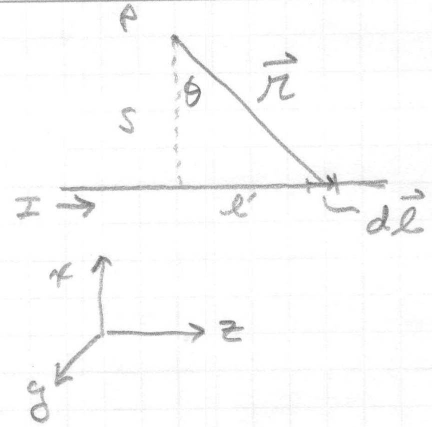
5-22)



$$\vec{A} = \frac{\mu_0}{4\pi}\int \frac{\vec{I}}{r}\,dl'$$

$$= \frac{\mu_0 I}{4\pi}\int \frac{d\vec{l}'}{r}$$

$$d\vec{l}' = dz'\,\hat{z}$$

$$r = \sqrt{s^2 + z'^2}$$

$$= \frac{\mu_0 I}{4\pi}\int_{z_1}^{z_2} \frac{dz'\,\hat{z}}{\sqrt{s^2+z'^2}}$$

$$\vec{A} = \frac{\mu_0 I}{4\pi}\log\left[z' + \sqrt{s^2+z'^2}\right]\Big|_{z_1}^{z_2}\,\hat{z}$$

$$\vec{A} = \frac{\mu_0 I}{4\pi}\log\left[\frac{z_2+\sqrt{s^2+z_2^2}}{z_1+\sqrt{s^2+z_1^2}}\right]\hat{z}$$

$$\vec{B} = \nabla\times\vec{A} = -\frac{\partial A_z}{\partial s}\,\hat{\phi}$$

$$= -\frac{\mu_0 I}{4\pi}\,\frac{\cancel{z_1+\sqrt{z_1^2+s^2}}}{z_2+\sqrt{z_2^2+s^2}}\times\left[\frac{s}{\left(\cancel{z_1+\sqrt{z_1^2+s^2}}\right)\sqrt{s^2+z_2^2}} + \frac{-s\left(z_2+\sqrt{z_2^2+s^2}\right)}{\left(z_1+\sqrt{z_1^2+s^2}\right)^{\cancel{2}}\sqrt{s^2+z_1^2}}\right]$$

from Mathematica

$$= -\frac{\mu_0 I s}{4\pi}\left[\frac{1}{\left(z_2+\sqrt{z_2^2+s^2}\right)\sqrt{s^2+z_2^2}} - \frac{1}{\left(z_1+\sqrt{z_1^2+s^2}\right)\sqrt{s^2+z_1^2}}\right]$$

TOPS 35500

$$\sin\theta_2 = \frac{z_2}{\sqrt{s^2+z_2^2}}$$

$$\sin\theta_1 = \frac{z_1}{\sqrt{s^2+z_1^2}}$$

1st term

$$\frac{1}{z_2+\sqrt{z_2^2+s^2}} \cdot \frac{1}{\sqrt{z_2^2+s^2}} \; \frac{z_2-\sqrt{z_2^2+s^2}}{z_2-\sqrt{z_2^2+s^2}} =$$

$$= \frac{z_2-\sqrt{z_2^2+s^2}}{(\cancel{z_2^2}-\cancel{z_2^2}-s^2)\sqrt{z_2^2+s^2}}$$

$$= -\frac{1}{s^2}\left[\frac{z_2}{\sqrt{z_2^2+s^2}} - 1\right]$$

2nd term

$$\frac{1}{(z_1+\sqrt{z_1^2+s^2})} \cdot \frac{1}{\sqrt{z_1^2+s^2}} \; \frac{z_1-\sqrt{z_1^2+s^2}}{z_1-\sqrt{z_1^2+s^2}} =$$

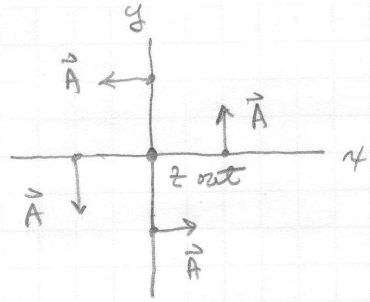
$$= \frac{z_1-\sqrt{z_1^2+s^2}}{(\cancel{z_1^2}-\cancel{z_1^2}-s^2]\sqrt{z_1^2+s^2}}$$

$$= -\frac{1}{s^2}\left[\frac{z_1}{\sqrt{z_1^2+s^2}} - 1\right]$$

$$\vec{B} = +\mu_0 I \cancel{s}\left(+\frac{1}{s^{\cancel{2}}}\right)\left[\frac{z_2}{\sqrt{z_2^2+s^2}} - \cancel{1} - \frac{z_1}{\sqrt{z_1^2+s^2}} + \cancel{1}\right]$$

$$= \frac{\mu_0 I}{4\pi s}\left[\sin\theta_2 - \sin\theta_1\right]$$

5-23) $\vec{A} = k\hat{\varphi} = \frac{\mu_0}{4\pi}\int \frac{\vec{I}}{r}\,dl$



$\vec{I} = I_0\,\hat{\varphi}$

$$\vec{B} = \nabla \times \vec{A}$$

$$= \frac{1}{s}\frac{\partial (sk)}{\partial s}\,\hat{z}$$

$$\vec{B} = \frac{k}{s}\,\hat{z}$$

$$\nabla \times \vec{B} = \mu_0 \vec{J}$$

$$\vec{J} = \frac{1}{\mu_0}\nabla \times \vec{B}$$

$$= \frac{1}{\mu_0}\left[-\frac{\partial B_z}{\partial s}\,\hat{\varphi}\right]$$

$$= -\frac{1}{\mu_0}\frac{\partial}{\partial s}(k s^{-1})\,\hat{\varphi}$$

$$= +\frac{1}{\mu_0}(+1)\,k s^{-2}\,\hat{\varphi}$$

$$\boxed{\vec{J} = \frac{k}{\mu_0}\frac{1}{s^2}\,\hat{\varphi}}$$

5 - 33)

$$\vec{B}_{dip} = \frac{\mu_0 m}{4\pi r^3} \left(2\cos\theta\, \hat{r} + \sin\theta\, \hat{\theta} \right)$$

Let $\vec{m} = m\hat{z}$

$$= m\left(\cos\theta\, \hat{r} - \sin\theta\, \hat{\theta}\right)$$

$$\vec{m} = (\vec{m}\cdot\hat{r})\hat{r} - m\sin\theta\, \hat{\theta}$$

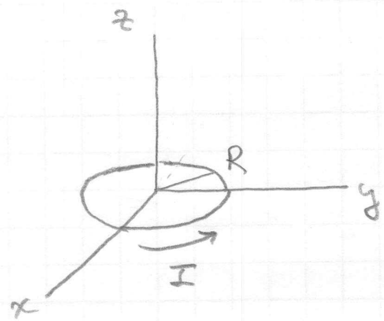
$$m\sin\theta\, \hat{\theta} = (\vec{m}\cdot\hat{r})\hat{r} - \vec{m}$$

$$\vec{B}_{dip} = \frac{\mu_0}{4\pi}\frac{1}{r^3}\left(2m\cos\theta\, \hat{r} + m\sin\theta\, \hat{\theta}\right)$$

$$= \frac{\mu_0}{4\pi}\frac{1}{r^3}\left(2(\vec{m}\cdot\hat{r})\hat{r} + (\vec{m}\cdot\hat{r})\hat{r} - \vec{m}\right)$$

$$= \frac{\mu_0}{4\pi}\frac{1}{r^3}\left[3(\vec{m}\cdot\hat{r})\hat{r} - \vec{m}\right]$$

5-34)



a. $\vec{m} = I \int d\vec{A}$

$= I \pi R^2 \hat{z}$

b. $\vec{B} = \frac{\mu_0 m}{4\pi r^3}\left(2\cos\theta\, \hat{r} + \sin\theta\, \hat{\theta}\right)$

$= \frac{\mu_0 I \pi R^2}{4\pi r^3}\left(2\cos\theta\, \hat{r} + \sin\theta\, \hat{\theta}\right)$

c. for $\theta = 0$, $\hat{r} = \hat{z}$, $r = z$

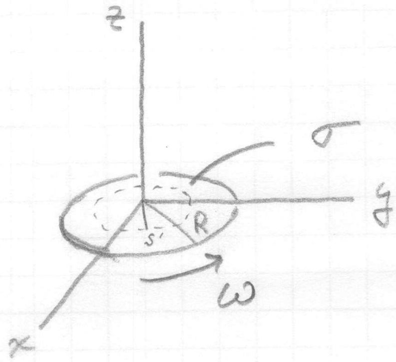
$\vec{B} = \frac{\mu_0 I \cancel{\pi} R^2}{{}_2\cancel{4}\cancel{\pi} r^3}\left(2(1)\hat{r} + 0\,\hat{\theta}\right)$

$= \frac{\mu_0 I R^2}{2 z^3}\hat{z}$

$\vec{B} = \frac{\mu_0 I}{2}\underbrace{\frac{R^2}{(R^2+z^2)^{3/2}}}_{\sim z^2}\hat{z} \qquad z >> R$

$\vec{B} = \frac{\mu_0 I R^2}{2z^3}\hat{z}$ same as above

5-35)



$$\vec{m} = I \int d\vec{A}$$

$$I = \sigma \frac{\omega}{2\pi} 2\pi s' ds'$$

$$= \sigma \omega s' ds'$$

$$\int d\vec{A} = \pi s'^2 \hat{z}$$

$$d\vec{m} = \sigma \omega s' ds' \pi s'^2 \hat{z}$$

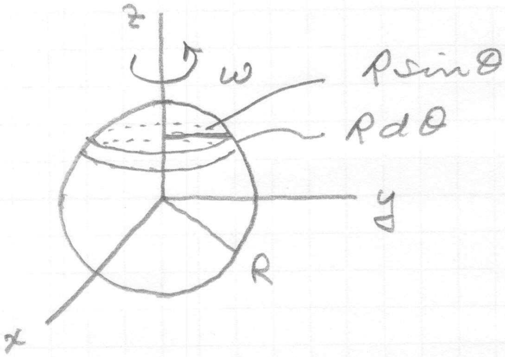
$$\vec{m} = \int_0^R \pi \sigma \omega s'^3 ds' \hat{z}$$

$$= \pi \sigma \omega \frac{s'^4}{4} \Big|_0^R \hat{z}$$

$$\boxed{\vec{m} = \frac{\pi \sigma \omega}{4} R^4 \hat{z}}$$

5.36)

5 MM, 0.8



$$dq = \sigma(2\pi R\sin\theta)\,R\,d\theta$$
$$= \sigma\, 2\pi R^2 \sin\theta\, d\theta$$

$$I = \frac{dq}{dt} = \sigma\,\frac{2\pi R^2 \sin\theta\, d\theta}{2\pi/\omega}$$

$$I = \sigma\omega R^2 \sin\theta\, d\theta$$

$$d\vec{m} = I\,\pi\,(R\sin\theta)^2\ \hat{z}$$
$$= \sigma\omega R^2 \sin\theta\, d\theta\ \pi R^2 \sin^2\theta\ \hat{z}$$
$$= \sigma\omega\pi R^4 \sin^3\theta\, d\theta\ \hat{z}$$

$$\vec{m} = \int_0^{\pi} \sigma\omega\pi R^4 \sin^3\theta\, d\theta\ \hat{z}$$

$$\boxed{\vec{m} = \frac{4}{3}\sigma\omega\pi R^4 \hat{z}}$$ from Mathematica

---

$$\vec{A} = \frac{\mu_0}{4\pi}\,\frac{m\sin\theta}{r^2}\,\hat{\varphi}$$

$$= \frac{\mu_0 \sin\theta}{4\pi r^2}\,\frac{4}{3}\sigma\omega\pi R^4\,\hat{\varphi}$$

TOPS 35500

$$\vec{A} = \mu_0 \frac{\sin\theta \, \sigma \omega R^4}{3 r^2} \hat{\varphi}$$

This is the same as the exact result in Eq. 5.67 in Griffiths