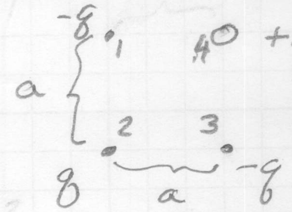
2-31)



$$W_4 = q V_{14} + q V_{24} + q V_{34}$$

$$V_{14} = \frac{1}{4\pi\varepsilon_0} \frac{-q}{a}$$

$$V_{24} = \frac{1}{4\pi\varepsilon_0} \frac{+q}{\sqrt{2}\,a}$$

$$V_{34} = \frac{1}{4\pi\varepsilon_0} \frac{-q}{a}$$

$$\therefore \text{answer} = \frac{1}{4\pi\varepsilon_0}\left(-\frac{q^2}{a} + \frac{q^2}{\sqrt{2}\,a} - \frac{q^2}{a} \right)$$

$$= \frac{1}{4\pi\varepsilon_0}\left(-\frac{2q^2}{a} + \frac{q^2}{\sqrt{2}\,a} \right)$$

$$= \frac{1}{4\pi\varepsilon_0} \frac{q^2}{a}\left(-2 + \frac{1}{\sqrt{2}} \right)$$

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1. place charge 1, $W_1 = 0$
2. place charge 2, $W_2 = \frac{1}{4\pi\varepsilon_0} \frac{-q^2}{a}$
3. place charge 3, $W_3 = \frac{1}{4\pi\varepsilon_0}\left(\frac{q^2}{\sqrt{2}\,a} - \frac{q^2}{a} \right)$

$$= \frac{1}{4\pi\varepsilon_0} \frac{q^2}{a}\left(-1 + \frac{1}{\sqrt{2}} \right)$$

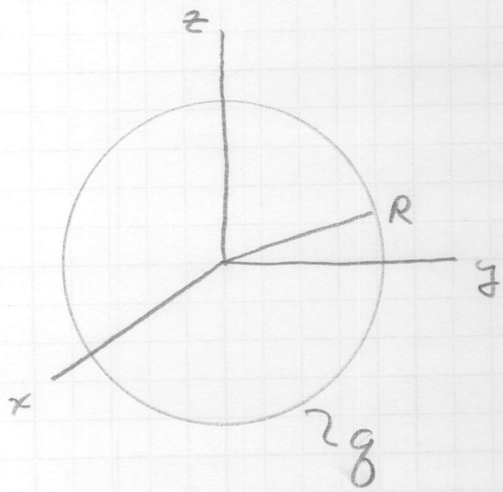
4. place charge 4, $W_4 = \frac{1}{4\pi\varepsilon_0} \frac{q^2}{a}\left(-2 + \frac{1}{\sqrt{2}} \right)$

$$\text{answer} = W_1 + W_2 + W_3 + W_4$$

$$= \frac{1}{4\pi\varepsilon_0} \frac{q^2}{a}\left(0 - 1 - 1 + \frac{1}{\sqrt{2}} - 2 + \frac{1}{\sqrt{2}} \right)$$

$$= \frac{1}{4\pi\varepsilon_0} \frac{q^2}{a}\left(-4 + \sqrt{2} \right)$$

2·32) a. $W = \frac{1}{2}\int \rho V d\tau$



$\rho = \frac{q}{\frac{4}{3}\pi R^3}$ $\quad r < R$

$\rho = \frac{3q}{4\pi R^3}$

$= 0 \quad r > R$

$d\tau = r^2 dr\, d\cos\theta\, d\phi$

$V = \frac{q}{4\pi\epsilon_0}\frac{1}{2R}\left(3 - \frac{r^2}{R^2}\right)$

from problem 2-21.

$W = \frac{1}{2}\int \rho\, d\tau$

$= \frac{1}{2}\int_0^R \rho \frac{q}{4\pi\epsilon_0}\frac{1}{2R}\left(3 - \frac{r^2}{R^2}\right) r^2 dr\, d\cos\theta\, d\phi$

$= \frac{1}{\cancel{2}}\rho \frac{1}{4\pi\epsilon_0}\frac{q}{2R} \cancel{4}\pi \int_0^R \left(3 - \frac{r^2}{R^2}\right) r^2 dr$

$= \rho \frac{1}{4\pi\epsilon_0}\frac{q}{\cancel{2}R} 2\pi \left[\frac{\cancel{3}r^3}{\cancel{3}} - \frac{r^5}{5R^2}\right]_0^R$

$= \frac{1}{4\pi\epsilon_0}\frac{\rho q \pi}{\cancel{R}}\left[R^{\cancel{3}\,2} - \frac{R^{\cancel{3}\,2}}{5} - 0\right]$

$= \frac{1}{4\pi\epsilon_0}\rho q \pi R^2 \frac{4}{5}$

$= \frac{1}{4\pi\epsilon_0}\frac{3q}{\cancel{4}\cancel{\pi}R^{\cancel{3}}} q \cancel{R^2}\cancel{\pi}\frac{\cancel{4}}{5}$

$\boxed{W = \frac{1}{4\pi\epsilon_0}\frac{3q^2}{5R}}$