A Star Is Born! 1

A Star Is Born! 2

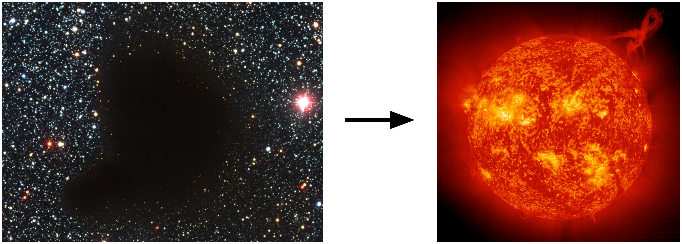
The photograph below shows a cloud of molecules called Bernard 68 (B68). It is located about 300 light-years (2.8×10^{15} *km*) away from us in the constellation Ophiuchus and is about 1.6 trillion kilometers across. It is made of molecules like CS, $\mathrm{N_2H}$, $\mathrm{H_2}$, and CO and is slowly rotating ($\omega = 9.4 \times 10^{-14}$ *rad*/*s*). The internal gravitational attraction of B68 may make the molecular cloud collapse far enough so it will ignite the nuclear fires and B68 will begin to shine.

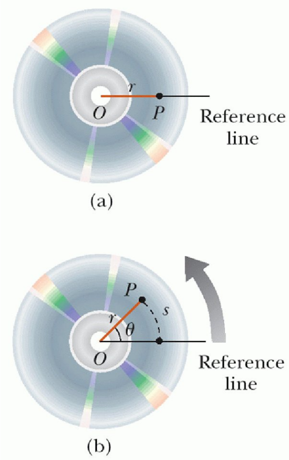
A Star Is Born 4

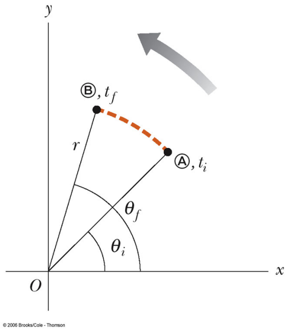
The molecular cloud B68 in the constellation Ophiuchus is rotating with an angular speed $\omega = 9.4 \times 10^{-14}\ rad/s$. The gravitational attraction among the atoms in the cloud may make it collapse until the core is hot enough to ignite nuclear reactions and B68 will begin to shine. If the final properties of B68 are the same as our Sun, *i.e.*, the same mass and size, then what will be its final angular velocity and period? Assume the lost mass carries away very little angular momentum. Compare this with the angular velocity of the Sun. Is your result reasonable? Why or why not?

$M_{B68} = 6.04 \times 10^{30}\ kg \quad I_{B68} = 2.7 \times 10^{54}\ kg - km^2 \quad M_{Sun} = 1.989 \times 10^{30}\ kg$

$R_{Sun} = 6.96 \times 10^{5}\ km \quad T_{Sun} = 25.4\ d$




r
O
P
Reference
line
(a)
P
s
r
θ
O
Reference
line
(b)

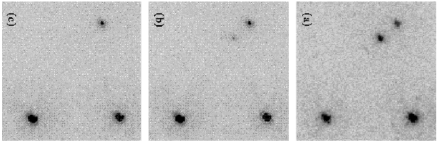

y
Ⓑ, t_f
r
Ⓐ, t_i
θ_f
θ_i
O
x
© 2006 Brooks/Cole - Thomson

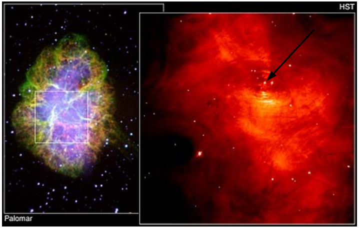
Linear Quantity	Connection	Rotational Quantity
s	$s = r\theta$	$\theta = \frac{s}{r}$
v_T	$v_T = r\omega$	$\omega = \frac{v_T}{r} = \frac{d\theta}{dt}$
a_T	$a_T = r\alpha$	$\alpha = \frac{a_T}{r} = \frac{d\omega}{dt}$
$KE = \frac{1}{2}mv^2$		$KE_R = \frac{1}{2}I\omega^2$
$\vec{F} = m\vec{a}$	$\tau = rF_\perp$	$\vec{\tau} = I\vec{\alpha}$
$\vec{p} = m\vec{v}$	$\vec{L} = \vec{r} \times \vec{p}$	$\vec{L} = I\vec{\omega}$

$$\vec{r}_{cm} = \frac{\sum m_i \vec{r}_i}{\sum m_i}$$

How Fast Will the Star Spin? 7

The pulsar in the Crab nebula has a period $T_0 = 0.033\ s$ and this period has been observed to be increasing by $\Delta T = 1.26 \times 10^{-5}\ s$ each year. Assuming constant angular acceleration what is the expression for the angular displacement of the pulsar? What are the values of the parameters in that expression?

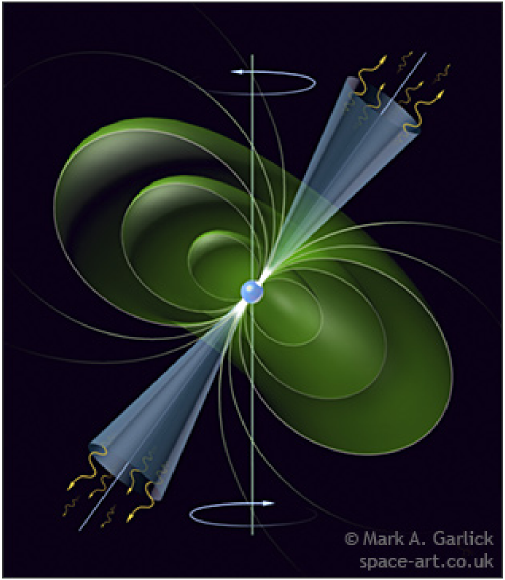




$$m_C = 3.4 \times 10^{30}\ kg$$

$$r_C = 25 \times 10^{3}\ m$$

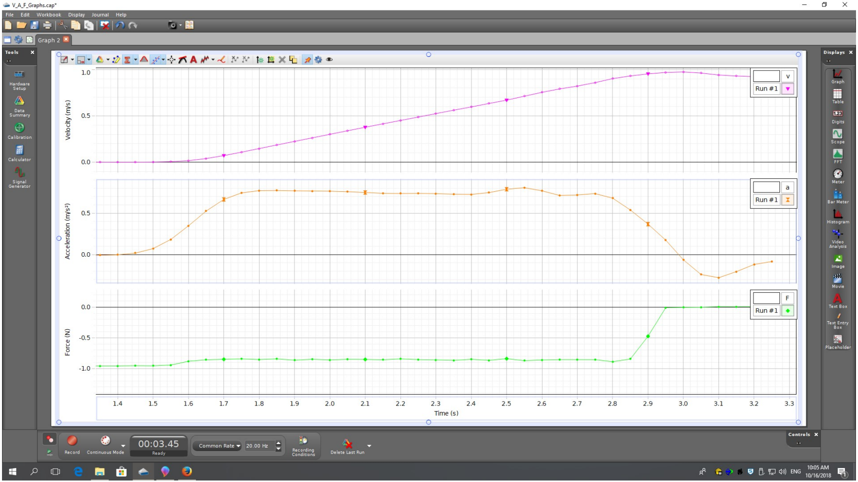
A Pulsar

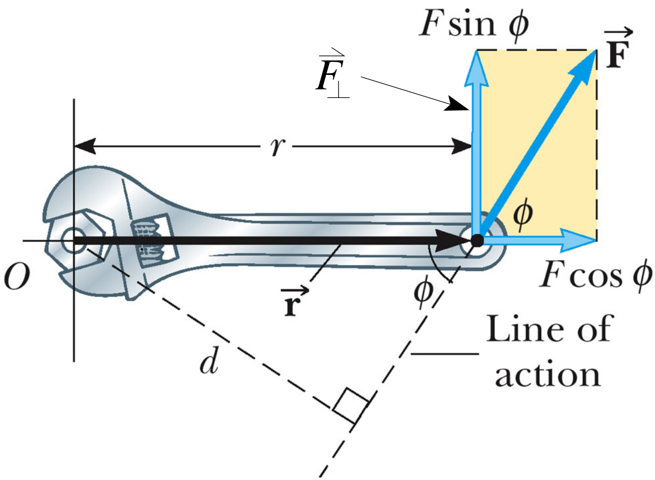


Linear Quantity	Connection	Rotational Quantity
s	$s = r\theta$	$\theta = \frac{s}{r}$
v_T	$v_T = r\omega$	$\omega = \frac{v_T}{r} = \frac{d\theta}{dt}$
a	$a_T = r\alpha$	$\alpha = \frac{a_T}{r} = \frac{d\omega}{dt}$
$KE = \frac{1}{2}mv^2$		$KE_R = \frac{1}{2}I\omega^2$
$\vec{F} = m\vec{a}$	$\tau = rF_\perp$	$\vec{\tau} = I\vec{\alpha}$
$\vec{p} = m\vec{v}$	$\vec{L} = \vec{r} \times \vec{p}$	$\vec{L} = I\vec{\omega}$

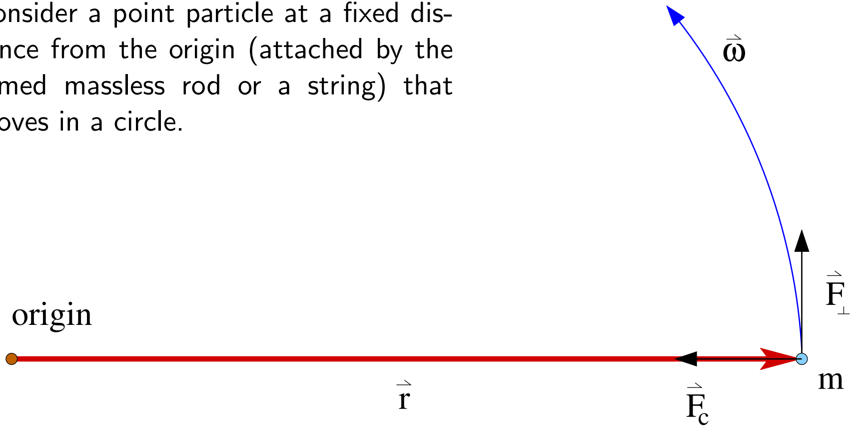
$$\vec{r}_{cm} = \frac{\sum m_i \vec{r}_i}{\sum m_i}$$

Force and Motion 1




$F \sin \phi$
$\vec{F}$
$F_\perp$
r
ϕ
O
$\vec{r}$
ϕ
$F \cos \phi$
Line of action
d

Consider a point particle at a fixed distance from the origin (attached by the famed massless rod or a string) that moves in a circle.



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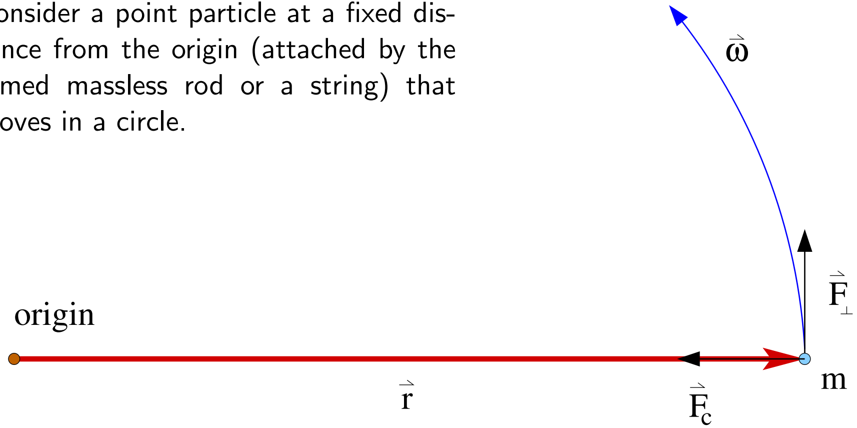
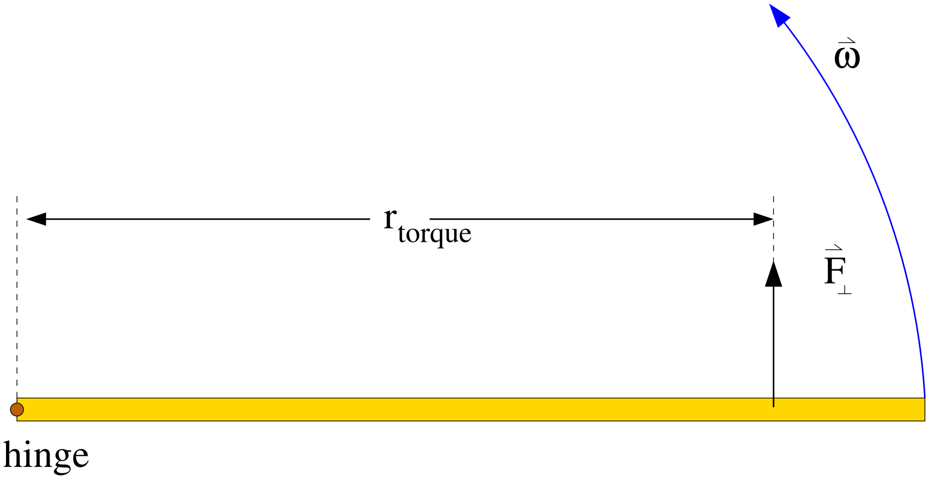
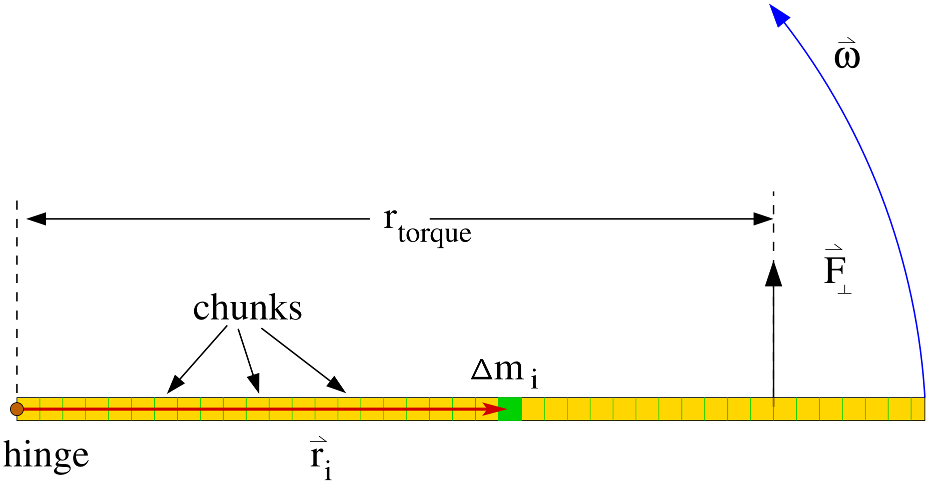


$$\vec{F} = m\vec{a} \rightarrow \vec{\tau} = r\vec{F}_\perp = \left(mr^2\right)\alpha$$

Torque - Rotating a Rigid Body (a door) 14



Torque - Rotating a Rigid Body (a door) 15



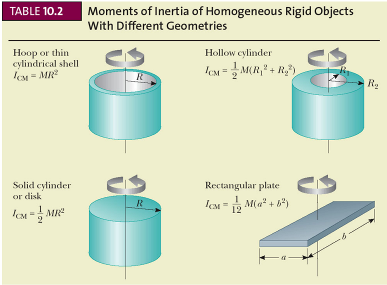
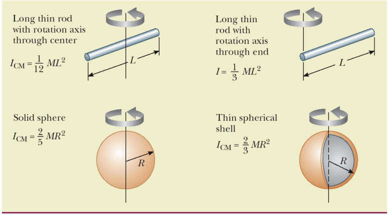


TABLE 10.2
Moments of Inertia of Homogeneous Rigid Objects With Different Geometries
Hoop or thin cylindrical shell
$I_{CM} = MR^2$
R
Hollow cylinder
$I_{CM} = \frac{1}{2}M(R_1^2 + R_2^2)$
R_1
R_2
Solid cylinder or disk
$I_{CM} = \frac{1}{2}MR^2$
R
Rectangular plate
$I_{CM} = \frac{1}{12}M(a^2 + b^2)$
b
a



Long thin rod with rotation axis through center
$I_{CM} = \frac{1}{12}ML^2$
L
Long thin rod with rotation axis through end
$I = \frac{1}{3}ML^2$
L
Solid sphere
$I_{CM} = \frac{2}{5}MR^2$
R
Thin spherical shell
$I_{CM} = \frac{2}{3}MR^2$
R

Linear → Rotational Quantities

Linear Quantity	Connection	Rotational Quantity
s	$s = r\theta$	$\theta = \frac{s}{r}$
v_T	$v_T = r\omega$	$\omega = \frac{v_T}{r} = \frac{d\theta}{dt}$
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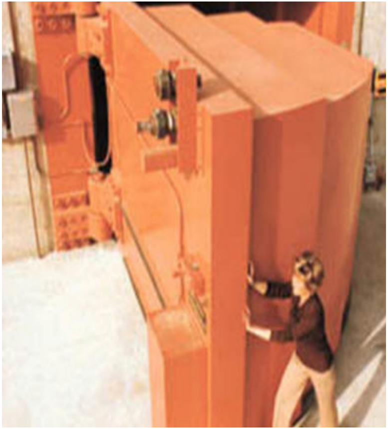
$\vec{F} = m\vec{a}$	$\tau = rF_\perp$	$\vec{\tau} = I\vec{\alpha}$
$\vec{p} = m\vec{v}$	$L = rp_\perp$	$\vec{L} = I\vec{\omega}$

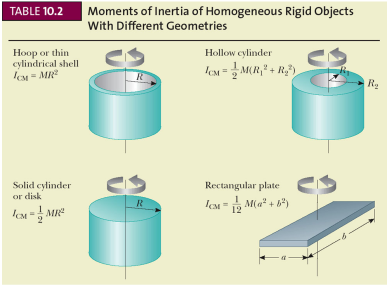
$$\vec{r}_{cm} = \frac{\sum m_i \vec{r}_i}{\sum m_i}$$

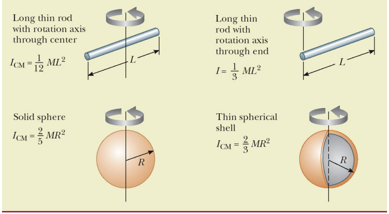
Torque - Rotational Force 18

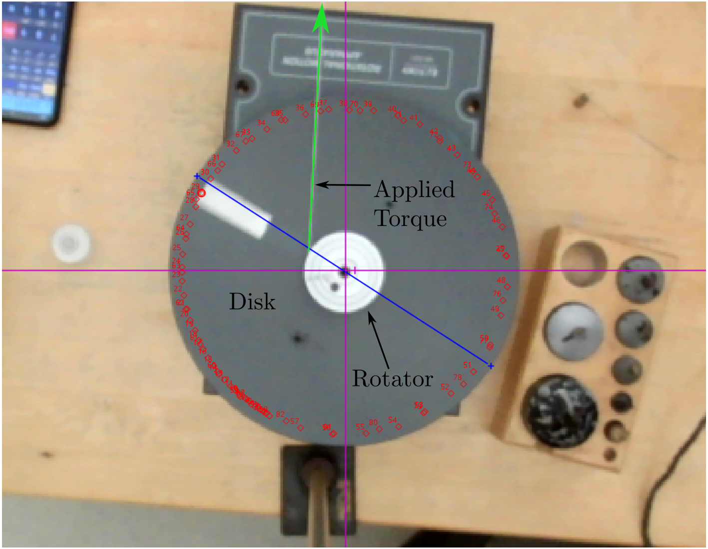
The shield door at a neutron test facility at Lawrence Livermore Laboratory is possibly the world's heaviest hinged door. It has a mass $m = 44,000\ kg$, a rotational inertia about a vertical axis through its hinges of $I = 8.7 \times 10^4\ kg - m^2$, and a (front) face width of $w = 2.4\ m$. A steady force $\vec{F}_a = 73\ N$, applied at its outer edge and perpendicular to the plane of the door, can move it from rest through an angle $\theta = 90^\circ$ in $\Delta t = 75\ s$. What is the torque exerted by the friction in the hinges?



Moments of Inertia

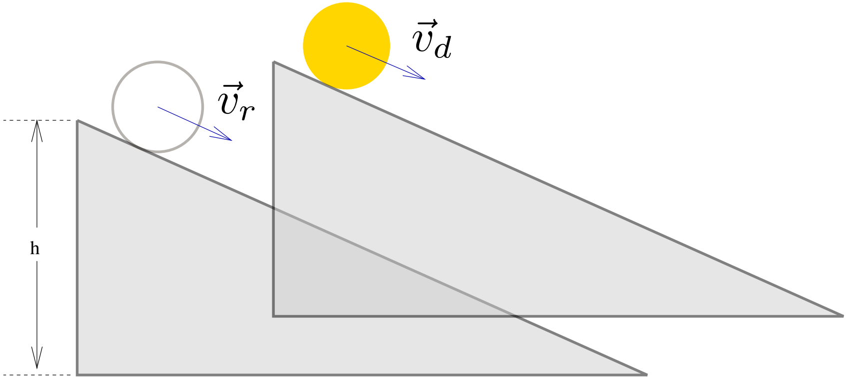


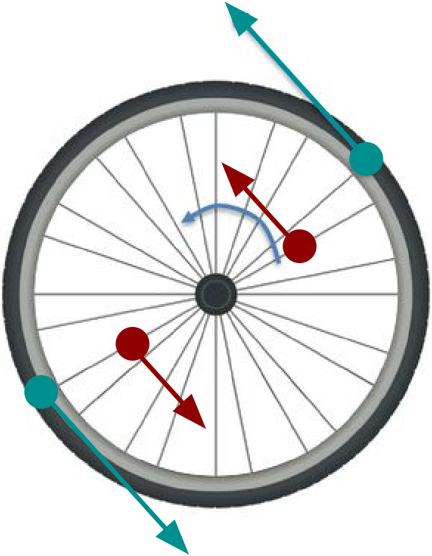




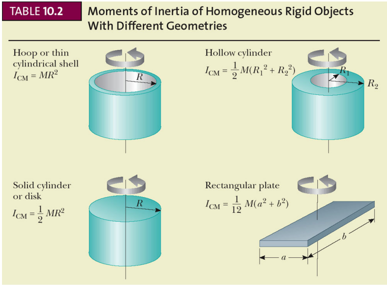
Applied
Torque
Disk
Rotator

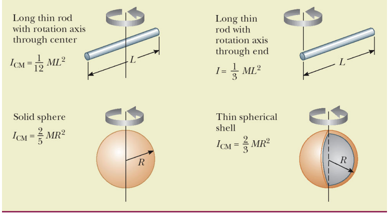
A wooden disk and a metal ring have the same mass *m* and radius *r*, start from rest, and roll down identical inclined planes (see figure). Which one wins?

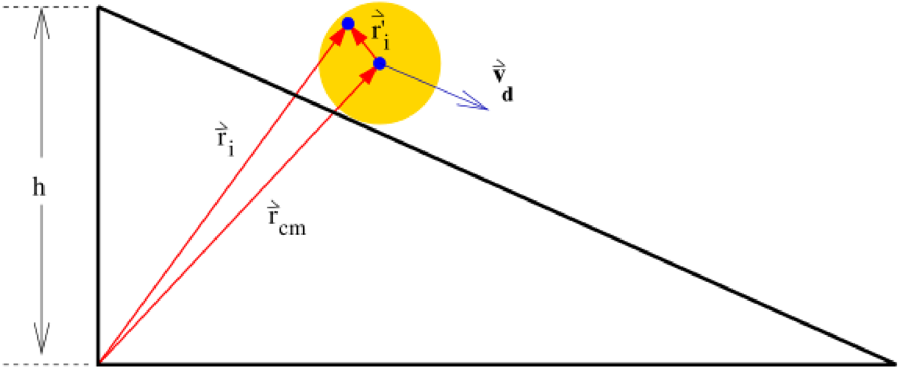




Moments of Inertia

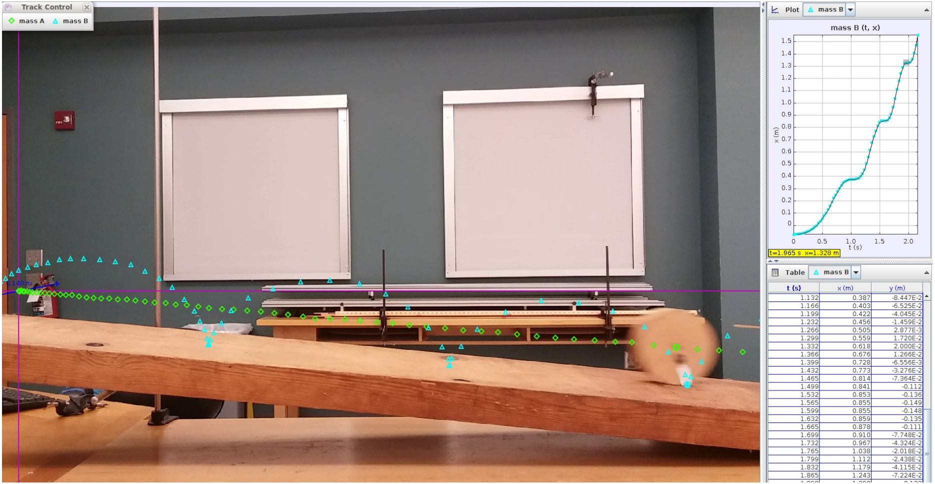




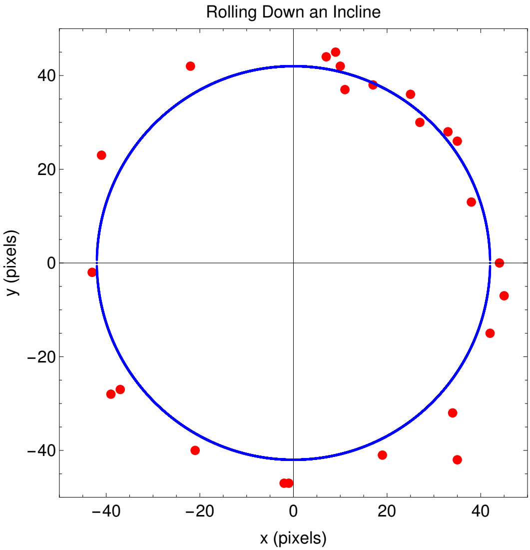


Link to video is here. Link to *Tracker* file is here.


Track Co
mass A
0.148 m
Plot
mass A
mass A (t, x)
1.5
1.4
1.3
1.2
1.1
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
x (m)
0
0.5
1.0
1.5
2.0
t (s)
t=0.600 s x=0.134 m
Table
t (s) x (m) y (m)
0.433 7.106E-2 -5.746E-3
0.466 8.227E-2 -7.302E-3
0.500 9.379E-2 -7.614E-3
0.533 0.106 -8.547E-3
0.566 0.120 -1.104E-2
0.600 0.134 -1.322E-2
0.633 0.149 -1.384E-2
0.666 0.164 -1.477E-2
0.699 0.180 -1.508E-2
0.733 0.197 -1.664E-2
0.766 0.214 -1.726E-2
0.799 0.232 -1.882E-2
0.833 0.251 -2.100E-2
0.866 0.270 -2.287E-2
0.899 0.292 -2.411E-2
0.933 0.312 -2.660E-2
0.966 0.333 -2.816E-2
0.999 0.356 -3.127E-2
1.032 0.378 -3.376E-2
1.066 0.404 -3.563E-2
1.099 0.427 -3.750E-2
1.132 0.453 -3.937E-2



Track Control
mass A
mass B
Plot
mass B
mass B (t, x)
x (m)
t (s)
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
1.1
1.2
1.3
1.4
1.5
0
0.5
1.0
1.5
2.0
t=1.965 s x=1.328 m
Table
mass B
t (s) x (m) y (m)
1.132 0.387 -8.447E-2
1.166 0.403 -6.525E-2
1.199 0.422 -4.045E-2
1.232 0.456 -1.459E-2
1.266 0.505 2.877E-3
1.299 0.559 1.720E-2
1.332 0.618 2.000E-2
1.366 0.676 1.266E-2
1.399 0.728 -6.556E-3
1.432 0.773 -3.276E-2
1.465 0.814 -7.364E-2
1.499 0.841 -0.112
1.532 0.853 -0.136
1.565 0.855 -0.149
1.599 0.855 -0.148
1.632 0.859 -0.135
1.665 0.878 -0.111
1.699 0.910 -7.748E-2
1.732 0.967 -4.324E-2
1.765 1.038 -2.018E-2
1.799 1.112 -2.438E-2
1.832 1.179 -4.115E-2
1.865 1.243 -7.224E-2



More here (pdf) and here (trz).

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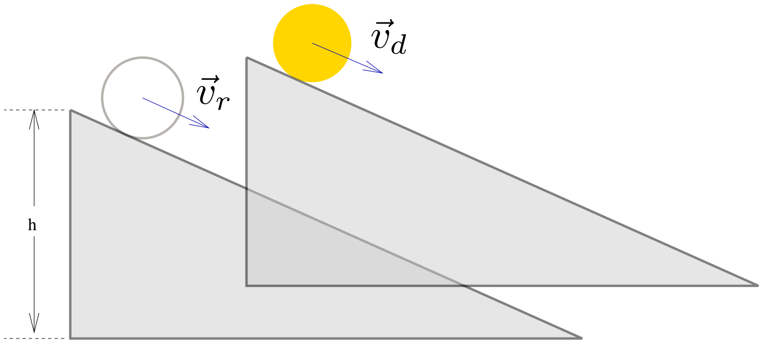

Linear Quantity	Connection	Rotational Quantity
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v_T	$v_T = r\omega$	$\omega = \frac{v_T}{r} = \frac{d\theta}{dt}$
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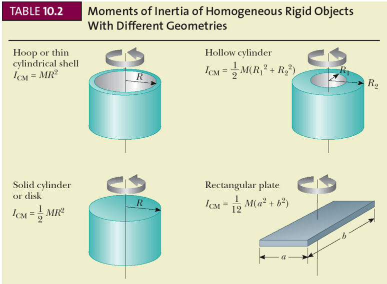
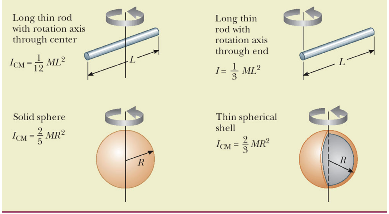
$\vec{p} = m\vec{v}$	$L = rp_\perp$	$\vec{L} = I\vec{\omega}$

$$\vec{r}_{cm} = \frac{\sum m_i \vec{r}_i}{\sum m_i}$$

Moments of Inertia

30





The molecular cloud B68 in the constellation Ophiuchus is rotating with an angular speed $\omega = 9.4 \times 10^{-14}\ rad/s$. The gravitational attraction among the atoms in the cloud may make it collapse until the core is hot enough to ignite nuclear reactions and B68 will begin to shine. If the final properties of B68 are the same as our Sun, *i.e.*, the same mass and size, then what will be its final angular velocity and period? Assume the lost mass carries away very little angular momentum. Compare this with the angular velocity of the Sun. Is your result reasonable? Why or why not?

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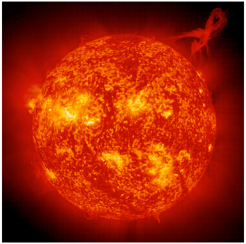
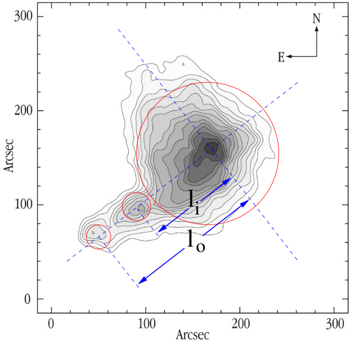
$R_{Sun} = 6.96 \times 10^{5}\ km$ $\quad T_{Sun} = 25.4\ d$



The plot below shows the 'obscuration' in the angular area around B68 based on measurements of background stars. The light in the center is 10^{14} dimmer than outside the edge of the cloud. To make life simple we will treat the mass distribution of B68 as three, rigid, uniform spheres that lie along the axis shown in the figure and rotate with $\omega = 9.4 \times 10^{-14}$ rad/s. The spheres do NOT rotate independently of the rest of the cloud. The origin is at the center of the central lobe. What is the moment of inertia of the cloud?

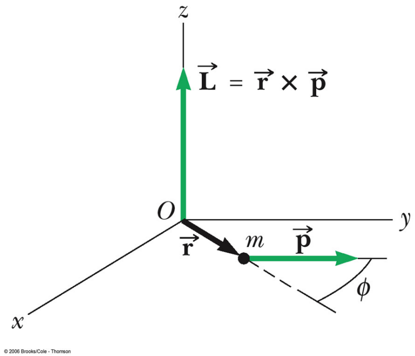
Lobe	Radius (km)	Mass (kg)
central	$R_c = 1.0 \times 10^{12}$	$m_c = 6.0 \times 10^{30}$
inner	$R_i = 2.0 \times 10^{11}$	$m_i = 4.6 \times 10^{28}$
outer	$R_o = 1.7 \times 10^{11}$	$m_o = 2.9 \times 10^{28}$

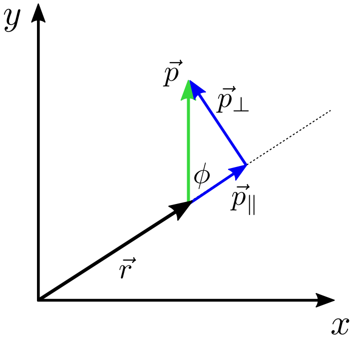
origin - inner cloud center	$l_i = 1.4 \times 10^{12}$ km
origin - outer cloud center	$l_o = 2.0 \times 10^{12}$ km

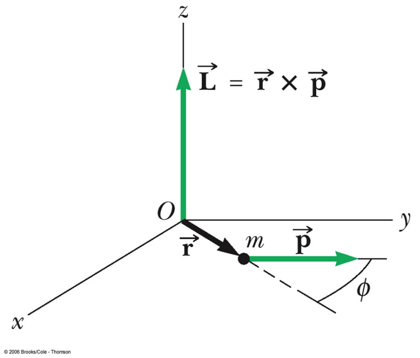


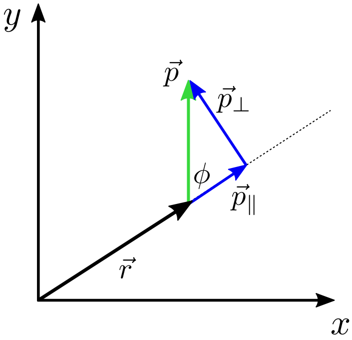
Map of the Obscuration in the Dark Cloud B68

ESO PR Photo 29c/99 (2 July 1999)


z
$\vec{L} = \vec{r} \times \vec{p}$
O
y
$\vec{r}$
m
$\vec{p}$
ϕ
x
© 2006 Brooks/Cole - Thomson


y
$\vec{p}$
$\vec{p}_\perp$
ϕ
$\vec{p}_\parallel$
$\vec{r}$
x





$$|\vec{L}| = rp_\perp = I\omega$$

Linear Quantity	Connection	Rotational Quantity
s	$s = r\theta$	$\theta = \frac{s}{r}$
v_T	$v = r\omega$	$\omega = \frac{v}{r} = \frac{d\theta}{dt}$
a	$a = r\alpha$	$\alpha = \frac{a}{r} = \frac{d\omega}{dt}$
$\vec{F} = m\vec{a} = \frac{d\vec{p}}{dt}$	$\tau = rF_\perp$	$\vec{\tau} = I\vec{\alpha} = \frac{d\vec{L}}{dt}$
$KE = \frac{1}{2}mv^2$		$KE_R = \frac{1}{2}I\omega^2$
$\vec{p} = m\vec{v}$	$L = rp_\perp$	$\vec{L} = I\vec{\omega}$

$$\vec{r}_{cm} = \frac{\sum m_i \vec{r}_i}{\sum m_i} \qquad I = \sum m_i r_i^2$$

Conservation of Momentum From $\vec{F}_{BA} = -\vec{F}_{AB}$ 36

$$\vec{F}_{AB} = -\vec{F}_{BA}$$

$$m_A \vec{a}_A = -m_B \vec{a}_B$$

$$m_A \frac{d\vec{v}_A}{dt} = -m_B \frac{d\vec{v}_B}{dt}$$

$$\frac{dm_A \vec{v}_A}{dt} = -\frac{dm_B \vec{v}_B}{dt}$$

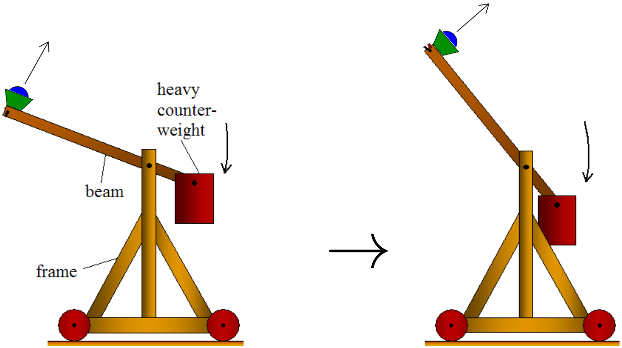
$$\frac{d\vec{p}_A}{dt} = -\frac{d\vec{p}_B}{dt}$$

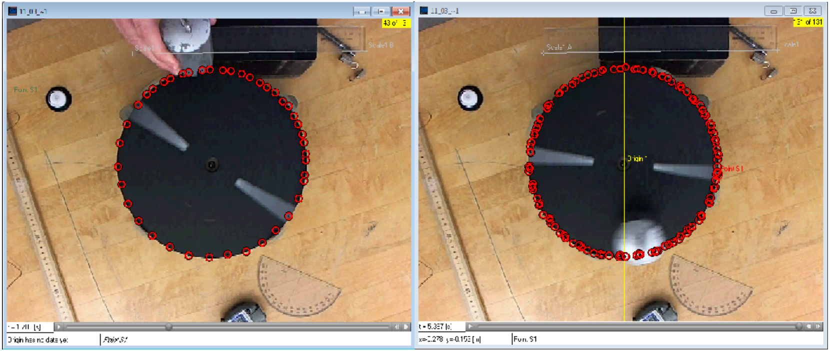
$$\frac{d\vec{p}_A}{dt} + \frac{d\vec{p}_B}{dt} = 0$$

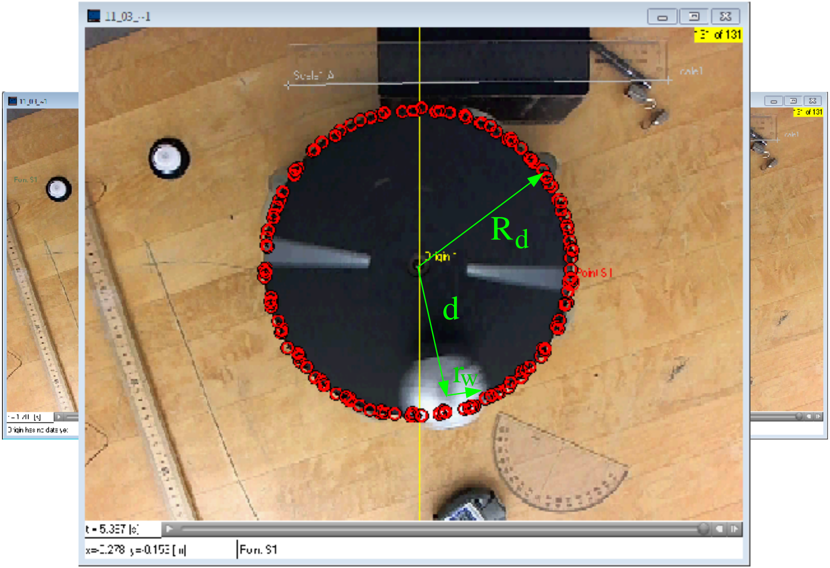
$$\frac{d}{dt}(\vec{p}_A + \vec{p}_B) = 0$$

$$\therefore \ \vec{p}_A + \vec{p}_B = const$$

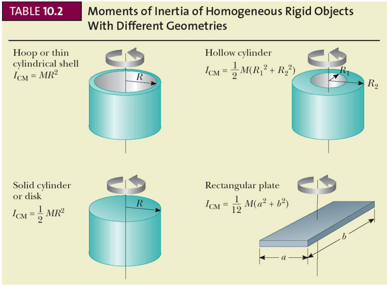
A trebuchet is a device used in the Middle Ages to throw big rocks at castles and is now used to throw other things like pumpkins, pianos, ... Consider the figures below. The trebuchet has a stiff wooden beam of mass $m_b = 15\ kg$ and length $l_b = 5\ m$ with masses $m_c = 700\ kg$ (the counterweight) and $m_p = 0.1\ kg$ (the payload) on it's ends. Treat these two masses as point particles. A frictionless axle is located a distance $d = 0.15\ m$ from the counterweight. The beam is released from rest in a horizontal position. We will launch the payload from a bucket at the end of the beam . What is the maximum speed the payload can reach before it leaves the bucket?

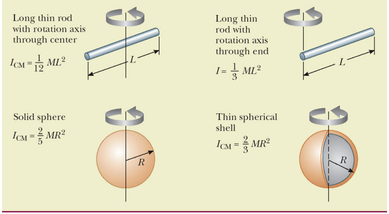




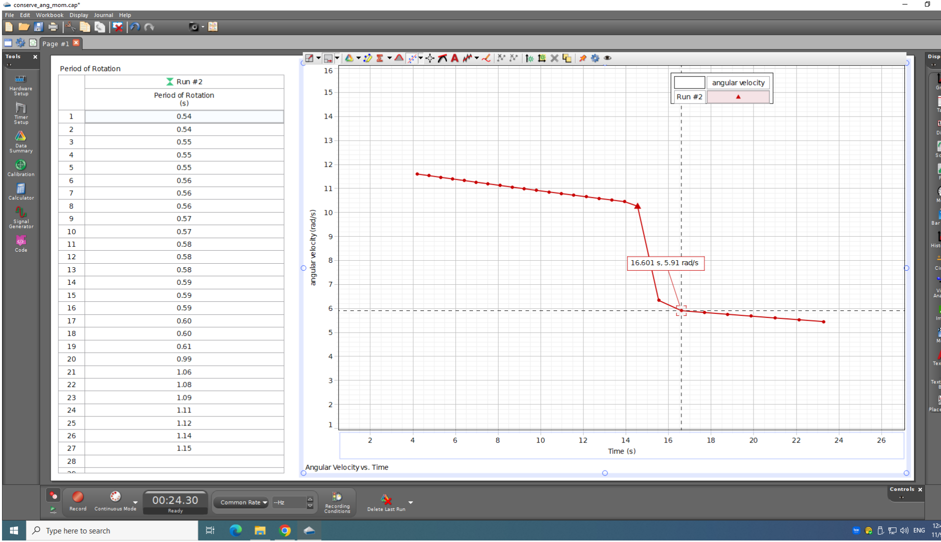

of 131
R_d
d
r_w
Point S1
t = 5.357 [s]
Fun. S1

Moments of Inertia 40





Angular Momentum Conservation

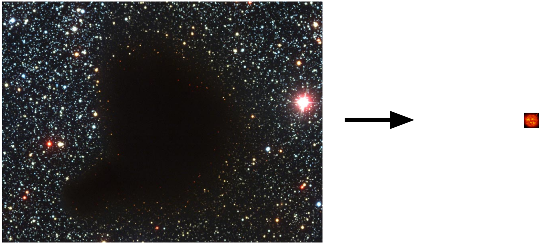


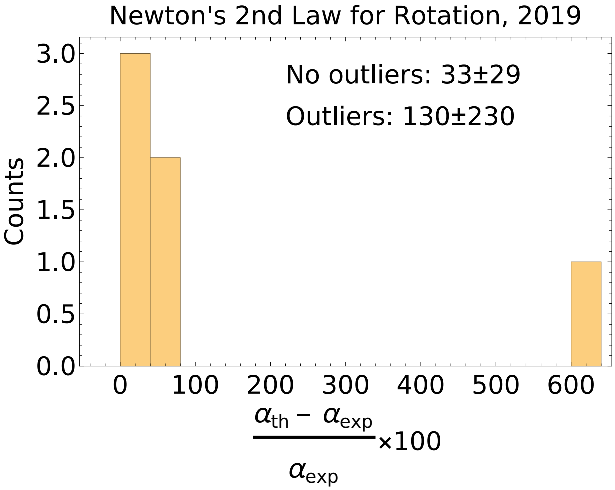
A Star Is Born 42

The molecular cloud B68 in the constellation Ophiuchus is rotating with an angular speed $\omega = 9.4 \times 10^{-14}\ rad/s$. The gravitational attraction among the atoms in the cloud may make it collapse until the core is hot enough to ignite nuclear reactions and B68 will begin to shine. If the final properties of B68 are the same as our Sun, *i.e.*, the same mass and size, then what will be its final angular velocity and period? Assume the lost mass carries away very little angular momentum. Compare this with the angular velocity of the Sun. Is your result reasonable? Why or why not?

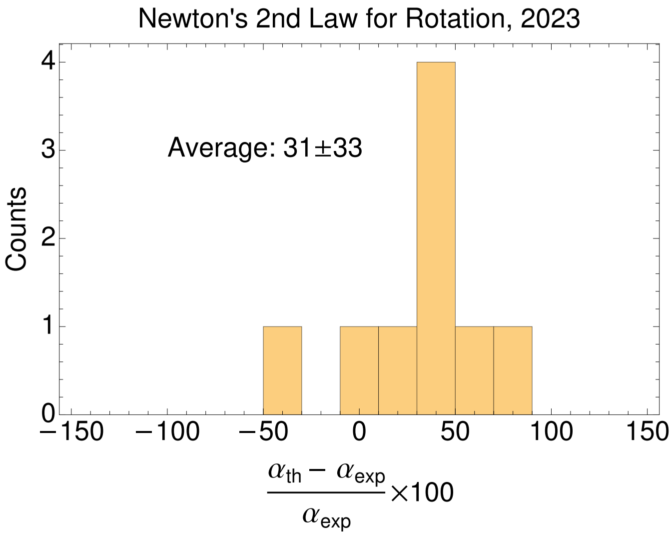
$M_{B68} = 6.04 \times 10^{30}\ kg$ $\quad I_{B68} = 2.7 \times 10^{54}\ kg - km^2$ $\quad M_{Sun} = 1.989 \times 10^{30}\ kg$

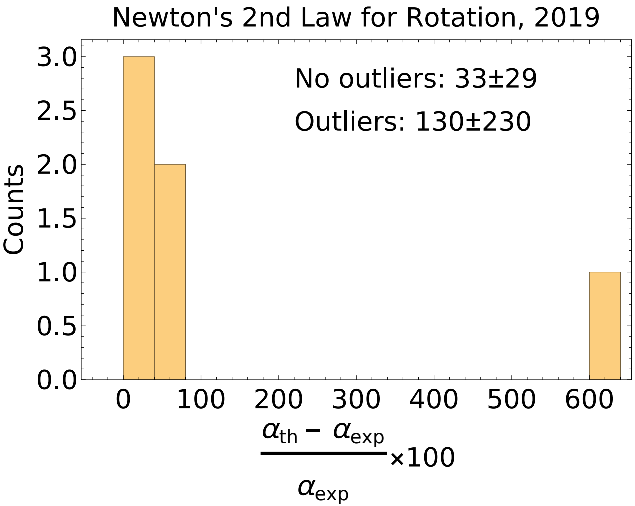
$R_{Sun} = 6.96 \times 10^{5}\ km$ $\quad T_{Sun} = 25.4\ d$





Newton's 2nd Law for Rotation, 2019
No outliers: 33±29
Outliers: 130±230
Counts
3.0
2.5
2.0
1.5
1.0
0.5
0.0
0
100
200
300
400
500
600
$\frac{\alpha_{th} - \alpha_{exp}}{\alpha_{exp}} \times 100$


Newton's 2nd Law for Rotation, 2023
Average: 31±33
Counts
0
1
2
3
4
−150
−100
−50
0
50
100
150
$\frac{\alpha_{th} - \alpha_{exp}}{\alpha_{exp}} \times 100$


Newton's 2nd Law for Rotation, 2019
No outliers: 33±29
Outliers: 130±230
Counts
3.0
2.5
2.0
1.5
1.0
0.5
0.0
0
100
200
300
400
500
600
$\frac{\alpha_{th} - \alpha_{exp}}{\alpha_{exp}} \times 100$

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