## Physics 131-01 Test 3

I pledge that I have neither given nor received unauthorized assistance during the completion of this work.

Name $\qquad$ Signature $\qquad$
Questions (5 for 8 pts. apiece) Answer in complete, well-written sentences WITHIN the spaces provided.

1. Suppose a ball of mass $m=0.20 \mathrm{~kg}$ is dropped and falls toward the surface of the moon so that it hits the ground with a speed of $v=40 \mathrm{~m} / \mathrm{s}$ and rebounds with the same speed. Will the astronaut notice the jerk as the moon recoils from him? Why or why not? Explain your reasoning.

2. The figure below shows a massless string wound around a spool of radius $r$. The mass falls with a constant acceleration, $a$. What is the equation for $y$ in terms of $\theta$ and $r$ ? Start from the expression for uniform acceleration in the $y$-direction for the falling mass and clearly show any steps or substitutions you make.

3. Does the period of simple harmonic motion (SHM) depend on the mass? Starting from the mathematical tools we developed to describe SHM (i.e. the equations on page 4) to determine the relationship between the period $T$ and the mass $m$.
4. Consider a rotational 'collision' like we explored in lab. A disk of mass $M_{d}$ and radius $R_{d}$ is spinning at an angular velocity $\omega_{i}$. A cylindrical weight of mass $m_{w}$ and radius $r_{w}$ is dropped onto the disk and sticks at a distance $d$ from the axis of rotation of the disk to the center of the weight. The final angular velocity after the collision is $\omega_{f}$. What is the moment of inertia of the system before ( $I_{\text {before }}$ ) and after $\left(I_{\text {after }}\right)$ the collision in terms of the quantities above. Explain your reasoning.
5. The solid cylinder ( 1 in the figure below) and the cylindrical shell ( 2 in the figure) below have the same mass $m$, same radius $r$, and turn on frictionless, horizontal axles. The cylindrical shell has lightweight spokes connecting the shell to the axle with essentially zero mass. A rope is wrapped around each cylinder and tied to a block. The blocks have the same mass $m_{b}$ and are held the same height above the ground. Both blocks are released simultaneously. Which hits the ground first? Or is it a tie? Explain.


Problems (3). Clearly show all reasoning for full credit. Use a separate sheet to show your work.

1. 10 pts. A block attached to a spring with unknown spring constant oscillates with a period of $T=2.0 \mathrm{~s}$. What is the period if the spring constant is doubled? Start from the equations on page 4.
2. $\quad 10$ pts. If the displacement $x$ of a mass $m$ on a spring is half the amplitude $A$, what is the fraction of the total energy that is in potential energy?
3. 20 pts. A tennis ball is a hollow sphere with a thin wall. It is set rolling without slipping at $5.0 \mathrm{~m} / \mathrm{s}$ on a horizontal section of a track as shown in the figure. It rolls around the inside of a vertical circular loop with radius $r=0.4 \mathrm{~m}$. What is the speed of the ball at the top of the loop?

4. 20 pts. A spacecraft is in empty space. It carries on board a gyroscope which is a device that changes the angular position of the spacecraft. It consists of a spinning wheel whose angular velocity is controlled by a motor. The one in the spacecraft has a moment of inertia of $I_{g}=20.0 \mathrm{~kg} \cdot \mathrm{~m}^{2}$ about the axis of the gyroscope. The moment of inertia of the rest of the spacecraft around the same axis is $I_{s}=5.00 \times 10^{5} \mathrm{~kg} \cdot \mathrm{~m}^{2}$. Neither the spacecraft nor the gyroscope is initially rotating. The gyroscope can be powered up in a negligible period of time to an angular speed of $100 \mathrm{rad} / \mathrm{s}$. With the gyroscope running, how long $\Delta t$ will it take to rotate the rest of the spacecraft by $\Delta \theta=30.0^{\circ}=0.52 \mathrm{rad}$ about the same axis?

## Physics 131-1 Equations and Constants, Test 3

$$
\begin{aligned}
& \Delta \vec{r}=\vec{r}_{\text {finish }}-\vec{r}_{\text {start }} \quad \Delta \vec{v}=\vec{v}_{\text {finish }}-\vec{v}_{\text {start }} \\
& \langle\vec{v}\rangle=\frac{\Delta \vec{r}}{\Delta t} \quad \vec{v}=\frac{d \vec{r}}{d t}=\lim _{\Delta t \rightarrow 0} \frac{\Delta \vec{r}}{\Delta t}=\lim _{\Delta t \rightarrow 0} \frac{\vec{r}(t+\Delta t)-\vec{r}(t)}{\Delta t} \\
& \langle\vec{a}\rangle=\frac{\Delta \vec{v}}{\Delta t} \quad \vec{a}=\frac{d \vec{v}}{d t}=\lim _{\Delta t \rightarrow 0} \frac{\Delta \vec{v}}{\Delta t}=\lim _{\Delta t \rightarrow 0} \frac{\vec{v}(t+\Delta t)-\vec{v}(t)}{\Delta t} \\
& x(t)=\frac{1}{2} a t^{2}+v_{0} t+y_{0} \quad v=a t+v_{0} \quad a=g \quad a_{c}=\frac{v^{2}}{r} \quad\left(\vec{v} \perp \vec{r} \quad \vec{v} \perp \vec{a}_{c}\right) \\
& \vec{F}_{n e t}=\sum_{i} \vec{F}_{i}=m \vec{a}=\frac{d \vec{p}}{d t} \quad \vec{F}_{A B}=-\vec{F}_{B A} \quad \vec{p}=\sum m_{i} \vec{v}_{i} \quad \vec{p}_{i}=\vec{p}_{f} \\
& \left|\vec{F}_{f}\right|=\mu N \quad\left|\vec{F}_{c}\right|=m \frac{v^{2}}{r} \quad\left|\vec{F}_{G}\right|=\frac{G m_{1} m_{2}}{r^{2}} \quad \vec{F}_{s}(x)=-k x \hat{i} \quad \vec{F}_{g}(y)=-m g \hat{j} \\
& W=\int \vec{F} \cdot d \vec{s}=\int|\vec{F}||\overrightarrow{d s}| \cos \theta=\Delta K E=-\Delta U \quad K E=\frac{1}{2} m v^{2} \quad K E_{i}=K E_{f} \quad \text { elastic } \\
& K E_{i}+U_{i}=K E_{f}+U_{f} \quad K E=K E_{c m}+K E_{\text {rot }} \quad K E_{r o t}=\frac{1}{2} I \omega^{2} \quad U_{s}(x)=\frac{1}{2} k x^{2} \quad U_{g}(y)=m g y \\
& \theta=\frac{s}{r} \quad \omega=\frac{v_{\perp}}{r}=\frac{d \theta}{d t} \quad \alpha=\frac{a_{\perp}}{r}=\frac{d \omega}{d t} \quad I=\sum m_{i} r_{i}^{2}=I_{c m}+M h^{2} \quad \vec{r}_{c m}=\frac{\sum m_{i} \vec{r}_{i}}{\sum m_{i}} \\
& \vec{\tau}=r F \sin \phi \hat{\theta}=I \vec{\alpha} \quad \vec{L}=\sum_{j} I_{j} \vec{\omega}_{j} \quad \vec{L}_{i}=\vec{L}_{f} \quad v_{c m}=r \omega \quad \theta=\frac{\alpha}{2} t^{2}+\omega_{i} t+\theta_{i} \quad \omega=\alpha t+\omega_{i} \\
& x(t)=A \cos (\omega t+\phi) \quad \omega^{2}=\frac{k}{m} \quad T=\frac{2 \pi}{\omega}=\frac{1}{f} \quad P E=\frac{1}{2} k x^{2} \quad M E=\frac{1}{2} k A^{2} \\
& \vec{A}=A_{x} \hat{i}+A_{y} \hat{j}+A_{z} \hat{k} \frac{d A}{d t}=0 \frac{d t}{d t}=1 \frac{d t^{2}}{d t}=2 t \frac{d}{d t} \cos \theta=-\sin \theta \frac{d}{d t} \sin \theta=\cos \theta \\
& \int f(x) d x=\lim _{\Delta x \rightarrow 0} \sum f\left(x_{i}\right) \Delta x \quad \int d x=x+c \quad \int x d x=\frac{x^{2}}{2}+c \\
& \sin \theta=\frac{o p p}{h y p} \quad \cos \theta=\frac{a d j}{h y p} \quad \tan \theta=\frac{o p p}{a d j} \quad x^{2}+y^{2}+z^{2}=R^{2} \quad \rho=\frac{m}{V} \\
& x=\frac{-b \pm \sqrt{b^{2}-4 a c}}{2 a} \quad C=2 \pi r \quad \text { Area }=\pi r^{2} \quad \text { Area }=\frac{1}{2} b h \quad \text { Area }=4 \pi r^{2}
\end{aligned}
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| Speed of Light $(c)$ | $2.9979 \times 10^{8} \mathrm{~m} / \mathrm{s}$ | proton/neutron mass | $1.67 \times 10^{-27} \mathrm{~kg}$ |
| :--- | :--- | :--- | :--- |
| $R$ | $8.31 \mathrm{~J} / \mathrm{K}-\mathrm{mole}$ | $g$ | $9.8 \mathrm{~m} / \mathrm{s}^{2}$ |
| Gravitation constant | $6.67 \times 10^{-11} \mathrm{~N}-\mathrm{m}^{2} / \mathrm{kg}^{2}$ | Earth's radius | $6.37 \times 10^{6} \mathrm{~m}$ |
| Earth-Moon distance | $3.84 \times 10^{8} \mathrm{~m}$ | Electron mass | $9.11 \times 10^{-31} \mathrm{~kg}$ |

## TABLE 10.2 Moments of Inertia of Homogeneous Rigid Objects With Different Geometries



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Long thin rod with rotation axis through end
$I=\frac{1}{3} M L^{2}$


Solid sphere
$I_{\mathrm{CM}}=\frac{2}{5} M R^{2}$


Thin spherical shell
$I_{\mathrm{CM}}=\frac{2}{3} M R^{2}$


