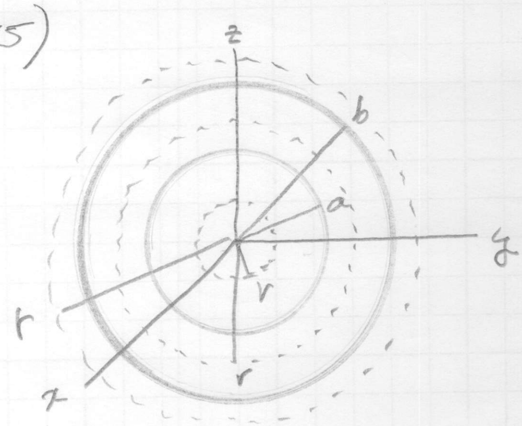
2-15)



$$\rho = \frac{k}{r^2} \qquad a \le r \le b$$
$$= 0 \qquad r < a$$
$$= 0 \qquad r > b$$

$$\vec{E} = ?$$

for $r < a$ $\quad \nabla \cdot \vec{E}_1 = \frac{\rho}{\varepsilon_0}$

$$\int_V \nabla \cdot \vec{E}_1 \, d\tau = \int_V \frac{\rho}{\varepsilon} \, d\tau$$

$$\oint_S \vec{E}_1 \cdot d\vec{A} = 0$$

$$d\vec{A} = r^2 \, d\cos\theta \, d\phi \, \hat{r}$$
$$\vec{E} = E(r)\,\hat{r} \quad \text{by symmetry}$$

$$\oint E_1(r)\,\hat{r} \cdot r^2 \, d\cos\theta \, d\phi \, \hat{r} = 0$$

$$E_1 r^2 \int_0^{2\pi} \int_{-1}^{1} d\cos\theta \, d\phi = 0$$

$$E_1 4\pi r^2 = 0$$

$$E_1 = 0$$

$$\boxed{\therefore \vec{E}_1 = 0}$$

for $a \le r \le b$ $\quad \nabla \cdot \vec{E}_2 = \frac{\rho}{\varepsilon_0}$

$$\int_V \nabla \cdot \vec{E}_2 \, d\tau = \int_V \frac{\rho}{\varepsilon_0} \, d\tau$$

$$\vec{E}_2 = E_2(r)\,\hat{r} \quad \text{by symmetry}$$

$$\oint_S \vec{E}_2 \cdot d\vec{A} = \frac{1}{\varepsilon_0} \int_0^{2\pi}\int_{-1}^{1}\int_a^{r} \frac{k}{\cancel{r'^2}} \cancel{r'^2}\, dr'\, d\cos\theta\, d\phi$$

Integrand only depends on r so

$$\int_0^{2\pi}\int_{-1}^{1} d\cos\theta\, d\phi = 2(2\pi) = 4\pi$$

$$\oint_S E_2 \hat{r} \cdot r^2 d\cos\theta\, d\phi\, \hat{r} = \frac{4\pi k}{\varepsilon_0} \int_a^r dr'$$

$$E_2 r^2 \cancel{4\pi} = \frac{\cancel{4\pi} k}{\varepsilon_0}(r-a)$$

$$E_2 = \frac{k}{\varepsilon_0 r^2}(r-a)$$

$$\boxed{\therefore \vec{E}_2 = \frac{k}{\varepsilon_0}\left(\frac{1}{r} - \frac{a}{r^2}\right)\hat{r}}$$

for $r > b$ $\qquad \nabla \cdot \vec{E}_3 = \frac{\rho}{\varepsilon_0}$

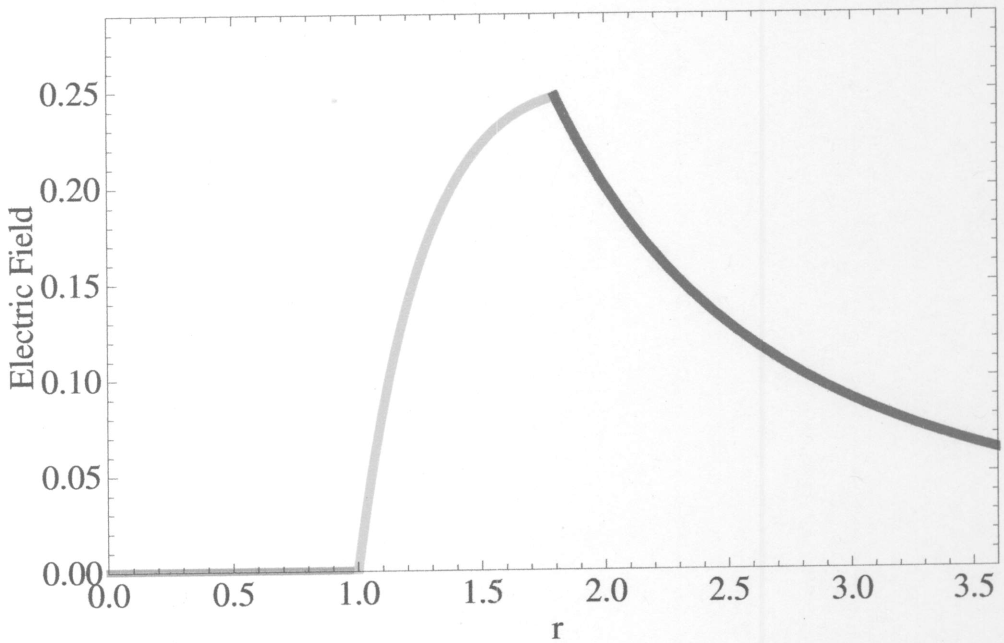
$$\int_V \nabla \cdot \vec{E}_3\, d\tau = \int \frac{\rho}{\varepsilon_0} d\tau$$

$$\oint_S \vec{E}_3 \cdot d\vec{A} = \frac{1}{\varepsilon_0} \int_0^{2\pi}\int_{-1}^{1}\int_a^{b} \frac{k}{\cancel{r^2}} \cancel{r^2}\, dr\, d\cos\theta\, d\phi$$

$$\int_0^{2\pi}\int_{-1}^{1} E_3 r^2 d\cos\theta\, d\phi = \frac{k}{\varepsilon_0}(2\pi)(2)\int_a^b dr$$

$$E_3 r^2 (2)(2\pi) = \frac{\cancel{4\pi} k}{\varepsilon_0}(b-a)$$

$$\vec{E}_3 = \frac{k}{\varepsilon_0}\left(\frac{b-a}{r^2}\right)\hat{r}$$


Electric Field
0.25
0.20
0.15
0.10
0.05
0.00
0.0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
r