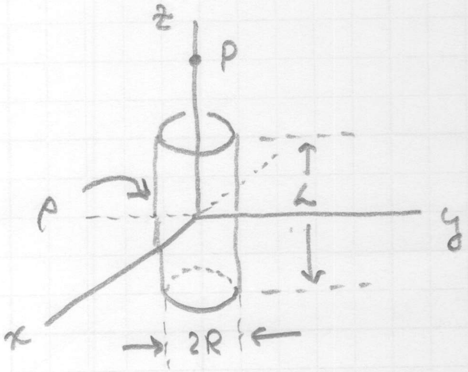
2-27)



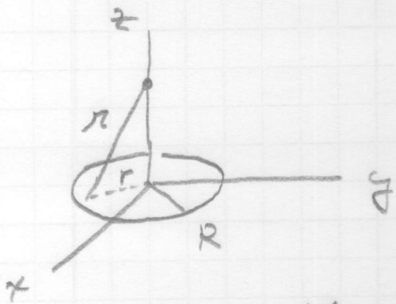
$z > \frac{L}{2}$

$$V(\vec{r}) = \frac{1}{4\pi\epsilon_0}\int \frac{\rho(\vec{r}\,')}{\mathscr{r}}\,d\tau'$$

Do a disk first. Use cylindrical coordinates.

$$\vec{E}_{disk} = \frac{1}{4\pi\epsilon_0}2\pi\sigma\left(1 - \frac{z}{\sqrt{z^2+R^2}}\right)\hat{z}$$

Problem 2-6.



$$V = -\int_0^z \vec{E}\cdot d\vec{l} \qquad d\vec{l} = dz\,\hat{z}$$

$$= -\int_\infty^z \frac{1}{4\pi\epsilon_0}2\pi\sigma\left(1 - \frac{z'}{\sqrt{z'^2+R^2}}\right)dz'\,\hat{z}$$

$$= -\frac{1}{4\pi\epsilon_0}2\pi\sigma\left[z'\Big|_\infty^z - \sqrt{z^2+R^2}\Big|_\infty^z\right]$$

let lower limit be z_0

$$[\;] = (z - z_0) - \left(\sqrt{z^2+R^2} - \sqrt{z_0^2+R^2}\right)$$

let z_0 get large

$$\cong z - \cancel{z_0} - \sqrt{z^2+R^2} + \cancel{z_0}$$

$$= z - \sqrt{z^2+R^2}$$

$$V = -\frac{1}{4\pi\epsilon_0}2\pi\sigma\left(z - \sqrt{z^2+R^2}\right)$$

$$V = \frac{1}{4\pi\varepsilon_0} 2\pi\sigma \left[\sqrt{z^2+R^2} - z\right]$$

Rewrite V in terms of the total charge.

$$V = \frac{1}{4\pi\varepsilon_0} 2\pi\sigma \frac{R^2}{R^2}\left(\sqrt{z^2+R^2} - z\right)$$

$$= \frac{1}{4\pi\varepsilon_0} \frac{2\pi}{R^2} Q \left(\sqrt{z^2+R^2} - z\right)$$

Now do the cylinder.

$$dV = \frac{1}{\cancel{4\pi}_{2}\varepsilon_0} \frac{2\pi}{R^2} dQ \left(\sqrt{z^2+R^2} - z\right)$$

$$dQ = \rho \pi R^2 dz$$

$$= \frac{1}{2\varepsilon_0} \frac{1}{\cancel{R^2}} \rho \pi \cancel{R^2} dz \left(\sqrt{z^2+R^2} - z\right)$$

$$= \frac{\pi\rho}{2\varepsilon_0}\left(\sqrt{z^2+R^2} - z\right) dz$$

$$V = \int dV$$

$$= \int_{z-\frac{L}{2}}^{z+\frac{L}{2}} \frac{\pi\rho}{2\varepsilon_0}\left(\sqrt{z'^2+R^2} - z'\right) dz'$$

$$= \frac{\pi\rho}{2\varepsilon_0}\left[\frac{z'}{2}\sqrt{z'^2+R^2} + \frac{R^2}{2}\ln\left(z' + \sqrt{R^2+z'^2}\right) - \frac{z'^2}{2}\right]_{z-\frac{L}{2}}^{z+\frac{L}{2}}$$

$$V = \frac{\pi \rho}{2\varepsilon_0}\Bigg[\left(\frac{z}{2}+\frac{L}{4}\right)\left(z^2+\frac{L^2}{4}+zL+R^2\right)^{1/2} +$$

$$\frac{R^2}{2}\ln\left(z+\frac{L}{2}+\sqrt{R^2+z^2+\frac{L^2}{4}+zL}\right) +$$

$$-\frac{1}{2}\left(\cancel{z^2}+\cancel{\frac{L^2}{4}}+zL\right)$$

$$-\left(\frac{z}{2}-\frac{L}{4}\right)\left(z^2+\frac{L^2}{4}-zL+R^2\right)^{1/2} +$$

$$-\frac{R^2}{2}\ln\left(z-\frac{L}{2}+\sqrt{R^2+z^2+\frac{L^2}{4}-zL}\right) +$$

$$+\left(\frac{1}{2}\right)\left(\cancel{z^2}+\cancel{\frac{L^2}{4}}-zL\right)\Bigg]$$

$$\boxed{V = \frac{\pi \rho}{2\varepsilon_0}\Bigg[\left(\frac{z}{2}+\frac{L}{4}\right)\left(z^2+\frac{L^2}{4}+zL+R^2\right)^{1/2} + \frac{R^2}{2}\ln\left[\frac{z+\frac{L}{2}+\sqrt{R^2+z^2+\frac{L^2}{4}+zL}}{z-\frac{L}{2}+\sqrt{R^2+z^2+\frac{L^2}{4}-zL}}\right] - \left(\frac{z}{2}-\frac{L}{4}\right)\left(z^2+\frac{L^2}{4}-zL+R^2\right)^{1/2} - zL\Bigg]}$$

$$\vec{E} = -\nabla V = -\frac{\partial V}{\partial z}\hat{z}$$

See Mathematica Output.