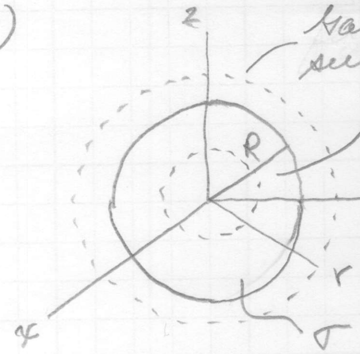
2-11)



for $r > R$

$$\nabla \cdot \vec{E} = \frac{\rho}{\varepsilon_0}$$

$\vec{E} = E(r)\,\hat{r}$ by symmetry

$$d\vec{A} = r^2\, d(\cos\theta)\, d\phi\, \hat{r}$$

$$\int_V \nabla \cdot \vec{E}_{out}\, d\tau = \int \frac{\rho}{\varepsilon_0}\, d\tau$$

$$\oint \vec{E}_{out} \cdot d\vec{A} = \frac{1}{\varepsilon_0} \underbrace{\int \rho\, d\tau}_{\text{total charge}}$$

$$\oint E_{out}\hat{r} \cdot r^2\, d\cos\theta\, d\phi\, \hat{r} = \frac{1}{\varepsilon_0}\, 4\pi R^2 \sigma$$

$$E_{out} r^2 \int_0^{2\pi}\int_{-1}^{1} d\cos\theta\, d\phi = \frac{1}{\varepsilon_0}\, 4\pi R^2 \sigma$$

$$E_{out} r^2 (2)(2\pi) = \frac{1}{\varepsilon_0}\, 4\pi R^2 \sigma$$

total charge Q

$$4\pi r^2 E_{out} = \frac{Q}{\varepsilon_0}$$

$$E_{out} = \frac{1}{4\pi\varepsilon_0} \frac{Q}{r^2}$$

$$\boxed{\therefore\ \vec{E}_{out} = \frac{1}{4\pi\varepsilon_0} \frac{Q}{r^2}\, \hat{r}}$$

same as 2.7

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for $r < R$ Everything is the same except

$$4\pi r^2 E_{in} = \frac{1}{\varepsilon_0} \int \rho\, d\tau$$

$$= 0$$

$\therefore\ \vec{E}_{in} = 0$ same as 2.7