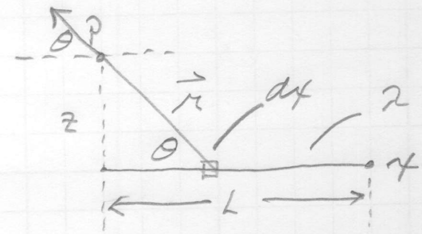
2-3)



$$\vec{E} = \int d\vec{E}$$

$$= \int dE_x \,\hat{x} + \int dE_z \,\hat{z}$$

$$|d\hat{E}| = \frac{1}{4\pi\varepsilon_0} \frac{\lambda\, dx}{(z^2+x^2)}$$

$$dE_x = -dE \cos\theta$$

$$= -dE \frac{x}{(x^2+z^2)^{1/2}}$$

$$dE_y = dE \sin\theta$$

$$= dE \frac{z}{(x^2+z^2)^{1/2}}$$

$$E_x = \int_0^L \frac{1}{4\pi\varepsilon_0} \frac{\lambda\, dx}{(x^2+z^2)} \frac{x}{(x^2+z^2)^{3/2}}$$

$$= \frac{\lambda}{4\pi\varepsilon_0} \int_0^L \frac{x}{(x^2+z^2)^{3/2}} dx$$

$$= \frac{\lambda}{4\pi\varepsilon_0} \left[-\frac{1}{\sqrt{x^2+z^2}} \Big|_0^L \right]$$

Mathematica

$$= -\frac{\lambda}{4\pi\varepsilon_0} \left[\frac{1}{\sqrt{L^2+z^2}} - \frac{1}{\sqrt{z^2}} \right]$$

$$E_x = \frac{\lambda}{4\pi\varepsilon_0} \left[\frac{1}{z} - \frac{1}{\sqrt{z^2+L^2}} \right]$$

for $z >> L$ $\quad E_x \to 0$

$$E_y = \int_0^L \frac{1}{4\pi\varepsilon_0} \frac{\lambda\, dx}{(x^2+z^2)} \frac{z}{(x^2+z^2)^{1/2}}$$

$$= \frac{\lambda z}{4\pi\varepsilon_0} \int_0^L \frac{1}{(x^2+z^2)^{3/2}} dx$$

$$= \frac{\lambda z}{4\pi\varepsilon_0} \left[\frac{x}{z^2\sqrt{x^2+z^2}} \right]_0^L$$

Mathematica

$$E_y = \frac{\lambda \cancel{z}}{4\pi\varepsilon_0} \frac{1}{z^{\cancel{2}}} \left[\frac{L}{\sqrt{L^2 + z^2}} - \frac{0}{\sqrt{0 + z^2}} \right]$$

$$= \frac{\lambda L}{4\pi\varepsilon_0} \frac{1}{z} \frac{1}{\sqrt{L^2 + z^2}}$$

for $z \gg L$ $\quad E_y = \frac{1}{4\pi\varepsilon_0} \frac{\overbrace{\lambda L}^{\text{total charge}}}{z^2}$

E_y looks like a point charge λL.