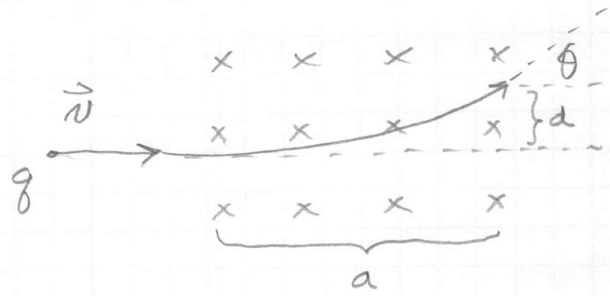
5-1)

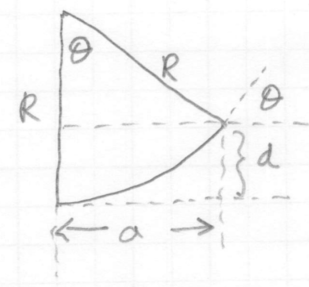


$\vec{p} = ?$

$q > 0$

$$\vec{F} = q\,\vec{v} \times \vec{B} \qquad \vec{v} \perp \vec{B}$$

$$|\vec{F}| = q\,v\,B = m\frac{v^2}{R}$$



$$\frac{q\cancel{p}B}{\cancel{m}} = \frac{p^{\cancel{2}}}{\cancel{m}R}$$

$$qBR = p \qquad R = ? \qquad \tan\theta = \frac{a}{R-d}$$

$$R^2 = a^2 + (R-d)^2$$

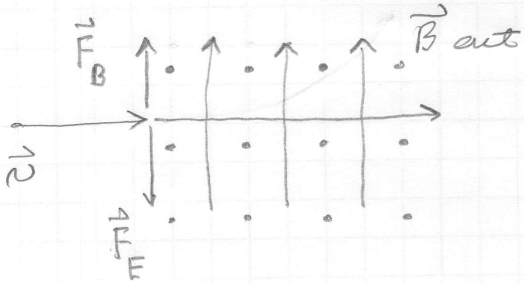
$$\cancel{R^2} = a^2 + \cancel{R^2} + d^2 - 2Rd$$

$$2Rd = a^2 + d^2$$

$$R = \frac{a^2+d^2}{2d}$$

$$\boxed{p = qB\left(\frac{a^2+d^2}{2d}\right)}$$

5-3)



a.

$$F_B = F_E$$

$$\cancel{q} v B = \cancel{q} E$$

$$\boxed{\vec{v} = \frac{E}{B}}$$

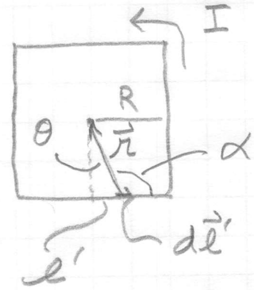
b.

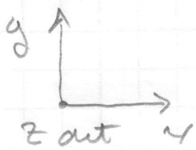
$$q v B = \frac{m v^2}{R}$$

$$q \frac{\cancel{E} B}{\cancel{B}} = \frac{m}{R} \frac{E^{\cancel{2}\,1}}{B^2}$$

$$\boxed{\frac{q}{m} = \frac{E}{R B^2}}$$

5-8)





for the bottom leg

$$d\vec{B} = \frac{\mu_0}{4\pi} \; I \frac{d\vec{l}' \times \hat{r}}{r^2}$$

$$r^2 = \frac{R}{4}^2 + l'^2$$

$$d\vec{l}' \times \hat{r} = dl' \sin\alpha \, \hat{z}$$

$$\pi - \left(\frac{\pi}{2} - \theta\right) = \alpha$$

$$\frac{\pi}{2} + \theta = \alpha$$

$$\therefore \sin\alpha = \cos\theta$$

$$d\vec{B}_b = \frac{\mu_0}{4\pi} \; \frac{I \, dl' \cos\theta \, \hat{z}}{R^2/4 + l'^2}$$

$$\cos\theta = \frac{R/2}{\sqrt{R^2/4 + l'^2}}$$

$$= \frac{\mu_0}{4\pi} \; \frac{I \, dl' \, R/2}{\left(R^2/4 + l'^2\right)^{3/2}} \; \hat{z}$$

$$\vec{B}_b = \int_{-R/2}^{R/2} \frac{\mu_0}{8\pi} \; \frac{I R}{\left(R^2/4 + l'^2\right)^{3/2}} \; dl' \; \hat{z}$$

$$= \frac{\mu_0 I R}{2\cancel{8}\pi} \; \frac{l'}{\frac{R^2}{\cancel{4}}\sqrt{R^2/4 + l'^2}} \Bigg|_{-R/2}^{R/2} \; \hat{z}$$

$$= \frac{\mu_0 I}{2\pi R} \left[\frac{R/2}{\sqrt{R^2/4 + R^2/4}} + \frac{(+R/2)}{\sqrt{R^2/4 + R^2/4}} \right] \hat{z}$$

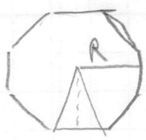
$$= \frac{\mu_0 I}{4\pi} \; \frac{\sqrt{2}}{R} \; \hat{z} = \frac{\mu_0 I}{2\sqrt{2}\,\pi R} \; \hat{z}$$

Each leg makes the same contribution to the field at the center.

$$\vec{B} = 4\,\vec{B}_b$$

$$\boxed{\vec{B} = \frac{2\sqrt{2}\,\mu_0 I}{\pi R}\,\hat{z}}$$

b.



for $n = 4 \quad -\frac{\pi}{4} < \theta < \frac{\pi}{4}$

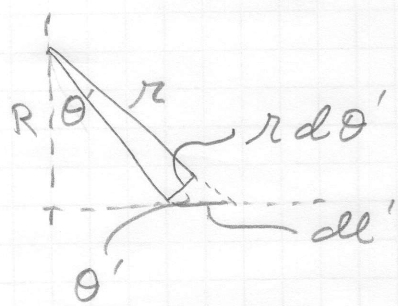
$n = 5 \quad -\frac{\pi}{5} < \theta < \frac{\pi}{5}$

$n = 6 \quad -\frac{\pi}{6} < \theta < \frac{\pi}{6}$

$\vdots$

$$d\vec{B}_b = \frac{\mu_0 I}{4\pi}\,\frac{dl'\cos\theta'}{r^2}\,\hat{z}$$

Rewrite dl' in terms of θ'



$$\cos\theta' = \frac{r\,d\theta'}{dl'}$$

$$dl' = \frac{r\,d\theta'}{\cos\theta'}$$

$$\cos\theta' = \frac{R}{r}$$

$$r = \frac{R}{\cos\theta'}$$

$$d\vec{B}_b = \frac{\mu_0 I}{4\pi}\,\frac{\cancel{R}}{\cancel{\cos\theta'}}\,\frac{d\theta'}{\cos\theta'}\,\cos\theta'\,\frac{\cancel{\cos^2\theta'}}{R^{\cancel{2}}}$$

$$d\vec{B}_b = \frac{\mu_0 I}{4\pi} \frac{1}{R} \cos\theta' \, d\theta'$$

$$\vec{B}_b = \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{\mu_0 I}{4\pi R} \cos\theta' \, d\theta' \, \hat{z} \quad \text{for part a.}$$

$$= \frac{\mu_0 I}{4\pi R} \sin\theta \Big|_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \hat{z}$$

$$= \frac{\mu_0 I}{4\pi R} \left(\frac{1}{\sqrt{2}} + \left(+ \frac{1}{\sqrt{2}} \right) \right) \hat{z}$$

$$= \frac{\mu_0 I}{2\sqrt{2}\,\pi R} \hat{z} \quad \text{as before.}$$

Now for an n-gon.

$$\vec{B}_b = \frac{\mu_0 I}{4\pi R} \sin\theta \Big|_{-\frac{\pi}{n}}^{+\frac{\pi}{n}}$$

$$= \frac{\mu_0 I}{4\pi R} \left(\sin\frac{\pi}{n} + \sin\frac{\pi}{n} \right)$$

$$= \frac{\mu_0 I}{2\pi R} \sin\frac{\pi}{n}$$

$$\therefore \quad \vec{B} = n\vec{B}_b = \frac{n \mu_0 I}{2\pi R} \sin\frac{\pi}{n}$$

c. Now let $n \to \infty$

$$\lim_{n\to\infty} n \cdot \sin\frac{\pi}{n} = \infty \cdot 0 = ?$$

use L'Hopital's rule

$$\lim_{n\to\infty} \frac{f(x)}{g(x)} = \lim_{n\to\infty} \frac{f'(x)}{g'(x)}$$

$$\frac{\sin\frac{\pi}{n}}{1/n} = \frac{0}{0} = ?$$

$$\frac{d}{dn} \sin \pi n^{-1} = (+\cos \pi n^{-1})(-1)\pi n^{-2}$$
$$= -\left(\cos\frac{\pi}{n}\right)\frac{\pi}{n^2}$$

$$\frac{d}{dn} n^{-1} = (-1)n^{-2} = -\frac{1}{n^2}$$

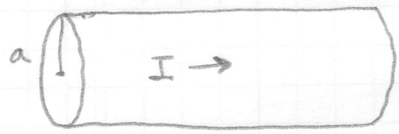
$$\frac{+\left(\cos\frac{\pi}{n}\right)\frac{\pi}{\cancel{n^2}}}{+\frac{1}{\cancel{n^2}}} = \pi \underbrace{\cos\frac{\pi}{n}}_{\to 1} \to \pi$$

so $\lim_{n\to\infty} n \sin\frac{\pi}{n} = \pi$

$$\vec{B} = \frac{\mu_0 I}{2\pi R}\pi\hat{z} = \frac{\mu_0 I}{2R}\hat{z}$$

agrees with result for a current loop.

5-13) a.

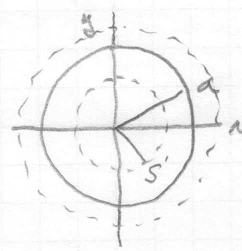


$\vec{B} = ?$

$\vec{J} = \vec{K} \left(= \dfrac{I}{2\pi a}\, \hat{z} \right)$

$$\nabla \times \vec{B} = \mu_0 \vec{J}$$

$$\int \nabla \times \vec{B} \cdot d\vec{A} = \mu_0 \int \vec{J} \cdot d\vec{A}$$



for $s < a$

$$\oint \vec{B} \cdot d\vec{l} = 0 \qquad \vec{B} = B(s)\,\hat{\varphi} \text{ by symmetry}$$

$$B \oint dl = 0 \qquad d\vec{l} = dl\,\hat{\varphi}$$

$$\boxed{\vec{B} = 0}$$

for $s > a$

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 \int \vec{K} \cdot d\vec{A} \qquad d\vec{A} = dA\,\hat{z}$$

$$B\, 2\pi s = \mu_0 K (2\pi a)$$

$$\vec{B} = \frac{\mu_0 K a}{s}\, \hat{\varphi}$$

$$K = \frac{I}{2\pi a}$$

$$\vec{B} = \mu_0 \frac{I}{2\pi a} \frac{a}{s}\, \hat{\varphi} = \frac{\mu_0 I}{2\pi s}\, \hat{\varphi}$$

b. $\vec{J} = \alpha s \hat{z}$ $\quad\quad \nabla \times \vec{B} = \mu_0 \vec{J}$

$$\int \nabla \times \vec{B} \cdot d\vec{A} = \mu_0 \int \vec{J} \cdot d\vec{A}$$

$$d\vec{A} = dA\,\hat{z}$$

$$= s'd\varphi' ds' \hat{z}$$

for $s > a$

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 \int_0^{2\pi}\int_0^{a} \alpha s'^2 ds' d\varphi'$$

$$\vec{B} = B(s)\,\hat{\varphi}$$ by symmetry

$$B\,2\pi s = \mu_0 \alpha \frac{s'^3}{3} 2\pi \Big|_0^a$$

$$B\,\cancel{2\pi} s = \frac{\cancel{2\pi}\mu_0 \alpha a^3}{3}$$

$$\vec{B} = \frac{\mu_0 \alpha a^3}{3s}\,\hat{\varphi}$$

for $s < a$

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 \int_0^{2\pi}\int_0^{s} \alpha s'^2 ds' d\varphi'$$

$$B\,2\pi s = \mu_0 \alpha\, 2\pi \frac{s'^3}{3}\Big|_0^s$$

$$B\,\cancel{2\pi}\cancel{s} = \mu_0 \alpha \cancel{2\pi} \frac{s^{\cancel{3}\,2}}{3}$$

$$\vec{B} = \frac{\mu_0 \alpha s^2}{3}\,\hat{\varphi}$$

Now get α

$$\int \vec{J} \cdot d\vec{A} = I$$

$$\int_0^{2\pi}\int_0^{a} \alpha s\; s\, d\varphi\, ds = I$$

$$2\pi\alpha \int_0^a s^2 ds = I$$

$$2\pi \frac{\alpha a^3}{3} = I$$

$$\alpha = \frac{3I}{2\pi a^3}$$

for $s < a$ $\vec{B} = \frac{\mu_0 s^2}{\cancel{3}} \frac{\cancel{3}I}{2\pi a^3} \hat{\phi}$

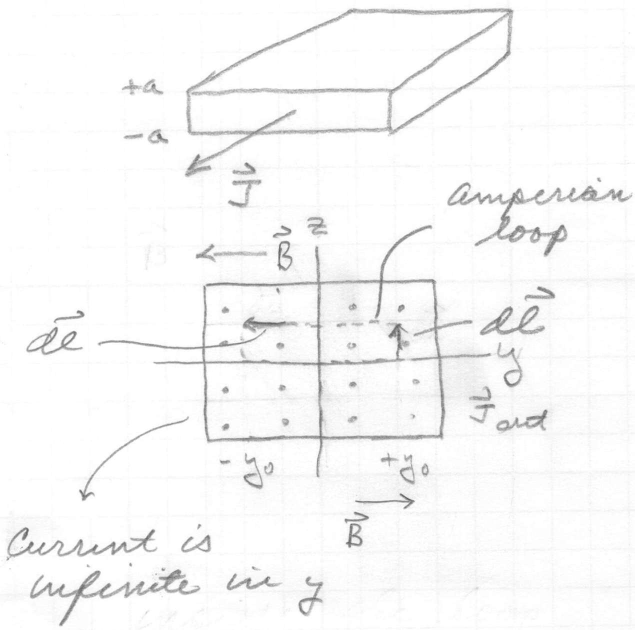
$$= \frac{\mu_0 I s^2}{2\pi a^3} \hat{\phi}$$

for $s > a$ $\vec{B} = \frac{\mu_0 \cancel{a^3}}{\cancel{3}s} \frac{\cancel{3}I}{2\pi \cancel{a^3}} \hat{\phi}$

$$= \frac{\mu_0 I}{2\pi s} \hat{\phi}$$

5-14)

$\vec{J} = J\hat{x}$



$\nabla \times \vec{B} = \mu_0 \vec{J}$

$\oint \vec{B}\cdot d\vec{l} = \mu_0 \int \vec{J}\cdot d\vec{A}$

$\vec{B} = -B(z)\hat{y}$

by Biot-Savart

$d\vec{A} = dy\,dz\,\hat{x}$

$\vec{B}\cdot d\vec{l} = 0$ on the sides

<u>inside</u>

$\oint \vec{B}\cdot d\vec{l} = \mu_0 J(2y_0)(z)$

$B(\cancel{2y_0}) = \mu_0 J(\cancel{2y_0})z$

$B = \mu_0 J z$

$$\boxed{\begin{array}{ll} \vec{B} = -\mu_0 J z\,\hat{y} & 0 < z < a \\ \vec{B} = +\mu_0 J z\,\hat{y} & 0 > z > -a \end{array}}$$

outside

$\oint \vec{B}\cdot d\vec{l} = \mu_0 J(2y_0)a$

$B(\cancel{2y_0}) = \mu_0 J(\cancel{2y_0})a$

$B = \mu_0 J a$

$$\boxed{\begin{array}{ll} \vec{B} = -\mu_0 J a\,\hat{y} & z > a \\ \vec{B} = +\mu_0 J a\,\hat{y} & z < -a \end{array}}$$