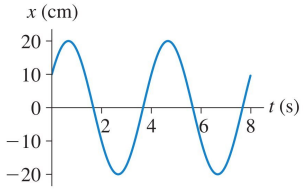
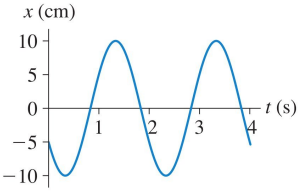
Homework 6 Periodic Motion

1. When a guitar string plays the note A, the string vibrates at 440 Hz. What is the period of the vibration?

2. What are the (a) amplitude, (b) frequency, and (c) phase constant of the oscillation shown in the figure?



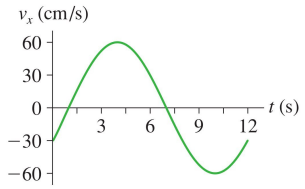
3. What are the (a) amplitude, (b) frequency, and (c) phase constant of the oscillation shown in the figure?



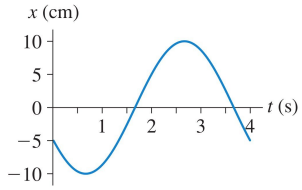
4. An object in simple harmonic motion has amplitude $A = 4.0\ cm$ and frequency $f = 4.0\ Hz$, and at $t = 0\ s$ it passes through the equilibrium point moving to the right. Write the function $x(t)$ that describes the objects position.

5. An air-track glider attached to a spring oscillates with a period of 1.5 s. At $t = 0\ s$ the glider is 5.00 cm left of the equilibrium position and moving to the right at 36.3 cm/s. (a) What is the phase constant? (b) What is the phase at $t = 0\ s$, 0.5 s, 1.0 s, and 1.5 s?

6. The figure to the right is the velocity-versus-time graph of a particle in simple harmonic motion. (a) What is the amplitude of the oscillation? (b) What is the phase constant? (c) What is the position at $t = 0\ s$?



7. The figure to the right is the position-versus-time graph of a particle in simple harmonic motion. (a) What is the phase constant? (b) What is the velocity at $t = 0\ s$? (c) What is v_{max}?



8. An object in SHM oscillates with a period of 4.0 s and an amplitude of 10 cm. How long does the object take to move from $x = 0.0\ cm$ to $x = 6.0\ cm$?

9. Starting from Newton's Second Law for Hooke's Law (the force exerted by a spring, $F_s = -kx$), show

$$\frac{d^2x}{dt^2} = -\frac{k}{m}x$$

10. What is the $d\sin t/dt$? What is $d\sin \omega t/dt$? What did you do differently from the first question in this problem?

11. What is the $d\cos t/dt$? What is $d\cos \omega t/dt$? What did you do differently from the first question in this problem?

12. What is $d^2\cos \omega t/dt^2$? Let $x(t) = \cos \omega t$ and combine this second derivative, Hooke's Law ($F_s = -kx$), and Newton's Second Law ($F_s = ma = md^2x/dt^2$) to show $\omega = \sqrt{k/m}$.

13. A block attached to a spring with unknown spring constant oscillates with a period of 2.0 s. What is the period if (a) The mass is doubled? (b) The mass is halved? (c) The amplitude is doubled? (d) The spring constant is doubled? Parts a-d are independent questions, each referring to the initial situation.

14. A 200 g air-track glider is attached to a spring. The glider is pushed in 10 cm and released. A student with a stopwatch finds that 10 oscillations take 12.0 s. What is the spring constant?

15. A $1.0 - kg$ block is attached to a spring with spring constant 16 N/m. While the block is sitting at rest, a student hits it with a hammer and almost instantaneously gives it a speed of 40 cm/s. What are (a) The amplitude of the subsequent oscillations? (b) The blocks speed at the point where $x = \frac{1}{2}A$?

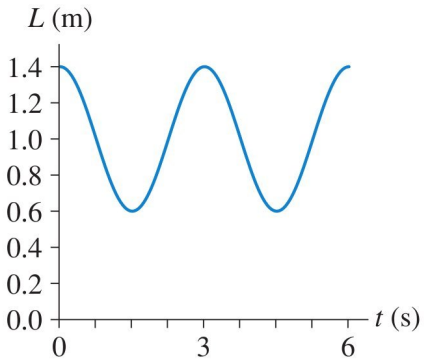
16. A spring is hung from the ceiling. When a block is attached to its end, it stretches 2.0 cm before reaching its new equilibrium length. The block is then pulled down slightly and released. What is the frequency of oscillation?

17. If the displacement x of a mass m on a spring is half the amplitude A, what is the fraction of the total energy that is in kinetic energy? What fraction is in potential energy?

18. What is the displacement x in terms of the amplitude A if half of the total energy is in kinetic energy and half is in potential energy?

19. An ultrasonic transducer, used in medical ultrasound imaging, is a very thin disk ($m = 0.10\ g$) oscillating back and forth at a frequency $f = 10^6\ Hz$ driven by an electromagnetic coil. The maximum restoring force that can be applied to the disk without breaking it is $F_{max} = 40,000\ N$. (a) What is the maximum oscillation amplitude that won't rupture the disk? (b) What is the disk's maximum speed at this amplitude?

20. Astronauts in space cannot weigh themselves by standing on a bathroom scale. Instead, they use a large spring. Suppose an astronaut attaches one end of a large spring to her belt and the other end to a hook on the wall of the space craft. A fellow astronaut then pulls her away from the wall and releases her. The spring's length as a function of time is shown in the figure. Ignore the mass of the belt and the spring. What is her mass m if the spring constant is $k = 220\ N/m$?



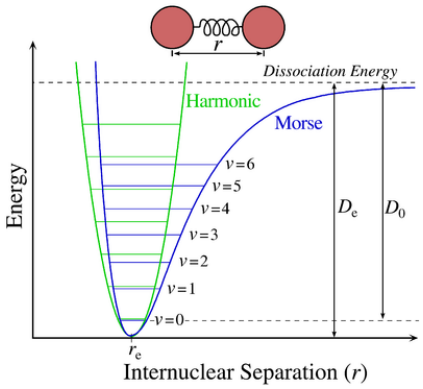
21. The motion of a particle is given by $x(t) = (25cm)\cos(10t)$, where t is in s. At what time is the kinetic energy twice the potential energy?

22. The end of the prong of a tuning fork that executes simple harmonic motion with a frequency of 1024 Hz has an amplitude $A = 0.4\ mm$. What is the maximum velocity v_{max} and maximum acceleration of the end of a prong? What is the speed of the end of the prong when the displacement from equilibrium is $x_1 = 0.1\ mm$?

23. Bungee jumpers bounce up and down freely on the bungee cord through many cycles. After a few cycles though, the cord does not go slack. Consider the following method to determine the mass of each person. An object of mass m_0 is oscillating freely on a light vertical spring with a period T_0. An object of unknown mass m_1 on the same spring oscillates with a period T_1. What is the spring constant and the unknown mass?

24. There have been studies of human cadavers to measure the moments of inertia of different body parts. This is useful for orthopedics and biomechanics. Consider the center of mass of a lower leg $m = 5.2\ kg$ was found to be $\ell = 0.19\ m$ from the knee. When the leg was allowed to pivot at the knee and swing freely as a pendulum, the oscillation frequency was $f = 1.6\ Hz$. What was the moment of inertia of the lower leg about the knee joint?

25. Carbon and oxygen are bound together by a force that can be modeled as a harmonic oscillator (see below). If the angular frequency is $\omega = 3.8 \times 10^{14}\ rad/s$ and the mass is $m = 1.14 \times 10^{-26}\ kg$, then what is the spring constant k? If the energy of the ground state is $E = 2 \times 10^{-20}\ J$, then what is the amplitude of the oscillation?



26. Starting with Newton's Second Law for a mass on a spring,

$$F_S = -kx$$

modify it to get a differential equation for the distance x from the equilibrium point. Is the expression

$$x(t) = A\sin(\omega t + \phi)$$

a solution to this differential equation? Under what conditions?