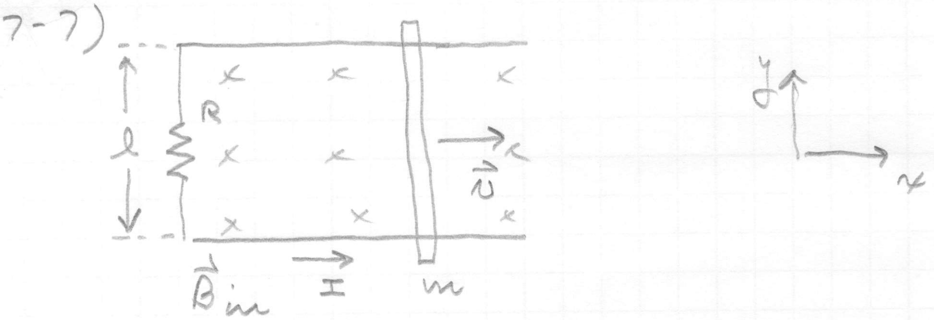
7-7)



a. $\vec{F}_m = Q\vec{v}\times\vec{B} = QvB$ here

$F_m = \lambda l v B$

$\vec{F}_e = Q\vec{E}$

$F_e = \lambda l E = \lambda V$

$F_m = F_e$

$\cancel{\lambda} l v B = \cancel{\lambda} V$

$V = lvB = IR$

$\therefore\ I = \frac{lvB}{R}$

b. $\vec{F}_B = Q\vec{v}\times\vec{B}$

$F_B = IlB = \frac{lv}{R}B\,lB = \frac{v\,l^2B^2}{R}$

$\vec{F}_B = -v\frac{l^2}{R}B^2\hat{x}$

c. $F_B = -v\frac{l^2B^2}{R}$ $\quad v = v_0$ at $t=0$

$m\frac{dv}{dt} = -\frac{v l^2B^2}{R}$

$= -\frac{l^2B^2}{R}v$

$\frac{1}{v}\frac{dv}{dt} = -\frac{l^2B^2}{mR}$

$\int_0^t \frac{1}{v}\frac{dv}{dt'}dt' = -\frac{l^2B^2}{mR}\int_0^t dt'$

$\int_{v_0}^{v}\frac{1}{v}dv = -\frac{l^2B^2}{mR}t$

$\ln\frac{v}{v_0} = -\frac{l^2B^2}{mR}t$

$$\frac{v}{v_0} = e^{-\lambda t} \qquad \lambda = \frac{\ell^2 B^2}{mR}$$

$$\boxed{v = v_0 e^{-\lambda t}}$$

d. $KE_0 = \frac{1}{2} m v_0^2$

$$P = IV = \left(\frac{\ell v B}{R}\right)(\ell v B) = \frac{\ell^2 v^2 B^2}{R}$$

$$W = \int P\,dt$$

$$= \int \frac{\ell^2 B^2}{R} v^2\,dt$$

$$= \int \frac{\ell^2 B^2}{R} v_0^2 e^{-2\lambda t}\,dt$$

$$= \frac{\ell^2 B^2}{R} v_0^2 \int_0^\infty e^{-2\lambda t}\,dt$$

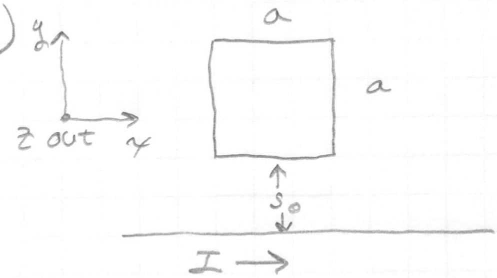
$$= \frac{\ell^2 B^2}{R} v_0^2 \left.\frac{e^{-2\lambda t}}{-2\lambda}\right|_0^\infty$$

$$= -\frac{\cancel{\ell^2}\cancel{B^2}}{\cancel{R}} v_0^2 \frac{m\cancel{R}}{2\cancel{\ell^2}\cancel{B^2}} \left[0 - 1\right]$$

$$= \frac{m}{2} v_0^2$$

$$\boxed{W = KE_0}$$

7-8)



$\vec{B} = \frac{\mu_0 I}{2\pi s}\hat{\phi}$

a. In the plane of the loop

$$\vec{B} = \frac{\mu_0 I}{2\pi y}\hat{z} \qquad d\vec{A} = dx\,dy\,\hat{z}$$

$$\Phi = \int \vec{B}\cdot d\vec{A}$$

$$= \int_{s_0}^{s_0+a}\int_{-\frac{a}{2}}^{\frac{a}{2}} \frac{\mu_0 I}{2\pi y}\hat{z}\cdot dx\,dy\,\hat{z}$$

$$= \frac{\mu_0 I}{2\pi}\int_{s_0}^{s_0+a}\int_{-\frac{a}{2}}^{\frac{a}{2}} \frac{1}{y}\,dx\,dy$$

$$= \frac{\mu_0 I}{2\pi}\,a\,\ln y\Big|_{s_0}^{s_0+a}$$

$$= \frac{\mu_0 I a}{2\pi}\ln\left(\frac{s_0+a}{s_0}\right)$$

$$= \frac{\mu_0 I a}{2\pi}\ln\left(1+\frac{a}{s_0}\right)$$

b. $\vec{v} = v\hat{y} = \frac{dy}{dt}\hat{y}$ $\quad s_0$ will change in $\vec{B}$.

$$\frac{d\Phi}{dt} = \frac{\mu_0 I a}{2\pi}\frac{d}{dt}\ln(1+a/s_0)$$

$$= \frac{\mu_0 I a}{2\pi}\frac{d\ln(1+a/s_0)}{ds_0}\frac{ds_0}{dt}$$

$$= \frac{\mu_0 I a v}{2\pi}\frac{1}{1+\frac{a}{s_0}}\cdot a(-1)s_0^{-2}$$

$$= -\frac{\mu_0 I v a}{2\pi}\frac{a s_0}{s_0+a}\frac{1}{s_0^2}$$

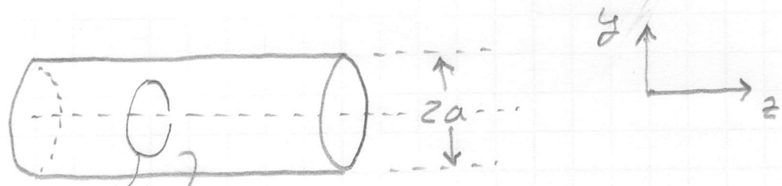
$$\frac{d\Phi}{dt} = -\frac{\mu_0 I v}{2\pi} \frac{a^2}{s_0(s_0+a)}$$

$$\mathcal{E} = -\frac{d\Phi}{dt}$$

$$\mathcal{E} = \frac{\mu_0 I a^2 v}{2\pi} \frac{1}{s_0(a+s_0)}$$

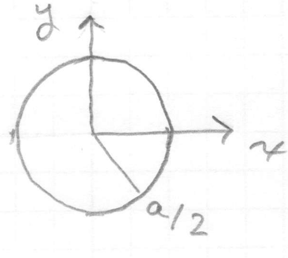
c. There will be no change in $\vec{B}$ and in the flux Φ so there will be no emf.

7-12)



$$\vec{B}(t) = B_0 \cos \omega t \, \hat{z}$$

$R, \; R_2 = \frac{a}{2}$



$$\Phi = \int \vec{B} \cdot d\vec{A}$$

$$\vec{B} = B_0 \cos\omega t \, \hat{z}$$

$$d\vec{A} = dx\,dy \, \hat{z}$$

$$\Phi = B_0 \cos \omega t \int dx\,dy$$

$$= B_0 \cos\omega t \; \pi \left(\frac{a}{2}\right)^2$$

$$\Phi = B_0 \frac{\pi a^2}{4} \cos\omega t$$

$$\mathcal{E} = -\frac{d\Phi}{dt}$$

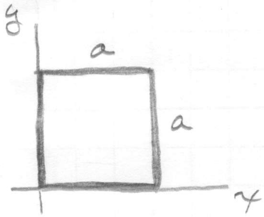
$$= - B_0 \frac{\pi a^2}{4} \left(-\sin\omega t (\omega)\right)$$

$$= B_0 \omega \frac{\pi a^2}{4} \sin \omega t$$

$$\mathcal{E} = IR$$

$$I = \frac{\mathcal{E}}{R} = B_0 \frac{\omega \pi a^2}{4R} \sin \omega t$$

7-13)



$\vec{B}(y,t) = k y^3 t^2 \hat{z}$

$\Phi = \int \vec{B} \cdot d\vec{A}$

$d\vec{A} = dx\, dy\, \hat{z}$

$\Phi = \int_0^a \int_0^a k y^3 t^2 \, dx\, dy$

$= k t^2 a \frac{y^4}{4} \Big|_0^a$

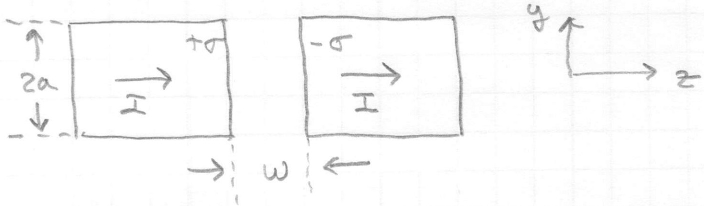
$= k \frac{t^2}{4} a^5$

$\mathcal{E} = - \frac{d\Phi}{dt}$

$= - \frac{d}{dt} \frac{k a^5 t^2}{4}$

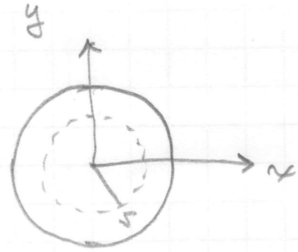
$\boxed{\mathcal{E} = - \frac{k a^5 t}{2}}$

7-31)



Between the plates: $\vec{E} = \frac{\sigma}{\varepsilon_0} \hat{z}$

$$= \frac{Q}{A\varepsilon_0} \hat{z}$$



$d\vec{A} = dx\,dy\,\hat{z}$

$$\nabla \times \vec{B} = \mu_0 \vec{J} + \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t}$$

$$\int \nabla \times \vec{B} \cdot d\vec{A} = \int \left[\mu_0 \vec{J} + \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t} \right] \cdot d\vec{A}$$

$$\int \vec{B} \cdot d\vec{l} = \cancelto{0}{\mu_0 I_{enc}} + \mu_0 \varepsilon_0 \int \frac{\partial \vec{E}}{\partial t} \cdot d\vec{A}$$

$$2\pi s B = \mu_0 \cancel{\varepsilon_0} \int \frac{\partial}{\partial t} \frac{Q}{A \cancel{\varepsilon_0}} \hat{z} \cdot dx\,dy\,\hat{z}$$

$$= \frac{\mu_0}{A} \int \underbrace{\frac{\partial Q}{\partial t}}_{I} dx\,dy$$

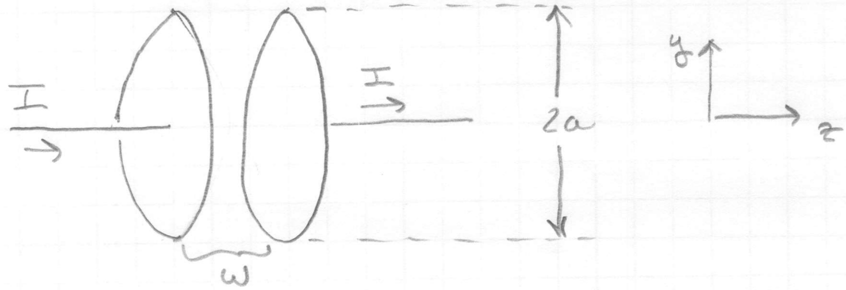
$$= \frac{\mu_0 I}{A} \int dx\,dy$$

$$= \frac{\mu_0 I}{A} \pi s^2 \qquad A = \pi a^2$$

$$2\pi \cancel{s} B = \frac{\mu_0 I \cancel{\pi} s^{\cancel{2}}}{\cancel{\pi} a^2}$$

$$\vec{B} = \frac{\mu_0 I s}{2\pi a^2} \hat{\varphi}$$

7-32)



a. for $\omega \ll a$

$$\vec{E} = \frac{\sigma}{\varepsilon_0}\hat{z} = \frac{Q}{A\varepsilon_0}\hat{z}$$

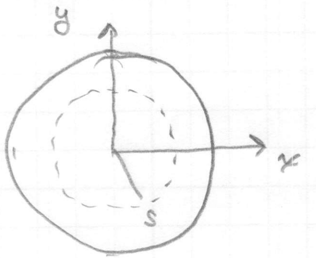
$$Q = \int I\,dt' = I\int_0^t dt'$$

$$Q = It$$

$$\boxed{\vec{E} = \frac{It}{A\varepsilon_0}\hat{z}}$$

$$A = \pi a^2$$

b.



$$d\vec{A} = dx\,dy\,\hat{z}$$

$$\nabla \times \vec{B} = \mu_0 \cancel{\vec{J}}^{\,0} + \mu_0\varepsilon_0\frac{\partial \vec{E}}{\partial t}$$

$$\int \nabla \times \vec{B}\cdot d\vec{A} = \mu_0\varepsilon_0\int\frac{\partial \vec{E}}{\partial t}\cdot d\vec{A}$$

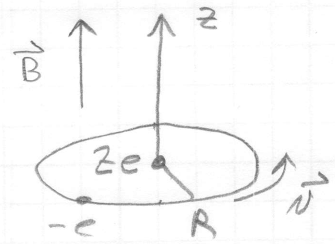
$$\int \vec{B}\cdot d\vec{l} = \mu_0\cancel{\varepsilon_0}\int\frac{\partial}{\partial t}\frac{It}{A\cancel{\varepsilon_0}}\hat{z}\cdot dx\,dy\,\hat{z}$$

$$B\,2\pi s = \frac{\mu_0 I}{A}\underbrace{\int dx\,dy}_{\pi s^2}$$

$$B\,2\cancel{\pi}\cancel{s} = \frac{\mu_0 I}{\cancel{\pi}a^2}\cancel{\pi}s^{\cancel{2}\,1}$$

$$\boxed{\vec{B} = \frac{\mu_0 I s}{2\pi a^2}\hat{\varphi}}$$

7-49)



$$F_c = \frac{1}{4\pi\varepsilon_0}\frac{Ze^2}{R^2} = \frac{m_e v_0^2}{R}$$

$$\vec{F}_{mag} = e\,\vec{v}_e \times \vec{B}$$
$$= -e v_e B\,\hat{r}$$

for $\vec{B} = dB\,\hat{z}$

$$F_c = \frac{1}{4\pi\varepsilon_0}\frac{Ze^2}{R'^2} + e v_e dB = \frac{m_e v^2}{R'}$$

$$T_0 = KE_0 = \frac{1}{2} m_e v_0^2 = \frac{1}{2}\frac{1}{4\pi\varepsilon_0}\frac{Ze^2}{R}$$

$$T_1 = \frac{1}{2} m_e v_1^2 = \frac{1}{2}\frac{1}{4\pi\varepsilon_0}\frac{Ze^2}{R'} + \frac{1}{2} e v_1 R' B$$

The induced field is

$$E_{in} = \frac{R}{2}\frac{dB}{dt}\,\hat{\varphi}$$

$$\therefore\ ma = m\frac{dv}{dt} = eE_{in} = \frac{eR}{2}\frac{dB}{dt}$$

$$\therefore\ m\,dv = \frac{eR}{2}\,dB$$

The change in KE is

$$dT = d\left(\frac{1}{2}mv^2\right) = mv\,dv.$$
$$= v\,m\,dv$$
$$= v\,\frac{eR}{2}\,dB$$

$$\therefore\ dT = \frac{e v_0 R}{2}\,dB \text{ here.}$$

$$T_1 = \frac{1}{2}\frac{1}{4\pi\varepsilon_0}\frac{Ze^2}{R'} + \frac{1}{2} e v_1 R' B$$

---

use $R' = R + dr = R\left(1 + \frac{dr}{R}\right)$

$v_1 = v_0 + dv$

$B = dB$

$$= \frac{1}{2}\left[\frac{1}{4\pi\varepsilon_0}\frac{Ze^2}{R\left(1+\frac{dr}{R}\right)} + e\left(v_0 + dv\right)R\left(1+\frac{dr}{R}\right)dB\right.$$

---

use $\left(1 + \frac{dr}{R}\right)^{-1} \approx \left(1 - \frac{dr}{R}\right)$

from Taylor series

$$T_1 = \frac{1}{2}\left[\frac{1}{4\pi\varepsilon_0}\frac{Ze^2}{R}\left(1 - \frac{dr}{R}\right) + \right.$$

$$e v_0 R\, dB + \cancel{e\, dv R\, dB}^{\,0} +$$

$$\left. e v_0 \cancel{dr\, dB}^{\,0} + e\, \cancel{dv\, dr\, dB}^{\,0}\right]$$

$$= \frac{1}{2}\frac{1}{4\pi\varepsilon_0}\frac{Ze^2}{R} - \frac{1}{2}\frac{1}{4\pi\varepsilon_0} Ze^2\frac{dr}{R^2} + \frac{1}{2} e v_0 R\, dB$$

$$= T_0 + dT \quad \text{(in general)}$$

$T_0 = \frac{1}{2}\frac{1}{4\pi\varepsilon}\frac{Ze^2}{R}$ see above.

$\therefore T_0$ is the first term.

$$T_1 - T_0 = dT = \frac{e v_0 R\, dB}{2} - \frac{1}{2}\frac{1}{4\pi\varepsilon_0} Ze^2\frac{dr}{R^2}$$

But we showed $dT = \frac{e v_0 R\, dB}{2}$ on the previous page so $dr = 0$!