## The Twins Paradox

Consider two twins. One sets out at the age of 25 on a spaceship from Earth at a speed of $0.99 c$ where $c$ is the speed of light. The Earthbound twin goes on about her/his business accumulating the normal accouterments of advancing age (gray hair, drooping body parts, etc.). After twenty years have passed for the Earthbound twin, the spacefaring one returns. When they finally meet the voyager is NOT twenty years older! She/He looks only a few years older than when she/he left and shows few signs of age. How much has she/he aged during the journey?


## Time Dilation



Electrons at the speed of light.
Time Dilation Measurement, CERN 1976


FERMIONS matter constituents

## spin $=1 / 2.3 / 2.5 / 2$.

| Leptons spin = 1/2 |  |  |
| :---: | :---: | :---: |
| Flavor | Mass GeV/c ${ }^{2}$ | Electric charge |
| $V_{\text {e }}^{\text {electron }}$ neutrino | $<1 \times 10^{-8}$ | 0 |
| e electron | 0.000511 | -1 |
| $v_{\mu} \text { muon }$ | <0.0002 | 0 |
| $\mu$ muon | 0.106 | -1 |
| $v_{7} \text { lau }$ | $<0.02$ | 0 |
| T tau | 1.7771 | -1 |


| Quarks spin $=1 / 7$ |  |  |
| :--- | ---: | ---: |
| Flavor | Approx <br> Mass <br> GeV/c | Electric <br> charge |
| U up | 0.003 | $2 / 3$ |
| d down | 0.006 | $-1 / 3$ |
| C charm | 1.3 | $2 / 3$ |
| S strange | 0.1 | $-1 / 3$ |
| t top | 175 | $2 / 3$ |
| b botrom | $\mathbf{4 . 3}$ | $-1 / 3$ |

Muon half-life: $2.2 \times 10^{-6}$ s

## The Postulates

- Physics is the same in all inertial reference frames (hopefully).
- The speed of light is the same in all inertial reference frames.



## Testing The Second Postulate

(1) Get on a very fast train. - At CERN in 1964 T. Alvager et al. created a beam of $\pi^{0}$ 's moving close to the speed of light ( $0.99975 c$ ) by hitting a beryllium target with a high-energy proton beam.
(2) The $\pi^{0 \prime}$ s almost immediately decayed into particles of light called photons ( $t_{1 / 2}=8.64 \times 10^{-17} \mathrm{~s}$ ).
(3) The photons were measured at different, known locations downstream from the target.
(4) $c^{\prime}=(2.9977 \pm 0.0004) \times 10^{8} \mathrm{~m} / \mathrm{s}$ versus $2.99792458 \times 10^{8} \mathrm{~m} / \mathrm{s}$.


Alvager et al, CERN, 1964

## Testing The Second Postulate

(1) Get on a very fast train. - At CERN in 1964 T. Alvager et al. created a beam of $\pi^{0}$ 's moving close to the speed of light ( $0.99975 c$ ) by hitting a beryllium target with a high-energy proton beam.
(2) The $\pi^{0 \prime}$ s almost immediately decayed into particles of light called photons ( $t_{1 / 2}=8.64 \times 10^{-17} \mathrm{~s}$ ).
(3) The photons were measured at different, known locations downstream from the target.
(9) $c^{\prime}=(2.9977 \pm 0.0004) \times 10^{8} \mathrm{~m} / \mathrm{s}$ versus $2.99792458 \times 10^{8} \mathrm{~m} / \mathrm{s}$.



Fig. 1. The experimental arrangement and typical time spectra of the $\gamma$ rays. recorded in the four detector positions $\mathbf{A}, \mathbf{A}^{\top}, \mathbf{B}, \mathbf{B}^{\prime}$. Channel width 0.55 nsec. The measuring time for 100000 counts in the peak wes about 10 min .
T.Alvager et al., Phys. Lett. 12, 260 (1964)

## Guidelines for Galilean Relativity Lab

- Use the video GalileanTransformation.mp4 available at the following site. https://facultystaff.richmond.edu/~ggilfoyl/genphys/131/links.html
- Measure separately the trajectory of the ball in the lab system (fixed origin) and in the launcher system (moving origin).
- To use a moving origin (1) click on the coordinates symbol in the toolbar. You should see the coordinates appear. (2) Click Coordinate Systems at the top of the Tracker GUI. (3) Make sure Fixed Origin is unchecked. (4) In each frame select the coordinate system in the drop-down menu in the toolbar (see figure) and set the origin. (5) Next, use the same drop-down menu to select the mass and then mark the pro-
 jectile in the usual way.


## Time Dilation



## Time Dilation



- 2006 Brocksicole-Thomsen


## Time Dilation


easem buescon Tinman


- 2006 Brocksicole-Thomsen



## Time Dilation


© 2006 Brooks'Cole -Themson


## Evidence for Time Dilation

(1) In 1971 Hafele and Keating at the old National Bureau of Standards (now National Institute for Standards and Technology) took four cesium-beam atomic clocks aboard commercial airliners and flew twice around the world, first eastward, then westward, and compared the clocks against those of the United States Naval Observatory.

|  | nanoseconds gained |  |  |  |
| :--- | :--- | :--- | :--- | :--- |
|  | predicted |  |  | measured |
|  | gravitational <br> (general relativity) | kinematic <br> (special relativity) | total |  |
| eastward | $144 \pm 14$ | $-184 \pm 18$ | $-40 \pm 23$ | $-59 \pm 10$ |
| westward | $179 \pm 18$ | $96 \pm 10$ | $275 \pm 21$ | $273 \pm 7$ |

(2) Mountaintop muon decay measurements.
(3) Electron beam at JLab.
(9) GPS and many others.

## The Twins Paradox

Consider two twins. One sets out at the age of 25 on a spaceship from Earth at a speed of $0.99 c$ where $c$ is the speed of light. The Earthbound twin goes on about her/his business accumulating the normal accouterments of advancing age (gray hair, drooping body parts, etc.). After twenty years have passed for the Earthbound twin, the spacefaring one returns. When they finally meet the voyager is NOT twenty years older! She/He looks only a few years older than when she/he left and shows few signs of age. How much has she/he aged during the journey?


## Another Twins Paradox (Length Contraction) 14

Consider the two twins again. One sets out at the age of 25 on a spaceship from Earth at a speed of $0.99 c$ where $c$ is the speed of light. After twenty years have passed for the Earthbound twin, the spacefaring one returns. What is the mileage on the spacefaring twin's spaceship? In other words, what distance did the spacefarer measure in traveling outward from the Earth at 0.99c, turning around at the midpoint of her/his trip, and returning directly to Earth?


Time Dilation Measurement, CERN 1976


## The Lorentz Transformations

| Galilean | Lorentz |
| :--- | :--- |
| $x^{\prime}=x-v t$ |  |
| $y^{\prime}=y$ |  |
| $z^{\prime}=z$ |  |
| $t^{\prime}=t$ |  |
| $u_{x}^{\prime}=u_{x}-v$ |  |
| $u_{y}^{\prime}=u_{y}$ |  |
| $u_{z}^{\prime}=u_{z}$ |  |

primes refer to the frame moving with velocity $v$.
$v$ - velocity of moving frame.
$u_{i}-i^{\text {th }}$ component of the velocity of an object in the stationary frame.
$u_{i}^{\prime}-i^{t h}$ component of the velocity of an object in the moving frame.
$\gamma=\frac{1}{\sqrt{1-v^{2} / c^{2}}}$ where $c$ is the speed of light.

## The Lorentz Transformations

| Galilean | Lorentz |
| :--- | :--- |
| $x^{\prime}=x-v t$ | $x^{\prime}=\gamma(x-v t)$ |
| $y^{\prime}=y$ | $y^{\prime}=y$ |
| $z^{\prime}=z$ | $z^{\prime}=z$ |
| $t^{\prime}=t$ | $t^{\prime}=\gamma\left(t-v x / c^{2}\right)$ |
| $u_{x}^{\prime}=u_{x}-v$ | $u_{x}^{\prime}=\frac{u_{x}-v}{1-u_{x} v / c^{2}}$ |
| $u_{y}^{\prime}=u_{y}$ | $u_{y}^{\prime}=u_{y}$ |
| $u_{z}^{\prime}=u_{z}$ | $u_{z}^{\prime}=u_{z}$ |

primes refer to the frame moving with velocity $v$. $v$ - velocity of moving frame.
$u_{i}-i^{\text {th }}$ component of the velocity of an object in the stationary frame.
$u_{i}^{\prime}-i^{t h}$ component of the velocity of an object in the moving frame.
$\gamma=\frac{1}{\sqrt{1-v^{2} / c^{2}}}$ where $c$ is the speed of light.

## Addition of Velocities

Quasars are galaxies in the early throes of birth (we think). They have been observed to be receding from us at high speeds and at great distances. Quasar $Q_{1}$ is found to have a recessional velocity $v_{0}=0.80 c$ relative to the Milky Way ( $c$ is the speed of light). Another quasar $Q_{2}$ is receding from the Earth at a speed of $v_{1}=0.90 c$ along approximately the same line of sight as measured from Earth (see figure below). An alien who lives in galaxy $Q_{1}$ measures the speed of quasar $Q_{2}$. What speed does the alien measure?


X-ray image of the quasar PKS 1127-145 10 billion light years from Earth. The jet is at least a million light years from the quasar.

## Addition of Velocities

A large armada of Federation spaceships moves with a speed $0.95 c$ relative to the nearby Kronos system. A scout ship launched from the trailing ship in the armada moves at a speed $0.7 c$ towards the front of the fleet. The scout ship's speed is measured relative to the fleet. What is the speed of the scout ship as measured on Kronos?


## Relativistic Energy

$$
E=m_{R} c^{2}=\frac{m c^{2}}{\sqrt{1-\frac{v^{2}}{c^{2}}}}
$$



## Relativistic Particles

An electron is accelerated to an energy $E=6 \mathrm{GeV}$ where $1 \mathrm{GeV}=10^{9} \mathrm{GeV}$ at the Thomas Jefferson National Accelerator Facility in Newport News. What is the electron's speed, relativistic mass, and kinetic energy?



## Adding Relativistic Velocities

A fast-moving train with speed $v_{0}=2.5 \times 10^{8} \mathrm{~m} / \mathrm{s}$ passes an observer standing on the ground. A girl on the train kicks a soccer ball at her big brother sitting in front of her with a speed $v_{1}=10^{8} \mathrm{~m} / \mathrm{s}$ as measured by her father (much to his horror!). What speed does the stationary observer measure for the speed $v_{2}$ of the thrown ball?


## The Universal Speed Limit (Part 1)

A spaceship (Observer 1 in the figure) is moving away from an Earth-bound observer (0) at a high speed $v_{0}$ as measured by Observer 0 . It emits a periodic light pulse the observer on the Earth (0) detects. The time between pulses measured by Observer 1 is $\Delta t_{1}$. The time between pulses measured by Observer 0 is $\Delta t_{0}$. How is $\Delta t_{0}$ related to $\Delta t_{1}$ ?


## The Universal Speed Limit (Part 2)

Two spaceships ( 1 and 2 in the figure) are moving away from an Earth-bound observer (0) at different speeds. The fast, lead ship (2) emits a periodic light pulse the observer on the second, slow ship (1) receives and immediately relays to Earth (0). The speeds and time intervals are defined below.

| $v_{0}:$ speed of 1 from 0 | $\Delta t_{0}:$ time interval on 0 |
| :---: | :---: |
| $v_{1}:$ speed of 2 from 1 | $\Delta t_{1}:$ time interval on 1 |
|  | $\Delta t_{2}:$ time interval on 2 |
| $v_{2}:$ speed of 2 from 0 |  |

(1) How is $\Delta t_{0}$ related to $\Delta t_{1}$ ?
(2) How is $\Delta t_{1}$ related to $\Delta t_{2}$ ?
(3) How is $\Delta t_{0}$ related to $\Delta t_{2}$ ?
(4) What is $v_{2}$ in terms of $v_{0}$ and $v_{1}$ ?


Observer 0


Observer 1

Spaceships with pulsing light


Observer 2

## Physics 131 Final Rules

(1) You will be able to have your notes, lab notebooks, your own handwritten solutions to homework problems, copies of the homework assignments for this clas, and the text available during the exam.
(2) All else from the web is forbidden during the exam especially my solutions or any others.
(3) The test will last for 185 minutes from when I send the email with the test until the single PDF with photos of your work is placed in the Box folder entitled final. If I don't get the photos in time, I will not accept them. You have extra time to take the photos of your work and upload them to Box. This is a standard procedure for online courses.
(4) Your webcam should be on during the entire exam. If you need to go out of the room, you have to inform me via the Chat.
(5) Your exam should be in a single pdf uploaded to Box. Failure to do so will be penalized.
(6) You are responsible for checking the pdf file of your exam is complete. Box will not let you check the file you uploaded. However, you can upload the CamScanner pdf file to a folder in your area of Box and view the file. You can also save the CamScanner file to your laptop and view it there. You will not receive credit for missing parts of your exam.
(7) If you want me to check your uploaded file you must first check the file as described above. Then, make the request via the Zoom chat and remain in the Zoom session until I respond. If you are missing parts of the test and leave the Zoom session before I respond, you will not receive credit for the missing parts.
(8) All of the above are subject to the honor code.

