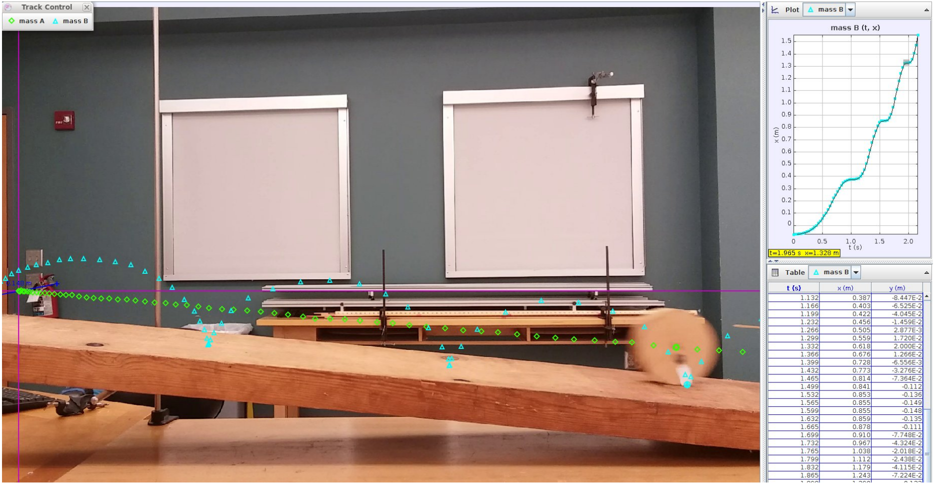
Moments of Inertia



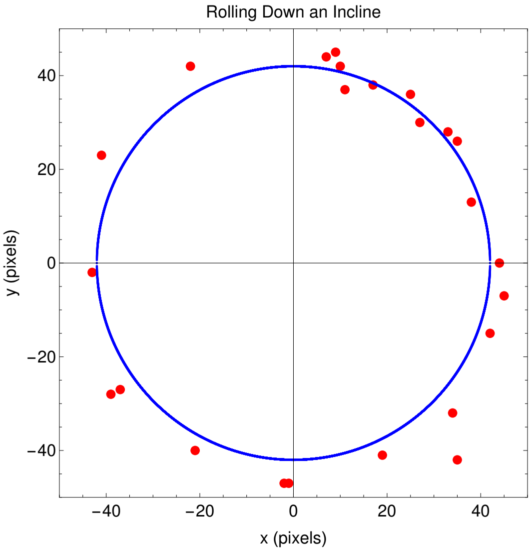



h
$\vec{r}_i$
$\vec{r}_{cm}$
$\mathbf{v_d}$

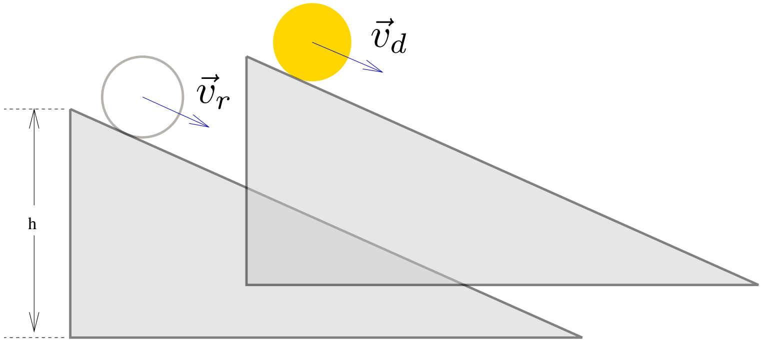
t (s)	x (m)	y (m)
0.433	7.106E-2	-5.746E-3
0.466	8.227E-2	-7.302E-3
0.500	9.379E-2	-7.614E-3
0.533	0.106	-8.547E-3
0.566	0.120	-1.104E-2
0.600	0.134	-1.322E-2
0.633	0.149	-1.384E-2
0.666	0.164	-1.477E-2
0.699	0.180	-1.508E-2
0.733	0.197	-1.664E-2
0.766	0.214	-1.726E-2
0.799	0.232	-1.882E-2
0.833	0.251	-2.100E-2
0.866	0.270	-2.287E-2
0.899	0.292	-2.411E-2
0.933	0.312	-2.660E-2
0.966	0.333	-2.816E-2
0.999	0.356	-3.127E-2
1.032	0.378	-3.376E-2
1.066	0.404	-3.563E-2
1.099	0.427	-3.750E-2
1.132	0.453	-3.937E-2



t (s)	x (m)	y (m)
1.132	0.387	-8.447E-2
1.166	0.403	-6.525E-2
1.199	0.422	-4.045E-2
1.232	0.456	-1.459E-2
1.266	0.505	2.877E-3
1.299	0.559	1.720E-2
1.332	0.618	2.000E-2
1.366	0.676	1.266E-2
1.399	0.728	-6.556E-3
1.432	0.773	-3.276E-2
1.465	0.814	-7.364E-2
1.499	0.841	-0.112
1.532	0.853	-0.136
1.565	0.855	-0.149
1.599	0.855	-0.148
1.632	0.859	-0.135
1.665	0.878	-0.111
1.699	0.910	-7.748E-2
1.732	0.967	-4.324E-2
1.765	1.038	-2.018E-2
1.799	1.112	-2.438E-2
1.832	1.179	-4.115E-2
1.865	1.243	-7.224E-2


Rolling Down an Incline
40
20
0
−20
−40
y (pixels)
−40
−20
0
20
40
x (pixels)

A wooden disk and a metal ring have the same mass *m* and radius *r*, start from rest, and roll down identical inclined planes (see figure). Which one wins?



Linear Quantity	Connection	Rotational Quantity
s	$s = r\theta$	$\theta = \frac{s}{r}$
v_T	$v_T = r\omega$	$\omega = \frac{v_T}{r} = \frac{d\theta}{dt}$
a	$a_T = r\alpha$	$\alpha = \frac{a_T}{r} = \frac{d\omega}{dt}$
$KE = \frac{1}{2}mv^2$		$KE_R = \frac{1}{2}I\omega^2$
$\vec{F} = m\vec{a}$	$\tau = rF_\perp$	$\vec{\tau} = I\vec{\alpha}$
$\vec{p} = m\vec{v}$	$L = rp_\perp$	$\vec{L} = I\vec{\omega}$

$$\vec{r}_{cm} = \frac{\sum m_i \vec{r}_i}{\sum m_i}$$

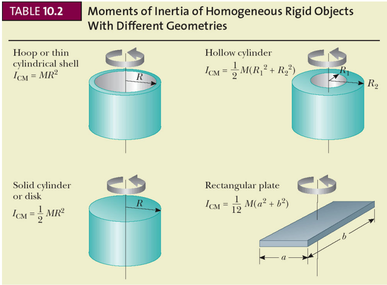
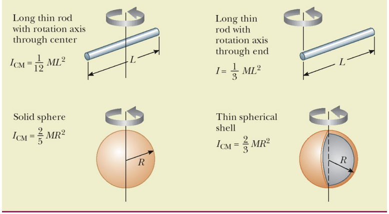


TABLE 10.2
Moments of Inertia of Homogeneous Rigid Objects With Different Geometries
Hoop or thin cylindrical shell
$I_{CM} = MR^2$
R
Hollow cylinder
$I_{CM} = \frac{1}{2}M(R_1^2 + R_2^2)$
R_1
R_2
Solid cylinder or disk
$I_{CM} = \frac{1}{2}MR^2$
R
Rectangular plate
$I_{CM} = \frac{1}{12}M(a^2 + b^2)$
a
b

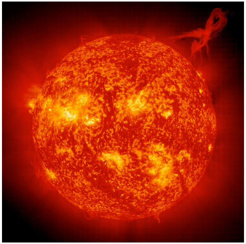

Long thin rod with rotation axis through center
$I_{CM} = \frac{1}{12}ML^2$
L
Long thin rod with rotation axis through end
$I = \frac{1}{3}ML^2$
L
Solid sphere
$I_{CM} = \frac{2}{5}MR^2$
R
Thin spherical shell
$I_{CM} = \frac{2}{3}MR^2$
R

The molecular cloud B68 in the constellation Ophiuchus is rotating with an angular speed $\omega = 9.4 \times 10^{-14}$ rad/s. The gravitational attraction among the atoms in the cloud may make it collapse until the core is hot enough to ignite nuclear reactions and B68 will begin to shine. If the final properties of B68 are the same as our Sun, *i.e.*, the same mass and size, then what will be its final angular velocity and period? Assume the lost mass carries away very little angular momentum. Compare this with the angular velocity of the Sun. Is your result reasonable? Why or why not?

$M_{B68} = 6.04 \times 10^{30}\ kg \quad I_{B68} = 2.7 \times 10^{54}\ kg - km^2 \quad M_{Sun} = 1.989 \times 10^{30}\ kg$

$R_{Sun} = 6.96 \times 10^{5}\ km \quad T_{Sun} = 25.4\ d$

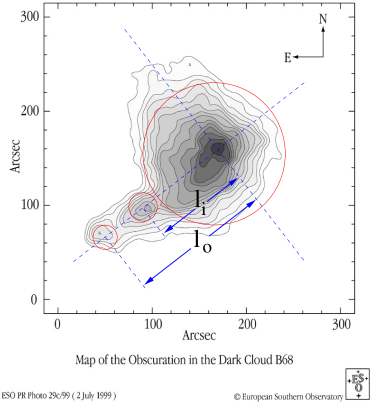
$\longrightarrow$



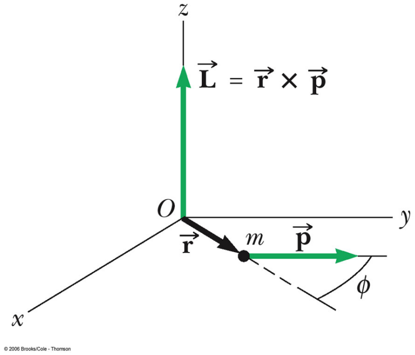
The plot below shows the 'obscuration' in the angular area around B68 based on measurements of background stars. The light in the center is 10^{14} dimmer than outside the edge of the cloud. To make life simple we will treat the mass distribution of B68 as three, rigid, uniform spheres that lie along the axis shown in the figure and rotate with $\omega = 9.4 \times 10^{-14}$ rad/s. The spheres do NOT rotate independently of the rest of the cloud. The origin is at the center of the central lobe. What is the moment of inertia of the cloud?

Lobe	Radius (km)	Mass (kg)
central	$R_c = 1.0 \times 10^{12}$	$m_c = 6.0 \times 10^{30}$
inner	$R_i = 2.0 \times 10^{11}$	$m_i = 4.6 \times 10^{28}$
outer	$R_o = 1.7 \times 10^{11}$	$m_o = 2.9 \times 10^{28}$

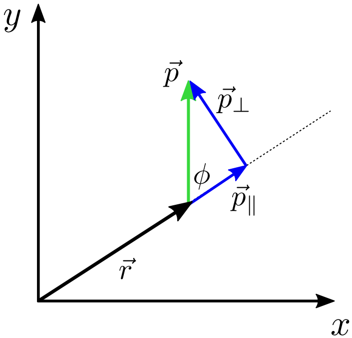
origin - inner cloud center	$l_i = 1.4 \times 10^{12}$ km
origin - outer cloud center	$l_o = 2.0 \times 10^{12}$ km



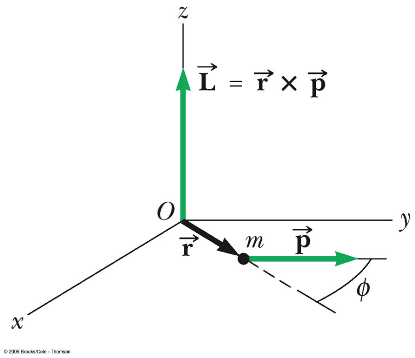
Map of the Obscuration in the Dark Cloud B68

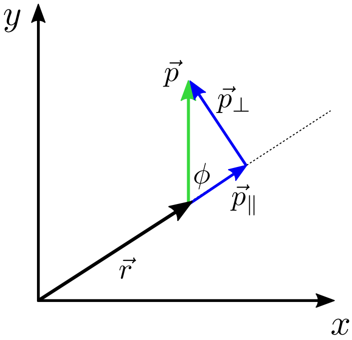


z
$\vec{L} = \vec{r} \times \vec{p}$
O
y
$\vec{r}$
m
$\vec{p}$
ϕ
x
© 2006 Brooks/Cole - Thomson



y
$\vec{p}$
$\vec{p}_\perp$
ϕ
$\vec{p}_\parallel$
$\vec{r}$
x





$$|\vec{L}| = rp_\perp = I\omega$$

Linear Quantity	Connection	Rotational Quantity
s	$s = r\theta$	$\theta = \frac{s}{r}$
v_T	$v = r\omega$	$\omega = \frac{v}{r} = \frac{d\theta}{dt}$
a	$a = r\alpha$	$\alpha = \frac{a}{r} = \frac{d\omega}{dt}$
$\vec{F} = m\vec{a} = \frac{d\vec{p}}{dt}$	$\tau = rF_\perp$	$\vec{\tau} = I\vec{\alpha} = \frac{d\vec{L}}{dt}$
$KE = \frac{1}{2}mv^2$		$KE_R = \frac{1}{2}I\omega^2$
$\vec{p} = m\vec{v}$	$L = rp_\perp$	$\vec{L} = I\vec{\omega}$

$$\vec{r}_{cm} = \frac{\sum m_i \vec{r}_i}{\sum m_i} \qquad I = \sum m_i r_i^2$$

$$\vec{F}_{AB} = -\vec{F}_{BA}$$

$$m_A \vec{a}_A = -m_B \vec{a}_B$$

$$m_A \frac{d\vec{v}_A}{dt} = -m_B \frac{d\vec{v}_B}{dt}$$

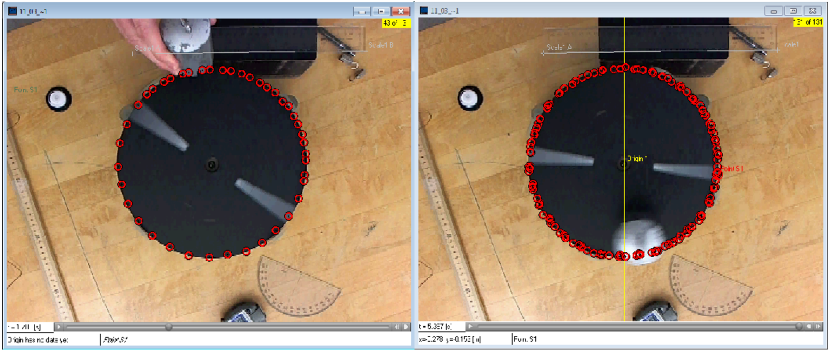
$$\frac{dm_A \vec{v}_A}{dt} = -\frac{dm_B \vec{v}_B}{dt}$$

$$\frac{d\vec{p}_A}{dt} = -\frac{d\vec{p}_B}{dt}$$

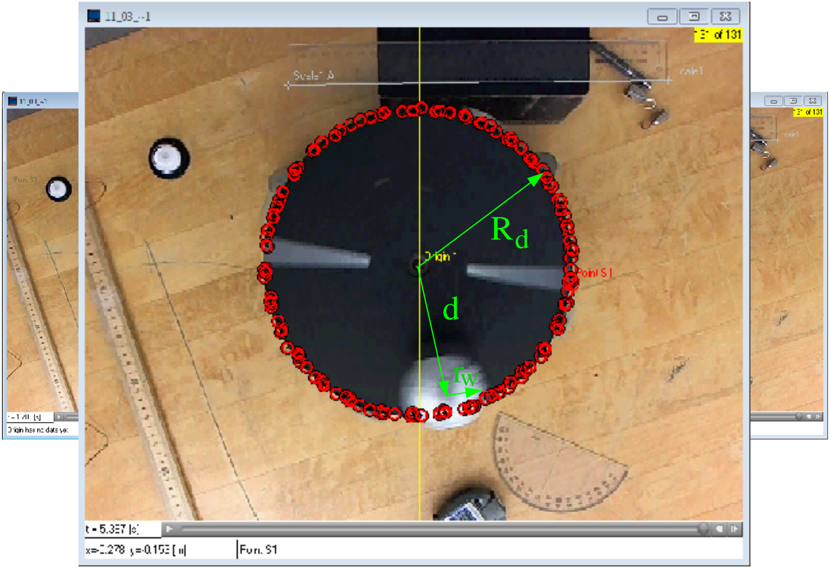
$$\frac{d\vec{p}_A}{dt} + \frac{d\vec{p}_B}{dt} = 0$$

$$\frac{d}{dt}\left(\vec{p}_A + \vec{p}_B\right) = 0$$

$$\therefore\ \vec{p}_A + \vec{p}_B = const$$



Angular Momentum Conservation



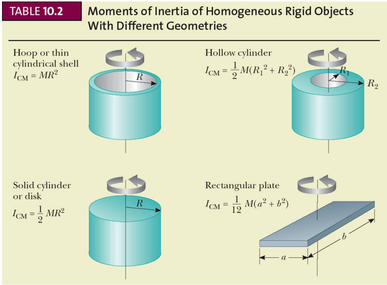
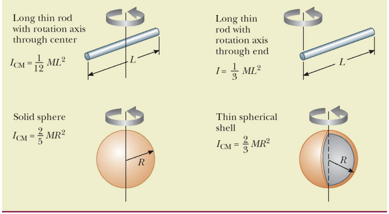


TABLE 10.2
Moments of Inertia of Homogeneous Rigid Objects With Different Geometries
Hoop or thin cylindrical shell
$I_{CM} = MR^2$
R
Hollow cylinder
$I_{CM} = \frac{1}{2}M(R_1^2 + R_2^2)$
R_1
R_2
Solid cylinder or disk
$I_{CM} = \frac{1}{2}MR^2$
R
Rectangular plate
$I_{CM} = \frac{1}{12}M(a^2 + b^2)$
b
a

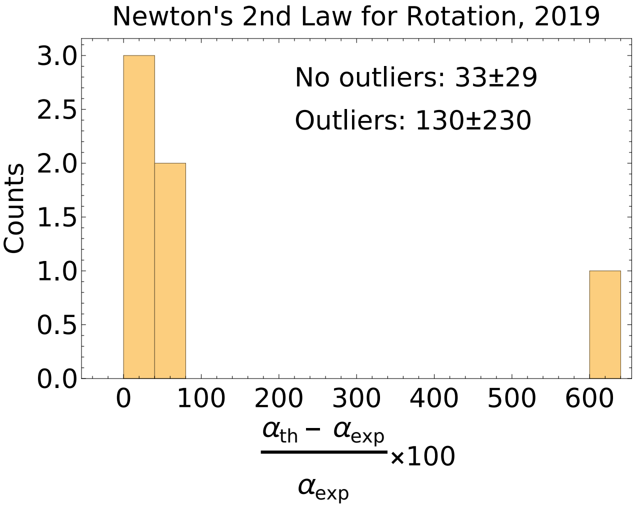

Long thin rod with rotation axis through center
$I_{CM} = \frac{1}{12}ML^2$
L
Long thin rod with rotation axis through end
$I = \frac{1}{3}ML^2$
L
Solid sphere
$I_{CM} = \frac{2}{5}MR^2$
R
Thin spherical shell
$I_{CM} = \frac{2}{3}MR^2$
R

A Star Is Born

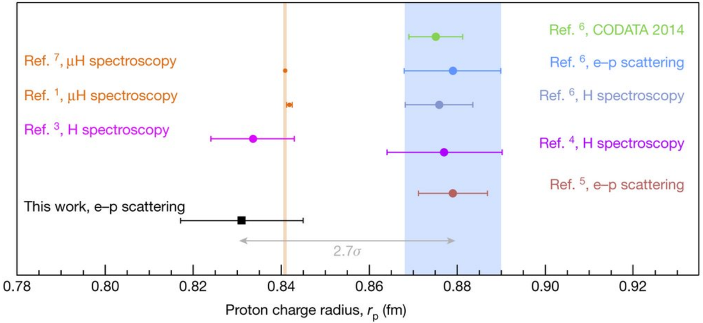
The molecular cloud B68 in the constellation Ophiuchus is rotating with an angular speed $\omega = 9.4 \times 10^{-14}\ rad/s$. The gravitational attraction among the atoms in the cloud may make it collapse until the core is hot enough to ignite nuclear reactions and B68 will begin to shine. If the final properties of B68 are the same as our Sun, *i.e.*, the same mass and size, then what will be its final angular velocity and period? Assume the lost mass carries away very little angular momentum. Compare this with the angular velocity of the Sun. Is your result reasonable? Why or why not?

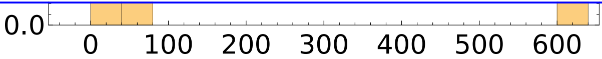
$M_{B68} = 6.04 \times 10^{30}\ kg$ $\quad I_{B68} = 2.7 \times 10^{54}\ kg - km^2$ $\quad M_{Sun} = 1.989 \times 10^{30}\ kg$

$R_{Sun} = 6.96 \times 10^{5}\ km$ $\quad T_{Sun} = 25.4\ d$


Newton's 2nd Law for Rotation, 2019
No outliers: 33±29
Outliers: 130±230
Counts
3.0
2.5
2.0
1.5
1.0
0.5
0.0
0
100
200
300
400
500
600
$\frac{\alpha_{th} - \alpha_{exp}}{\alpha_{exp}} \times 100$

Nature volume 575, pages 147-150, Nov 6 (2019)





$$\frac{\alpha_{th} - \alpha_{exp}}{\alpha_{exp}} \times 100$$