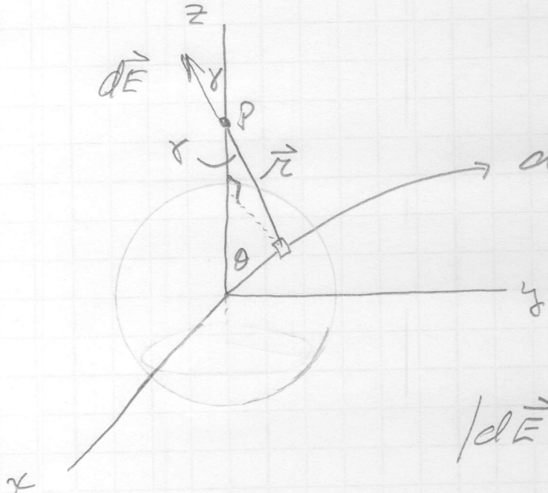
2-7)



$$dq = \sigma\, dA = \sigma R^2 \, d\cos\theta \, d\phi$$

$$r^2 = R^2 + z^2 - 2Rz\cos\theta$$

$$|d\vec{E}| = \frac{1}{4\pi\varepsilon_0} \frac{\sigma^2 R^2 d\cos\theta\, d\phi}{R^2 + z^2 - 2Rz\cos\theta}$$

The transverse components will cancel so get the z component.

$$dE_z = dE\cos\gamma \qquad \cos\gamma = \frac{z - R\cos\theta}{r}$$

$$E_z = \int dE_z$$

$$= \int \frac{1}{4\pi\varepsilon_0} \frac{\sigma R^2 d\cos\theta\, d\phi}{R^2 + z^2 - 2Rz\cos\theta} \left(\frac{z - R\cos\theta}{\sqrt{R^2 + z^2 - 2Rz\cos\theta}}\right)$$

$$= \frac{1}{4\pi\varepsilon_0} \sigma R^2 \left[z \int_0^{2\pi}\int_{-1}^{1} \frac{d(\cos\theta)\, d\phi}{(R^2 + z^2 - 2Rz\cos\theta)^{3/2}} - R\int_0^{2\pi}\int_{-1}^{1} \frac{\cos\theta\, d\cos\theta\, d\phi}{(R^2 + z^2 - 2Rz\cos\theta)^{3/2}} \right]$$

$$\int_0^{2\pi} d\phi = 2\pi, \quad \text{let } u = \cos\theta$$

$$= \frac{1}{4\pi\varepsilon_0} 2\pi\sigma R^2 \left[\int_{-1}^{1} \frac{(z - Ru)\, du}{(R^2 + z^2 - 2Rzu)^{3/2}} \right]$$

$$E_z = \frac{1}{4\pi\varepsilon_0} 2\pi\sigma R^2 \left[\frac{zu - R}{z^2\sqrt{z^2 + R^2 - 2Rzu}} \right]_{-1}^{1}$$

$$= \frac{1}{4\pi\varepsilon_0} 2\pi\sigma R^2 \left[\frac{z - R}{z^2\sqrt{z^2 + R^2 - 2Rz}} - \frac{z(-1) - R}{z^2\sqrt{z^2 + R^2 + 2Rz}} \right]$$

$$= \frac{1}{4\pi\varepsilon_0} 2\pi\sigma R^2 \frac{1}{z^2} \left[\frac{z - R}{\sqrt{z^2 + R^2 - 2Rz}} + \frac{z + R}{\sqrt{z^2 + R^2 + 2Rz}} \right]$$

$$\sqrt{z^2 + R^2 - 2Rz} = R - z \quad \text{for } R > z$$

$$= z - R \quad \text{for } R < z$$

$$\sqrt{z^2 + R^2 + 2Rz} = R + z \quad \text{always}$$

for $R > z$

$$E_z = \frac{1}{4\pi\varepsilon_0} 2\pi\sigma R^2 \frac{1}{z^2} \left[\frac{z - R}{R - z} + \frac{z + R}{z + R} \right]$$

$$= 0 \,!$$

for $R < z$

$$E_z = \frac{1}{4\pi\varepsilon_0} \, 2\pi\sigma R^2 \, \frac{1}{z^2} \left[\frac{z-R}{z-R} + \frac{z+R}{z+R} \right]$$

$$= \frac{1}{4\pi\varepsilon_0} \frac{4\pi R^2 \sigma}{z^2} \Big\} \text{ total charge}$$

It looks like a point charge!