Homework 5 Rotational Motion

1. During a certain period of time, the angular position of a swinging door is described by $\theta = 5.00 + 10.0t + 2.00t^2$, where θ is in radians and t is in seconds. Determine the angular position, angular speed, and angular acceleration of the door (a) at $t = 0$ and (b) at $t = 3.00\ s$.

2. An electric motor rotating a grinding wheel at 100 rev/min is switched off. The wheel then moves with a constant negative angular acceleration of magnitude 2.00 rad/s^2. (a) During what time interval does the wheel come to rest? (b) Through how many radians does it turn while it is slowing down?

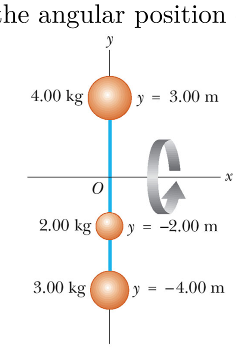
3. The tub of a washer goes into its spin cycle, starting from rest and gaining angular speed steadily for 8.00 s, at which time it is turning at 5.00 rev/s. At this point, the person doing the laundry opens the lid and a safety switch turns off the washer. The tub smoothly slows to rest in 12.0 s. Through how many revolutions does the tub turn while it is in motion?

4. Make an order-of-magnitude estimate of the number of revolutions through which a typical automobile tire turns in 1 yr. State the quantities you measure or estimate and their values.

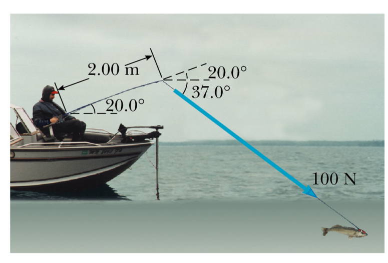
5. A disk 8.00 cm in radius rotates at a constant rate of 1200 rev/min about its central axis. Determine (a) its angular speed, (b) the tangential speed at a point 3.00 cm from its center, (c) the radial acceleration of a point on the rim, and (d) the total distance a point on the rim moves in 2.00 s.

6. A wheel 2.00 m in diameter lies in a vertical plane and rotates with a constant angular acceleration of 4.00 rad/s^2 . The wheel starts at rest at $t = 0$, and the radius vector of a certain point P on the rim makes an angle of 57.3° with the horizontal at this time. At $t = 2.0\ s$, find (a) the angular speed of the wheel, (b) the tangential speed and the total acceleration of the point P, and (c) the angular position of the point P.

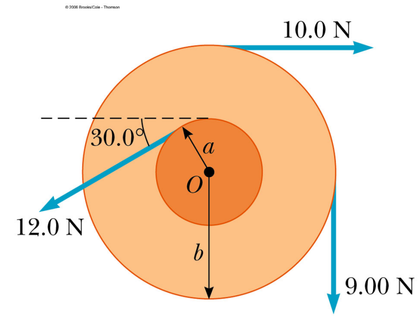
7. Rigid rods of negligible mass lying along the y axis connect three particles (see figure). The system rotates about the x axis with an angular speed of 2.0 rad/s. Treat the particles as points. Find (a) the moment of inertia about the x axis and the total rotational kinetic energy evaluated using $\frac{1}{2}I\omega^2$ and (b) the tangential speed of each particle and the total kinetic energy evaluated using $\sum \frac{1}{2}m_i v_i^2$.



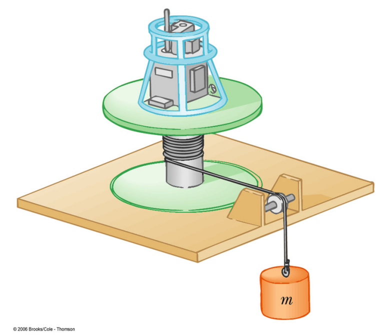
8. The fishing pole in the figure makes an angle of 20.0° with the horizontal. What is the torque exerted by the fish about an axis perpendicular to the page and passing through the anglers hand?



9. Find the net torque on the wheel in the figure about the axle through O, taking $a = 10.0\ cm$ and $b = 25.0\ cm$.

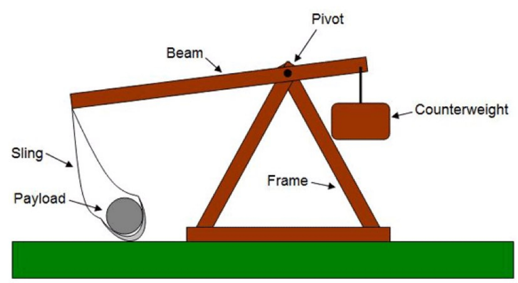


10. This problem describes one experimental method for determining the moment of inertia of an irregularly shaped object such as the payload for a satellite. The figure shows a counterweight of mass m suspended by a cord wound around a spool of radius r, forming part of a turntable supporting the object. The turntable can rotate without friction. When the counterweight is released from rest, it descends through a distance h, acquiring a speed v. Show that the moment of inertia I of the rotating apparatus (including the turntable) is $mr^2(2gh/v^2 - 1)$.

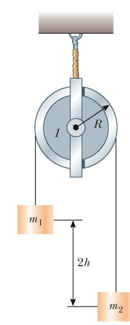


11. Big Ben, the Parliament tower clock in London, has an hour hand 2.70 m long with a mass of 60.0 kg and a minute hand 4.50 m long with a mass of 100 kg (see the figure). Calculate the total rotational kinetic energy of the two hands about the axis of rotation. (You may model the hands as uniform long, thin rods.)

12. A trebuchet is a device used during the Middle Ages to throw rocks at castles and sometimes now used to fling pianos as a sport. A simple trebuchet is shown in the figure. Model it as a stiff rod of negligible mass, 3.00 m long, joining particles of mass 60.0 kg and 0.120 kg at its ends. It can turn on a frictionless horizontal axle perpendicular to the rod and 14.0 cm from the large-mass particle. The rod is released from rest in a horizontal orientation. Find the maximum speed that the small-mass object attains.



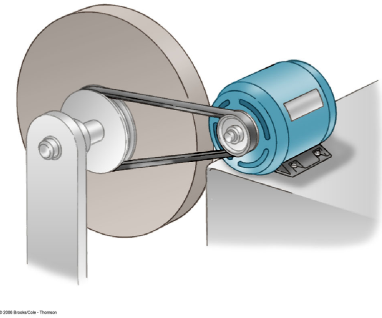
13. Consider two objects with $m_1 > m_2$ connected by a light string that passes over a pulley having a moment of inertia of I about its axis of rotation as shown in the figure. The string does not slip on the pulley or stretch. The pulley turns without friction. The two objects are released from rest separated by a vertical distance $2h$. (a) Use the principle of conservation of energy to find the translational speeds of the objects as they pass each other. (b) Find the angular speed of the pulley at this time.



14. The combination of an applied force and a friction force produces a constant total torque of 36.0 N-m on a wheel rotating about a fixed axis. The applied force acts for 6.00 s. During this time the angular speed of the wheel increases from 0 to 10.0 rad/s. The applied force is then removed, and the wheel comes to rest in 60.0 s. Find (a) the moment of inertia of the wheel, (b) the magnitude of the frictional torque, and (c) the total number of revolutions of the wheel.

15. A potter's wheel - a thick stone disk of radius 0.500 m and mass 100 kg is freely rotating at 50.0 rev/min. The potter can stop the wheel in 6.00 s by pressing a wet rag against the rim and exerting a radially inward force of 70.0 N. Find the effective coefficient of kinetic friction between wheel and rag.

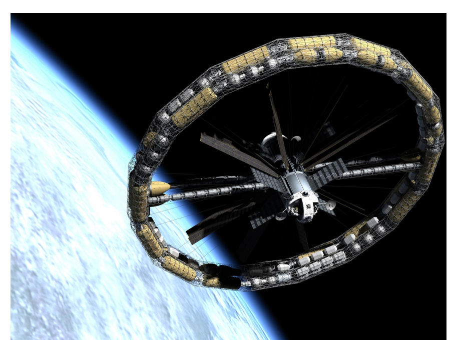
16. An electric motor turns a flywheel through a drive belt that joins a pulley on the motor and a pulley that is rigidly attached to the flywheel, as shown in the figure. The flywheel is a solid disk with a mass of 80.0 kg and a diameter of 1.25 m. It turns on a frictionless axle. Its pulley has much smaller mass and a radius of 0.230 m. The tension in the upper (taut) segment of the belt is 135 N, and the flywheel has a clockwise angular acceleration of 1.67 rad/s^2 . Find the tension in the lower (slack) segment of the belt.



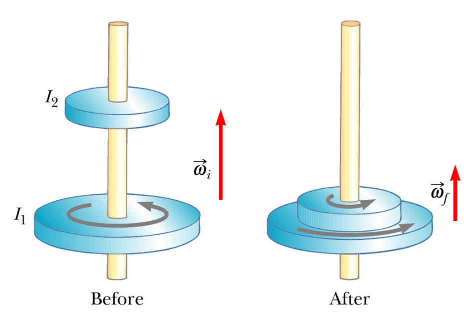
17. A uniform solid disk and a uniform hoop are placed side by side at the top of an incline of height h. If they are released from rest and roll without slipping, which object reaches the bottom first? Verify your answer by calculating their speeds when they reach the bottom in terms of h.

18. A particle of mass 0.400 kg is attached to the 100-cm mark of a meter stick of mass 0.100 kg. The meter stick rotates on a horizontal, frictionless table with an angular speed of 4.00 rad/s. Calculate the angular momentum of the system when the stick is pivoted about an axis (a) perpendicular to the table through the 50.0-cm mark and (b) perpendicular to the table through the 0-cm mark.

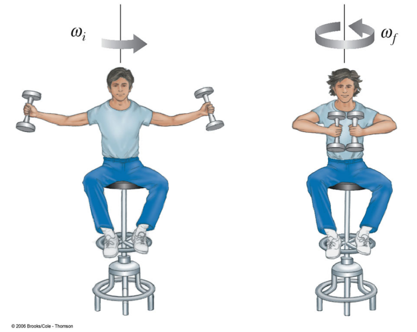
19. A von Braun wheel is a space station constructed in the shape of a hollow ring. Imagine the one in the figure has a mass 5.00×10^4 kg. Members of the crew walk on a deck formed by the inner surface of the outer cylindrical wall of the ring, with radius 100 m. At rest when constructed, the ring is set rotating about its axis so that the people inside experience an effective free-fall acceleration equal to g. (the figure shows the ring together with some other parts that make a negligible contribution to the total moment of inertia.) The rotation is achieved by firing two small rockets attached tangentially to opposite points on the outside of the ring. (a) What angular momentum does the space station acquire? (b) How long must the rockets be fired if each exerts a thrust of 125 N?



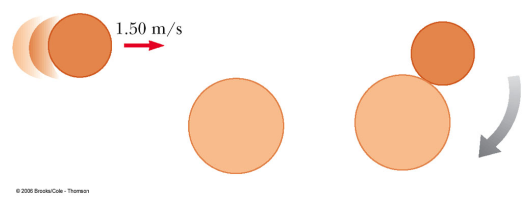
20. A cylinder with moment of inertia I_1 rotates about a vertical, frictionless axle with angular speed ω_i. A second cylinder, this one having moment of inertia I_2 and initially not rotating, drops onto the first cylinder (see figure). Because of friction between the surfaces, the two eventually reach the same angular speed ω_f. (a) Calculate ω_f. (b) Show that the kinetic energy of the system decreases in this interaction and calculate the ratio of the final to the initial rotational energy.



21. A student sits on a freely rotating stool holding two weights, each of mass $m = 3.00\ kg$ as shown in the figure. When his arms are extended horizontally, the weights are $\ell_1 = 1.00\ m$ from the axis of rotation and he rotates with an angular speed of $\omega_1 = 0.750\ rad/s$. The moment of inertia of the student plus stool is $I_s = 3.00 kg - m^2$ and is assumed to be constant. The student pulls the weights inward horizontally to a position $\ell_2 = 0.300\ m$ from the rotation axis. (a) Find the new angular speed of the student. (b) Find the kinetic energy of the rotating system before and after he pulls the weights inward.



22. A puck of mass $m_1 = 80.0\ g$ and radius $r_1 4.0\ cm$ slides along an air table at a speed of $v_1 = 1.50\ m/s$ as shown in the figure. It makes a glancing collision with a second puck of radius $r_2 = 6.0\ cm$ and mass $m_2 = 120\ g$ (initially at rest) such that their rims just touch. Because their rims are coated with instant-acting glue, the pucks stick together and spin after the collision. (a) What is the angular momentum of the system relative to the center of mass? (b) What is the angular speed about the center of mass?



23. Heading straight toward the summit of Pikes Peak, an airplane of mass $12,000\ kg$ flies over the plains of Kansas at nearly constant altitude $4.30\ km$ with constant velocity $175\ m/s$ west. (a) What is the airplanes vector angular momentum relative to a wheat farmer on the ground directly below the airplane? (b) Does this value change as the airplane continues its motion along a straight line? (c) What is its angular momentum relative to the summit of Pikes Peak?

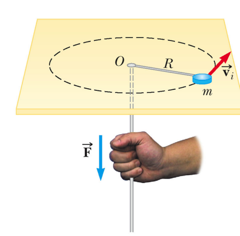
24. Big Ben, the Parliament tower clock in London, has hour and minute hands with lengths of $2.70\ m$ and $4.50\ m$ and masses of $60.0\ kg$ and $100\ kg$, respectively. Calculate the total angular momentum of these hands about the center point. Treat the hands as long, thin, uniform rods.

25. A playground merry-go-round of radius $R = 2.00\ m$ has a moment of inertia $I = 250\ kg \cdot m^2$ and is rotating at $10.0\ rev/min$ about a frictionless vertical axle. Facing the axle, a $25.0 - kg$ child hops onto the merry-go-round and manages to sit down on the edge. What is the new angular speed of the merry-go-round?

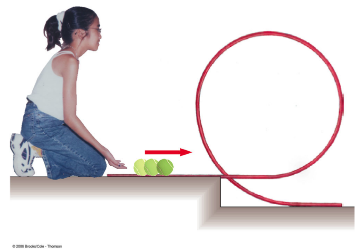
26. A $60.0 - kg$ woman stands at the rim of a horizontal turntable having a moment of inertia of $500\ kg \cdot m^2$ and a radius of $2.00\ m$. The turntable is initially at rest and is free to rotate about a frictionless, vertical axle through its center. The woman then starts walking around the rim clockwise (as viewed from above the system) at a constant speed of $1.50\ m/s$ relative to the Earth. (a) In what direction and with what angular speed does the turntable rotate? (b) How much work does the woman do to set herself and the turntable into motion?

27. A space station shaped like a giant wheel has a radius of $100\ m$ and a moment of inertia of $5.00 \times 10^8\ kg \cdot m^2$. A crew of 150 is living on the rim, and the stations rotation causes the crew to experience an apparent free-fall acceleration of g. When 100 people move to the center of the station for a union meeting, the angular speed changes. Assume that the average mass for each inhabitant is $65.0\ kg$. What apparent free-fall acceleration is experienced by the managers remaining at the rim?

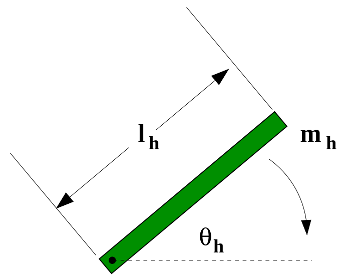
28. The puck in the figure has a mass of 0.120 kg. The distance of the puck from the center of rotation is originally 40.0 cm, and the puck is sliding with a speed of 80.0 cm/s. The string is pulled downward 15.0 cm through the hole in the frictionless table. Determine the work done on the puck. (Suggestion: Consider the change of kinetic energy.)



29. A tennis ball is a hollow sphere with a thin wall. It is set rolling without slipping at 4.03 m/s on a horizontal section of a track as shown in the figure. It rolls around the inside of a vertical circular loop 90.0 cm in diameter and finally leaves the track at a point 20.0 cm below the horizontal section. (a) Find the speed of the ball at the top of the loop. Demonstrate that it will not fall from the track. (b) Find its speed as it leaves the track. (c) Suppose static friction between ball and track were negligible so that the ball slid instead of rolling. Would its speed then be higher, lower, or the same at the top of the loop? Explain.

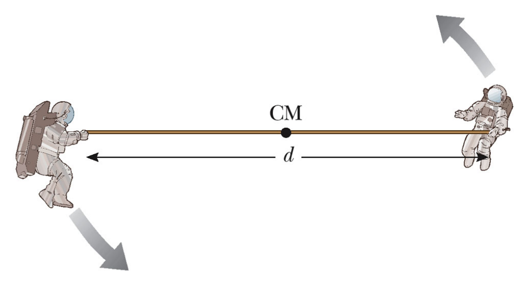


30. A construction worker was injured when a wooden beam that was partially supporting a porch broke off, swung down, and hit the worker in the head just as the person was walking by. You are an expert witness in the court case. The initial configuration of the beam is shown in the figure below with a length l_h, mass m_h, and an initial angle θ_h. It is attached to a pivot at the bottom. The top of the worker's head is the same height as the pivot. Treat the beam as a uniform rod and neglect friction. What is the angular speed of the beam as it strikes the worker in the head in terms of l_h, mass m_h, and θ_h and any other constants?

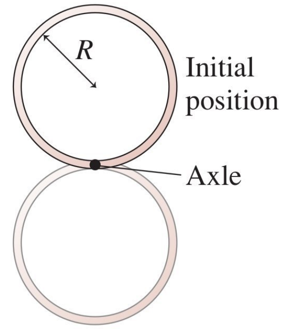


31. A spacecraft is in empty space. It carries on board a gyroscope with a moment of inertia of $I_g = 20.0\ kg \cdot m^2$ about the axis of the gyroscope. The moment of inertia of the spacecraft around the same axis is $I_s = 5.00 \times 10^5\ kg \cdot m^2$. Neither the spacecraft nor the gyroscope is originally rotating. The gyroscope can be powered up in a negligible period of time to an angular speed of 100 s^{-1}. If the orientation of the spacecraft is to be changed by 30.0°, for how long should the gyroscope be operated?

32. Two astronauts (see figure), each having a mass M, are connected by a rope of length d having negligible mass. They are isolated in space, orbiting their center of mass at speeds v. Treating the astronauts as particles, calculate (a) the magnitude of the angular momentum of the system and (b) the rotational energy of the system. By pulling on the rope, one of the astronauts shortens the distance between them to $d/2$. (c) What is the new angular momentum of the system? (d) What are the astronauts new speeds? (e) What is the new rotational energy of the system? (f) How much work does the astronaut do in shortening the rope?



33. The figure below shows a hoop of mass M and radius R rotating about an axle at the edge of the hoop. The hoop starts at its highest position and is given a very small push to start it rotating. At its lowest position, what are (a) the angular velocity ω and (b) the speed $v_\perp$ of the lowest point on the hoop?

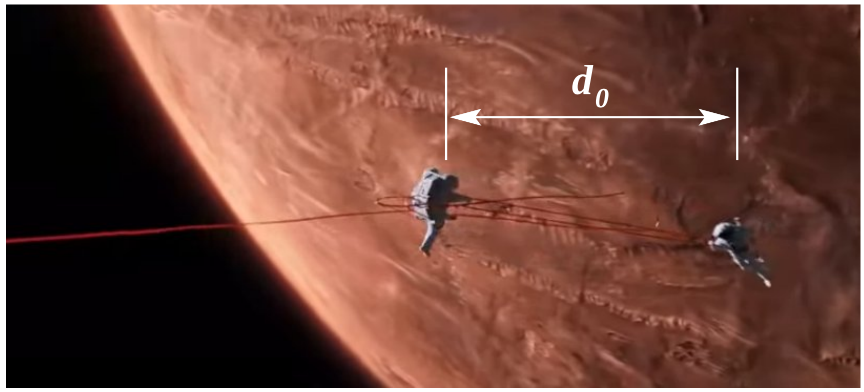


34. In the final rescue scene of the movie *The Martian* Commander Lewis and Mark Watney are spinning around each other on a massless tether of length d_0. They orbit the center-of-mass of the tether located at the midpoint. Commander Lewis pulls Watney toward her until she can grab him. Assume the masses of each astronaut are m (the commander is lighter, but she has a manned maneuvering unit attached to her

spacesuit) and their initial tangential speed is $v_{\perp 0}$. Treat the astronauts as point particles as they orbit around one another. What is the astronauts' angular momentum in terms of d_0, m, and $v_{\perp 0}$? Commander Lewis pulls Watney toward her until she can reach him at a distance d_1. What is the tangential velocity $v_{\perp 1}$ when she grabs him in terms of d_0, d_1, m, and $v_{\perp 0}$? What is the change in the kinetic energy ΔKE of the astronauts? Use the following values to obtain a numerical value for ΔKE ONLY. Compare your result with the energy of a car going 55 mph - 3×10^5 J. Could she do it or is Hollywood violating the laws of physics?

$m = 320$ kg $\quad d_0 = 7$ m

$v_{\perp 0} = 2$ m/s $\quad d_1 = 0.5$ m



35. A spacecraft is in empty space. It carries on board a gyroscope which is a device that changes the angular position of the spacecraft. It consists of a spinning wheel whose angular velocity is controlled by a motor. The one in the spacecraft has a moment of inertia of $I_g = 20.0$ $kg \cdot m^2$ about the axis of the gyroscope. The moment of inertia of the rest of the spacecraft around the same axis is $I_s = 5.00 \times 10^5$ $kg \cdot m^2$. Neither the spacecraft nor the gyroscope is initially rotating. The gyroscope can be powered up in a negligible period of time to an angular speed of 100 rad/s. With the gyroscope running, how long Δt will it take to rotate the rest of the spacecraft by $\Delta\theta = 30.0° = 0.52$ rad about the same axis?

36. A spinning wheel requires a time $\Delta t = 3$ s to rotate $\theta = 20$ rev. Its angular speed at the end of Δt is $\omega = 90$ rad/s. What is the angular acceleration of the wheel?

37. A model airplane like the ones we used in lab has a mass $m = 0.75$ kg and is suspended from a string and flying in a circle of radius $r = 0.25$ m. The model airplane engine provides a thrust of $F_a = 0.8$ N perpendicular to the string. Treat the airplane like a point particle here. (a) What is the torque produced by the engine about the center of the circle? (b) What is the angular acceleration of the airplane in level flight? (c) What is the linear acceleration of the airplane tangent to its flight path?

38. A car is driving on a flat, circular track and accelerates uniformly from rest with a tangential acceleration $a_T = 1.6$ m/s^2 The car makes it one-quarter of the way around the circle and then skids off the track. What is the coefficient of static friction between the car and the track?