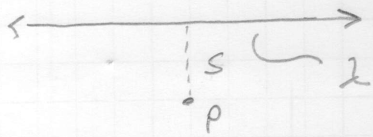
2-21) See notes p. 19-22

2-22)



$$\vec{E} = \frac{1}{2\pi\varepsilon_0}\frac{\lambda}{s}\hat{s}$$

Use cylindrical coordinates $\quad V(s) = -\int_\infty^s \vec{E}\cdot d\vec{l}$

$$d\vec{l} = ds\,\hat{s}$$

$$V(s) = -\int_\infty^s \frac{1}{2\pi\varepsilon_0}\frac{\lambda}{s'}\hat{s}'\cdot ds'\,\hat{s}'$$

$$= -\frac{1}{2\pi\varepsilon_0}\lambda\int_\infty^s \frac{1}{s'}ds'$$

$$= -\frac{\lambda}{2\pi\varepsilon_0}\ln s'\Big|_\infty^s$$

$$= -\frac{\lambda}{2\pi\varepsilon_0}\left(\ln s - \ln\infty\right)$$

↳ EEEKK!

We chose poorly. The problem here is the infinitely long wire which is not realistic. Pick some arbitrary point like $s = s_0$

$$= -\frac{\lambda}{2\pi\varepsilon_0}\left(\ln s - \ln s_0\right)$$

$$\boxed{V(s) = -\frac{\lambda}{2\pi\varepsilon_0}\ln\left(\frac{s}{s_0}\right)}$$

$$\nabla V = \frac{\partial V}{\partial s}\hat{s} + \frac{1}{s}\frac{\partial V}{\partial\varphi}\hat{\varphi} + \frac{\partial V}{\partial z}\hat{z}$$

$$= -\frac{\lambda}{2\pi\varepsilon_0}\frac{1}{s/s_0}\cdot\frac{1}{s_0}\hat{s} + 0 + 0$$

$$\nabla V = -\frac{\lambda}{2\pi\epsilon_0}\frac{1}{s}\hat{s}$$

$$\therefore \vec{E} = -\nabla V = \frac{\lambda}{2\pi\epsilon_0}\frac{1}{s}\hat{s}$$ as before.