Where’s the heat?

The Earth’s troposphere - where we live - is getting warmer. The science of this phenomenon is complex, but we can start by building ‘simple’ models with physics ideas like conservation of energy and momentum that derive from Newton’s laws. We will focus on two questions.

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2. What is the average temperature of the Earth’s surface?
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## Module structure

<table>
<thead>
<tr>
<th>F1</th>
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<td>Chemistry</td>
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<td>Chemistry</td>
<td>Biology</td>
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</table>

## Module S1: Typical Week

<table>
<thead>
<tr>
<th>Monday</th>
<th>Tuesday</th>
<th>Wednesday</th>
<th>Thursday</th>
<th>Friday</th>
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<tbody>
<tr>
<td>1:30 – 3:30 Workshop</td>
<td>1:30 – 3:30</td>
<td>1:30 – 3:30 Lab</td>
<td></td>
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</tbody>
</table>
Where’s the Heat?

Reconstructed Temperature

Medieval Warm Period

Little Ice Age

Temperature Anomaly (°C)

0 200 400 600 800 1000 1200 1400 1600 1800 2000

2004∗
Where’s the heat?

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1. How much heat has gone into the atmosphere?
2. What is the average temperature of the Earth’s surface?
The Plan

Newton’s Laws

Conservation Laws

Energy and momentum

Kinematics

Laws of Thermodynamics

Atoms

Kinetic Theory

Calorimetry

Specific heats

Heat in the Atmosphere

Temperature of the Earth

Stefan’s Law
A cart is pulled across a flat surface with a rope at an angle $\theta = 60^\circ$ to the horizontal for a distance $x = 3 \text{ m}$. The magnitude of the force is $|\vec{F}| = 3 \text{ N}$ and the mass of the cart is $m = 5 \text{ kg}$. Assume the cart rolls with no effect due to friction. What is the work done by the force?
Work and Variable Forces
Work and Variable Forces

\[ F(x) \]

Jerry Gilfoyle

Where’s the heat?
Work and Variable Forces

\[ F(x) \]

\[ F(x) \]

Jerry Gilfoyle

Where's the heat?
A hanging spring, when stretched, exerts a restoring force that pulls the spring back to its equilibrium position.

\[ \vec{F}_s = -k\vec{y} \]

The vector \( \vec{y} \) is the displacement of the end of the spring from its equilibrium position. A one-dimensional force \( F_1 = 5 \, N \) is applied to a spring stretching it from its relaxed, equilibrium state a distance of \( |\vec{y}_1| = y_1 = 0.12 \, m \). Then, an additional force \( F_2 = 2 \, N \) is added and the spring stretches another \( |\Delta y| = 0.05 \, m \). What is the work done by the spring for this last part? The spring constant is \( k = 42 \, N/m \).
Mechanical Energy Conservation

Position
(m)

Time (s)

Velocity
(m/s)

Time (s)

Where's the heat?
‘Proof’ of Mechanical Energy Conservation

Red – Total energy $\langle E \rangle = 0.35 \pm 0.03 \; J$

Blue – Potential energy

Green – Kinetic energy

Where’s the heat?
Explaining the Scatter in the Data

Start

End
Quarks on Springs

Two quarks, an up and an anti-bottom are bound together to form a B meson. The force between the quarks can be modeled as a spring force. What is the form of the potential energy? If the spring with the up quark attached is stretched a distance $x_i$ from equilibrium and released from rest, then how is the kinetic energy related to the initial potential energy when it passes through the equilibrium point? If $x_i = 1.2 \times 10^{-15}$ m, what is the speed of the up quark when the spring passes through its equilibrium point? The anti-bottom quark is fixed. The spring constant is $k = 6.0 \times 10^{17}$ N/m and the up quark has $m_q = 1.4 \times 10^{-28}$ kg.
A subatomic particle known as a $\Lambda_0$ decays from rest by emitting a proton of kinetic energy $E_1 = 10 \text{ MeV}$ and a second unknown particle of kinetic energy $E_2 = 67 \text{ MeV}$. Identify the unknown particle $x$ using the table of particle masses below.

<table>
<thead>
<tr>
<th>Particle</th>
<th>Mass (MeV/c^2)</th>
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</thead>
<tbody>
<tr>
<td>Electron (e)</td>
<td>0.551</td>
</tr>
<tr>
<td>Muon ($\mu^\pm$)</td>
<td>106</td>
</tr>
<tr>
<td>Pion ($\pi^\pm$)</td>
<td>139</td>
</tr>
<tr>
<td>Kaon ($K^\pm$)</td>
<td>494</td>
</tr>
<tr>
<td>Eta ($\eta$)</td>
<td>549</td>
</tr>
<tr>
<td>Proton (p)</td>
<td>938</td>
</tr>
<tr>
<td>Neutron (n)</td>
<td>939</td>
</tr>
<tr>
<td>Lambda ($\Lambda_0$)</td>
<td>1116</td>
</tr>
</tbody>
</table>
‘Proof’ of Newton’s Third Law
‘Proof’ of Newton’s Third Law

Both moving, opposite directions, same mass
One heavy and moving, one light and stationary
One light and moving, one heavy and stationary
One moving, one stationary, same mass
One heavy and moving slow, one light and moving fast, same direction

Jerry Gilfoyle
Where’s the heat?
What Happened To The Dinosaurs?

Dinosaurs were the dominant vertebrate animals of terrestrial ecosystems for over 160 million years from about 230 million years ago to 65 million years ago. Recent research indicates that theropod dinosaurs are most likely the ancestors of birds and many were active animals with elevated metabolisms often with adaptations for social interactions. What caused them to largely disappear?
Evidence of an Asteroid Strike

1. The dinosaurs disappeared at the boundary between the Cretaceous and Tertiary Periods (the KT Boundary) about 65 million years ago.

2. The data show the abundance of iridium which is commonly found in meteorites and not on Earth. The horizontal axis is the iridium abundance and the vertical axis is the age of the sample with increasing age going down.

3. The large peak implies a large infusion of the atom coincident with the KT boundary. This peak was observed in rocks from Italy, Denmark, and New Zealand.

4. An impact crater the right size and age for a 10-km asteroid has been found on the Yucatan Peninsula near Chicxulub in Mexico.

The End of the Dinosaurs

It is now believed the dinosaurs and many other species were driven to extinction 65 million years ago by an ecological disaster brought on by the collision of an asteroid with the Earth. Consider the following scenario. The asteroid collides with the Earth as the Earth orbits the Sun and sticks to the surface as shown in the figure (a perfectly inelastic collision). How much does the velocity of the Earth change? How much energy is released in the collision? How does this compare with the energy released by the Hiroshima atomic bomb ($6.8 \times 10^{13} \text{ J}$)?

Asteroid mass: $m_A = 3.4 \times 10^{14} \text{ kg}$
Asteroid speed: $v_A = 2.5 \times 10^4 \text{ m/s}$
Earth mass: $m_E = 6.0 \times 10^{24} \text{ kg}$
Earth speed: $v_A = 3.0 \times 10^4 \text{ m/s}$
Angle: $\theta = 30^\circ$
Effects of the Chicxulub Asteroid Strike

1. Megatsunamis as high as 5 kilometers (3.1 mi); enough to completely inundate even large islands such as Madagascar.

2. Excavated material along with pieces of the impactor, ejected out of the atmosphere by the blast, would have been heated to incandescence upon re-entry, broiling the Earth’s surface and possibly igniting wildfires.

3. Colossal shock waves would have triggered global earthquakes and volcanic eruptions.

4. The emission of dust and particles could have covered the entire surface of the Earth for years, possibly a decade. Photosynthesis by plants would be interrupted, affecting the entire food chain.

5. Sunlight would have been blocked from reaching the surface of the earth by the dust particles in the atmosphere, cooling the surface dramatically.

6. It is estimated that 75% or more of all species on Earth vanished.
‘Proof’ of Newton’s Third Law

Where’s the heat?
Measurement and Uncertainty

Average and Standard Deviation

Same number of measurements with different standard deviations

Same average

Number of Measurements

X
Precision versus Accuracy

Not precise.
Average and Standard Deviation

Precise, but not accurate.
Average and Standard Deviation

Precise and accurate.
Average and Standard Deviation
More on Precision versus Accuracy-2

Physics 131, fall, 2011

\[ \bar{g} = 11.6 \pm 1.2 \text{ m/s}^2 \]
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For ‘simple’ distributions the average and standard deviation are useful. For other distributions, more information is needed.
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The Plan

- Newton’s Laws
- Conservation Laws
- Energy and momentum

- Kinematics
- Laws of Thermodynamics
- Atoms
- Kinetic Theory
- Calorimetry
- Specific heats

- Heat in the Atmosphere
- Temperature of the Earth
- Stefan’s Law
The Plan

- Newton’s Laws
  - Conservation Laws
    - Energy and momentum
  - Kinematics
    - Laws of Thermodynamics
      - Atoms
        - Kinetic Theory
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  - Stefan’s Law
Heat is the thermal energy transferred from one place or body to another due to a difference in temperature. Thermal energy is the mechanical energy (kinetic and potential) associated with the motion of the atoms within an object.
Measuring Temperature (Constant Volume)

Where's the heat?
Temperature and Heat - 1

Jerry Gilfoyle

Where's the heat?

Temperature versus Time

- Temperature, ChA Run #1
- Water and steam
- Water only
- Ice and water mixture
- Ice gone
- Stirred
Heat is thermal energy transferred from one place or body to another due to a difference in temperature. Thermal energy is the mechanical energy (kinetic and potential) associated with the atomic motion within an object.
Two ice cubes each with mass $m_i = 0.050 \text{ kg}$ are taken from a freezer at $T_0 = -15^\circ\text{C}$ and dropped into a container holding $m_w = 1.0 \text{ kg}$ of water at $T_1 = 25^\circ\text{C}$. What will be the final temperature of the liquid? Assume the container absorbs no heat.
Calorimetry

Two ice cubes each with mass \( m_I = 0.050 \ kg \) are taken from a freezer at \( T_0 = -15^\circ C \) and dropped into a container holding \( m_w = 1.0 \ kg \) of water at \( T_1 = 25^\circ C \). What will be the final temperature of the liquid? Assume the container absorbs no heat.
The First Law of Thermodynamics

Let 1.00 kg of liquid water at 100° C be converted to steam at 100° C. The water is contained in a cylinder with a movable piston of negligible mass that sits right on top of the water at the start. The volume changes from an initial value of 1.00 × 10⁻³ m² as a liquid to 1.671 m³ as steam. The latent heat of vaporization of water is \( L_V = 2.26 \times 10^6 \) J/kg and atmospheric pressure is \( P_{atm} = 1.01 \times 10^5 \) Pa.

1. How much work is done by this process?
2. How much heat must be added?
3. What is the change in the water’s internal energy?
Ideal Gases

A weather balloon is loosely inflated to a volume $V_0 = 2.2 \ m^3$ with helium at a pressure of $P_0 = 1.0 \times 10^5 \ Pa$ and a temperature $T_0 = 20^\circ C$. At an elevation of 20,000 ft the atmospheric pressure is down to $P_1 = 0.5 \times 10^5 \ Pa$ and the temperature is $T_1 = -48^\circ C$. The bag can expand freely. What is the new volume of the bag? What is the mass of the gas?
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The Plan

Newton’s Laws → Conservation Laws

Kinematics

Laws of Thermodynamics

Atoms → Kinetic Theory

Calorimetry → Specific heats

Heat in the Atmosphere → Temperature of the Earth

Stefan’s Law

Energy and momentum
The Kinetic Model of Ideal Gases

1. The gas consists of a large number of small, mobile particles and their average separation is large.
2. The particles obey Newton’s Laws and the conservation laws, but their motion can be described statistically.
3. The particles’ collisions are elastic.
4. The inter-particle forces are small until they collide.
5. The gas is pure.
6. The gas is in thermal equilibrium with the container walls.
The Plan

The Model

- Newtons’ Laws
- Ideal Gas Law
- First Law

Pressure of an Ideal Gas

Temperature of an Ideal Gas

Specific Heat of an Ideal Gas

Compare with data!!
The Data

Molar Specific Heat of Gases

Molecule

\[ \frac{C_v}{N_A k_B} \]

\( C_v \)

\( N_A \)

\( k_B \)

He Ar Ne Kr

\( \text{H}_2 \)\( \text{N}_2 \)\( \text{O}_2 \)\( \text{CO} \)

\( \text{Cl}_2 \)\( \text{H}_2\text{O} \)\( \text{CO}_2 \)\( \text{CH}_4 \)

Where's the heat?
A helium atom is moving straight up from the floor of the lab that is at room temperature $T = 300 \, K$. Miraculously, the atom never strikes another atom or molecule until it reaches the ceiling at a height $h = 4.0 \, m$ above the floor. What is the helium atom’s rms speed when it hits the ceiling? How much has its speed changed from the initial speed?
Applying Quantum Mechanics

How much heat does it take to increase the temperature of \( n = 4.0 \) moles of \( \text{H}_2 \) gas by \( \Delta T = 25 \) K at room temperature \( T = 25^\circ \text{C} \) if the gas is held at constant volume? Would the answer change if the gas were \( \text{N}_2 \)? What about \( \text{He} \)?
How much heat has gone into the atmosphere?

The troposphere is warming because the atmosphere is largely transparent to light from the Sun (≈ 70%) while the light emitted by Earth is largely blocked (≈ 20%) by the tropopause itself. This is the greenhouse effect.

Since 1985 the heat added to the Earth is about \( Q_E = 1.63 \times 10^{22} \) J. What effect would adding the amount of heat \( Q_E \) on the average temperature of the Earth’s atmosphere? The mass of the Earth’s atmosphere is \( m_A = 5.15 \times 10^{18} \) kg. The measured change in the Earth’s atmosphere is shown in the plot.
Where’s the heat?

The heat added to the Earth since 1985 would cause a much higher temperature increase than what is observed. Some other sink of heat is necessary to explain the measured temperature increase in the atmosphere.
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Where has the heat gone??!!
Where’s the heat?

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The oceans!!
Where’s the heat?

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Where has the heat gone??!!

THE OCEANS!!

\[ C_V(\text{water}) \approx 4 C_V(\text{N}_2) \]
\[ m(\text{oceans}) \approx 270 m_A \]
Where’s the heat?

The heat added to the Earth since 1985 would cause a much higher temperature increase than what is observed. Some other sink of heat is necessary to explain the measured temperature increase in the atmosphere.

Where has the heat gone??!!

THE OCEANS!!

\[
C_V(\text{water}) \approx 4C_V(\text{N}_2)
\]

\[
m(\text{oceans}) \approx 270m_A
\]

What is $\Delta T$ if we include the oceans?
Heat Transfer

1. Conduction - Heat flow via molecular agitation within a material without any motion of the material as a whole.

2. Convection - Heat flow via mass motion of a fluid when the hot fluid moves away from the source.

3. Radiation - Emission of electromagnetic waves (i.e. light) that carry energy away from the emitter.
Conduction - Heat flow via molecular agitation within a material without any motion of the material as a whole.

Convection - Heat flow via mass motion of a fluid when the hot fluid moves away from the source.

Radiation - Emission of electromagnetic waves (i.e. light) that carry energy away from the emitter. 

Main driver of climate change.
The Plan

- Newton’s Laws
- Conservation Laws
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- Kinematics
- Laws of Thermodynamics
- Atoms
- Kinetic Theory
- Calorimetry
- Specific heats
- Heat in the Atmosphere
- Temperature of the Earth
- Stefan’s Law

Where’s the heat?
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Newton’s Laws

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Heat in the Atmosphere

Temperature of the Earth

Stefan’s Law
How hot is the Sun?

The energy of the Sun is transmitted by radiation/light and this is the main mechanism for energy transfer. The solar ‘constant’ $I_S$ is the energy flux of the Sun on the Earth. It has a value $I_S = 1370 \ J/s - m^2$. What is the temperature of the Sun’s surface? Assume it has an emissivity of one $e_S = 1$. The Earth-Sun distance is $R_{ES} = 1.496 \times 10^{11} \ m$, the Sun’s radius is $R_S = 6.96 \times 10^8 \ m$, and the Stefan-Boltzmann constant is $\sigma = 5.67 \times 10^{-8} \ J/s - m^2 - K^4$. 

Jerry Gilfoyle

Where’s the heat?
What is the temperature of the Earth’s surface - v1?

The radiation from the Sun strikes the Earth and is reflected or absorbed. The absorbed part heats the Earth so that it starts to radiate. Eventually a steady state is reached with the power being absorbed by the Earth is radiated out into space. About 30% of the Sun’s light striking the Earth is immediately reflected. What is the average temperature of the Earth? This is the no-atmosphere model. The Earth’s radius $R_E$, emissivity $e_E$, and other constants are below.

$$R_E = 6.37 \times 10^6 \text{ m}$$

$$\sigma = 5.67 \times 10^{-8} \text{ J/s} - \text{m}^2 - \text{K}^4$$

$$I_S = 1370 \text{ J/s} - \text{m}^2$$

$$e_E = 1$$
The Lapse Rate

\[ \langle \gamma_t \rangle = -6.5 \, K/\text{km} \]

The lapse rate applies only to the troposphere. The temperature of the stratosphere is roughly constant.
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The lapse rate applies only to the troposphere. The temperature of the stratosphere is roughly constant.
The Lapse Rate - An Example

If the temperature at Base Camp in Nepal is $T_0 = 65^\circ F = 18.3^\circ C$ at an altitude of $y_0 = 18,000 ft = 5,500 m$, then what is the temperature at the summit of Mt. Everest where $y_1 = 29,029 ft = 8,848 m$? The lapse rate is $\gamma_t = -6.5 \ K/km$. 

Jerry Gilfoyle
Where's the heat?
The Lapse Rate - An Example

If the temperature at Base Camp in Nepal is $T_0 = 65^\circ F = 18.3^\circ C$ at an altitude of $y_0 = 18,000 ft = 5,500 m$, then what is the temperature at the summit of Mt. Everest where $y_1 = 29,029 ft = 8,848 m$? The lapse rate is $\gamma_t = -6.5 \, K/km$. 

[Diagram showing temperature decrease with elevation]
What is the average temperature of the Earth’s surface?

The Earth’s atmosphere has several components including the troposphere where we live, the stratosphere where the air density is low, and the narrow boundary between the two - the tropopause. The energy of the Sun is transmitted by radiation/light and this is the main mechanism for energy transfer between the atmosphere’s parts. Using the lapse rate, Stefan’s Law, and the conservation of energy, what is the average temperature at the surface of the Earth?
What is the average temperature of the Earth’s surface?

The stratosphere is mostly transparent to radiation from the Sun and the infrared radiation from the Earth. The troposphere absorbs most of the Earth’s infrared radiation. Assume the atmosphere is 70% transparent for light with $\lambda \leq 5 \, \mu m$ and the troposphere completely absorbs light with $\lambda > 5 \, \mu m$.

1. Consider the relationship between the incoming radiation from the Sun striking the Earth and the outgoing heat radiation from the Earth. What is the average temperature of the troposphere?

2. The thin air of the stratosphere means its temperature is roughly constant. What is the temperature of the stratosphere?

3. How does the temperature vary with height in the troposphere? Knowing the temperature of the stratosphere, what is the temperature at the surface of the Earth?
What is the average temperature of the Earth’s surface?

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Consider the relationship between the incoming radiation from the Sun striking the Earth and the outgoing heat radiation from the Earth. What is the average temperature of the troposphere?

The thin air of the stratosphere means its temperature is roughly constant. What is the temperature of the stratosphere?

How does the temperature vary with height in the troposphere?

Knowing the temperature of the stratosphere, what is the temperature at the surface of the Earth?
1. Average temperature of the troposphere

Consider the relationship between the incoming radiation from the Sun striking the Earth and the outgoing heat radiation from the Earth. What is the average temperature of the troposphere?

\[
I_s \quad \text{Solar constant} \quad 1370 \text{J/s} - \text{m}^2
\]
\[
A_c \quad \text{Earth’s cross-sectional area} \quad \pi R_E^2
\]
\[
t_E \quad \text{fraction of light transmitted} \quad 0.7
\]
\[
R_E \quad \text{Earth’s radius} \quad 6.37 \times 10^6 \text{ m}
\]
\[
e_E \quad \text{emissivity of Earth} \quad 1.0
\]
\[
A_s \quad \text{Earth’s surface area} \quad 4\pi R_E^2
\]
\[
\sigma \quad \text{Stefan-Boltzmann constant} \quad 5.67 \times 10^{-8} \frac{\text{J}}{\text{s} \cdot \text{m}^2 \cdot \text{K}^4}
\]
2. Temperature of the stratosphere

The thin air of the stratosphere means its temperature is roughly constant. What is the temperature of the stratosphere?

\[ a_s \text{ fraction of light absorbed in stratosphere} < 0.01 \]
\[ T_t \text{ average troposphere temperature} 255 \, K \]
\[ e_s \text{ emissivity of stratosphere} < 0.01 \]
3. Temperature at the surface

How does the temperature vary with height in the troposphere? Knowing the temperature of the stratosphere, what is the temperature at the surface of the Earth?

\[
\begin{align*}
T_s & \quad \text{stratosphere temperature} \quad 214 \text{ K} \\
h_t & \quad \text{height of troposphere} \quad 11 \text{ km} \\
\gamma_t & \quad \text{lapse rate of troposphere} \quad -6.5 \text{ K/km}
\end{align*}
\]
Summary of Solution

1. Use conservation of energy to get average temperature of the troposphere $T_t$ using $t_E I S A_c = e_E \sigma A_s T_t^4$. 
1. Use conservation of energy to get average temperature of the troposphere $T_t$ using $t_E I_S A_c = e_E \sigma A_s T_t^4$.

2. Use conservation of energy to get the constant temperature of the stratosphere $T_s$ using $a_s \sigma A T_t^4 = 2e_s \sigma A T_s^4$. 
Summary of Solution

1. Use conservation of energy to get average temperature of the troposphere $T_t$ using $t_E l_s A_c = e_E \sigma A_s T_t^4$.

2. Use conservation of energy to get the constant temperature of the stratosphere $T_s$ using $a_s \sigma A T_t^4 = 2 e_s \sigma A T_s^4$.

3. Use the lapse rate $\gamma_t$ to extrapolate from the top/bottom edge of the troposphere/stratosphere to the Earth’s surface.
## What can we change?

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$l_s$</td>
<td>Solar constant</td>
<td>$1370 J/s - m^2$</td>
</tr>
<tr>
<td>$R_E$</td>
<td>Earth's radius</td>
<td>$6.37 \times 10^6 \text{ m}$</td>
</tr>
<tr>
<td>$t_a$</td>
<td>Fraction of light transmitted</td>
<td>0.7</td>
</tr>
<tr>
<td>$e_a$</td>
<td>Emissivity of Earth</td>
<td>1.0</td>
</tr>
<tr>
<td>$T_t$</td>
<td>Average troposphere temperature</td>
<td>255 $K$</td>
</tr>
<tr>
<td>$e_s$</td>
<td>Emissivity of stratosphere</td>
<td>$&lt; 0.01$</td>
</tr>
<tr>
<td>$T_s$</td>
<td>Stratosphere temperature</td>
<td>214 $K$</td>
</tr>
<tr>
<td>$h_t$</td>
<td>Height of troposphere</td>
<td>11 $km$</td>
</tr>
<tr>
<td>$l_t$</td>
<td>Lapse rate of troposphere</td>
<td>$-6.5 \text{ K/km}$</td>
</tr>
</tbody>
</table>
What if there are more clouds?

Average Earth Temperature

Absorption Coefficient

Surface Temperature

286
287
288
289

Absorption Coefficient

0.69 0.70 0.71 0.72 0.73 0.74 0.75

Average Earth Temperature

288
Additional Slides
More Climate Data

Reconstructed Temperature

Medieval Warm Period

Little Ice Age

Temperature Anomaly (°C)

2004

Where's the heat?